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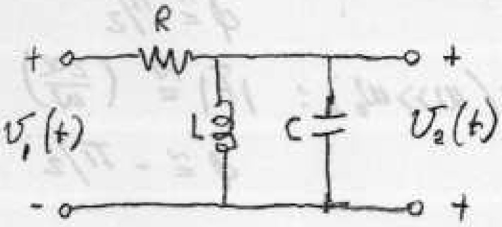
NOTES FOR 6.002 LECTURE #16, APRIL 8, 2008

SINUSOIDAL STEADY STATE IN RLC CIRCUITS: READ: CH 14.



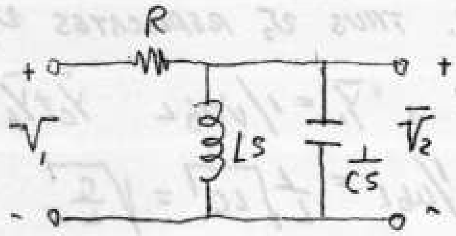
DRIVE THIS NETWORK WITH A SINUSOID

$$v_1(t) = V_1 \cos \omega t$$

$v_2(t)$ IS A SINUSOID IN THE STEADY STATE $v_2(t) = V_2(\omega)\cos[\omega t + \phi(\omega)]$

DETERMINE $V_2(\omega)$ AND $\phi(\omega)$.

FOR ANALYSIS, DRIVE THE CIRCUIT WITH $Ve^{j\omega t}$, EMPLOY ELEMENT IMPEDANCES AND FOCUS ON COMPLEX AMPLITUDES.



FIRST THINK ABOUT WHAT MUST HAPPEN FOR VERY LOW AND VERY HIGH FREQUENCIES.

ANALYSIS: THINK VOLTAGE DIVIDER: $\bar A(s) = \dfrac{\bar V_2}{\bar V_1} = \dfrac{Ls \,\|\, \frac{1}{Cs}}{Ls \,\|\, \frac{1}{Cs} + R}$

$$\bar A(s) = \frac{\dfrac{Ls\left(\frac{1}{Cs}\right)}{Ls + 1/Cs}}{\dfrac{Ls\left(\frac{1}{Cs}\right)}{Ls + 1/Cs} + R} = \frac{Ls\left(\frac{1}{Cs}\right)}{Ls\left(\frac{1}{Cs}\right) + RLs + \frac{R}{Cs}} = \frac{Ls}{RLCs^2 + Ls + R}$$

OR:

$$\bar A(s) = \frac{\frac{1}{RC}s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

DEFINE: $2\alpha = 1/RC$, $\omega_0^2 = 1/LC$ } THESE DEFINED PARAMETERS HAVE USEFUL INTERPRETATIONS LATER

$$\bar A(s) = \frac{2\alpha s}{s^2 + 2\alpha s + \omega_0^2} \quad \text{OR FOR } s = j\omega$$

$$\bar A(j\omega) = \frac{2\alpha(j\omega)}{-\omega^2 + 2\alpha(j\omega) + \omega_0^2} = \underbrace{\frac{2\alpha\omega}{\sqrt{(\omega_0^2-\omega^2)^2 + (2\alpha\omega)^2}}}_{\frac{|\bar V_2(\omega)|}{V_1}} e^{j\overbrace{\left(\frac{\pi}{2} - \theta\right)}^{\phi(\omega)}}$$

WHERE $\theta = \tan^{-1}\left(\dfrac{2\alpha\omega}{\omega_0^2 - \omega^2}\right)$

FOR VERY LOW FREQUENCIES ($\omega << \omega_0$): $|\bar{A}| \cong \left(\frac{2\alpha}{\omega_0^2}\right)\omega$ ↑ Q, $\phi \cong \pi/2$

FOR VERY HIGH FREQUENCIES ($\omega >> \omega_0$): $|\bar{A}| \cong \left(\frac{2\alpha}{\omega}\right)$, $\phi \cong -\pi/2$

INTUITI[ON] CONFIRMED

INTERESTING BEHAVIOR NEAR ω_0:

FOR $\omega = \omega_0$ $\quad |\bar{A}| = 1 \quad \phi = 0 \quad \underline{v_2(t) = v_1(t)}$

ω_0 IS THE RESONANT FREQUENCY. AT THIS FREQUENCY, THE PARALLEL LC COMBINATION HAS ZERO ADMITTANCE AND THERE IS NO VOLTAGE DROP IN R. THUS v_2 REPLICATES v_1.

AT RESONANCE $\bar{Y}_C = j\omega_0 C$, $\bar{Y}_L = 1/j\omega_0 L$ $\quad \bar{Y}_C + \bar{Y}_L = j(\omega_0 C - 1/\omega_0 L)$

$$\omega_0 C = C\sqrt{\frac{1}{LC}} = \sqrt{\frac{C}{L}} \qquad 1/\omega_0 L = \frac{1}{L}\sqrt{LC} = \sqrt{\frac{C}{L}} \qquad \underline{\bar{Y}_C + \bar{Y}_L \equiv 0}$$

THERE IS CURRENT IN C AND L NONETHELESS. EVALUATE THE CURRENTS DOWN IN L, C:

$$\left.\begin{aligned} \bar{I}_C &= \bar{V}_2(j\omega_0 C) = j\sqrt{\frac{C}{L}}\bar{V}_2 \\ \bar{I}_L &= \bar{V}_2\left(1/j\omega_0 L\right) = -j\sqrt{\frac{C}{L}}\bar{V}_2 \end{aligned}\right\}$$ THIS IS A CIRCULATING CURRENT IN L & C

NOTE THAT $\bar{V}_2$ AND $\bar{I}_L$ ARE ORTHOGONAL ($\frac{\pi}{2}$ OUT OF PHASE)

THUS WHEN $v_2(t)$ IS A MAXIMUM, $i_L(t)$ IS ZERO AND VICE-VERSA

AT RESONANCE STORED ENERGY IS FLOWING BACK AND FORTH BETWEEN L AND C.

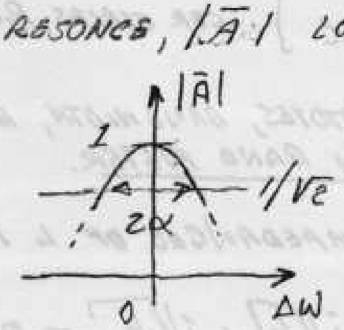
BEHAVIOR NEAR ω_0 LET $\underline{\omega = \omega_0 + \Delta\omega}$ $\quad \Delta\omega << \omega_0$

$$|\bar{A}(j\omega)| = \frac{2\alpha(\omega_0 + \Delta\omega)}{\sqrt{[(\omega_0 + \omega_0 + \Delta\omega)(\omega_0 - \omega_0 - \Delta\omega)]^2 + [2\alpha(\omega_0 + \Delta\omega)]^2}}$$

$$\cong \frac{2\alpha\omega_0}{\sqrt{(2\omega_0)^2(-\Delta\omega)^2 + (2\alpha\omega_0)^2}} = \frac{2\alpha}{\sqrt{(-2\Delta\omega)^2 + (2\alpha)^2}}$$

WHEN $\Delta\omega = 0$ $|\bar{A}| = 1$ WHEN $\underline{\Delta\omega = \pm\alpha}.$ $|\bar{A}| = 1/\sqrt{2}$

NEAR RESONCE, $|\bar{A}|$ LOOKS LIKE



$\omega_0 \pm \alpha$ ARE REFERRED TO AS <u>HALF-POWER FREQUENCIES</u> BECAUSE $|\bar{V}_2|^2 = \frac{1}{2}|\bar{V}_2|^2_{MAX}$

2α IS THE <u>BANDWIDTH</u> OF THE RESONANT PEAK.

<u>CONSTRUCT THE BODE PLOT:</u>

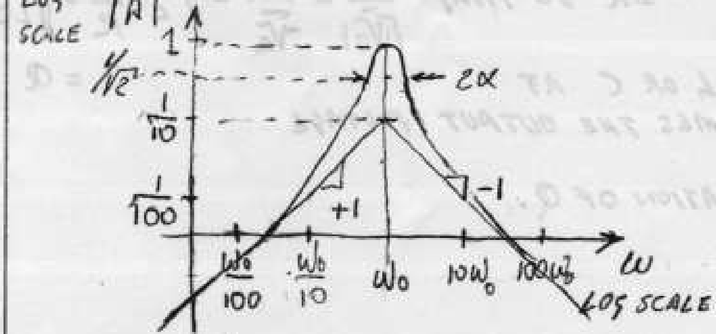
$Q = \dfrac{\omega_0}{2\alpha}$ ANOTHER INTERPRETATION OF Q

WE HAVE SYMPTOTES ON p2. TOP

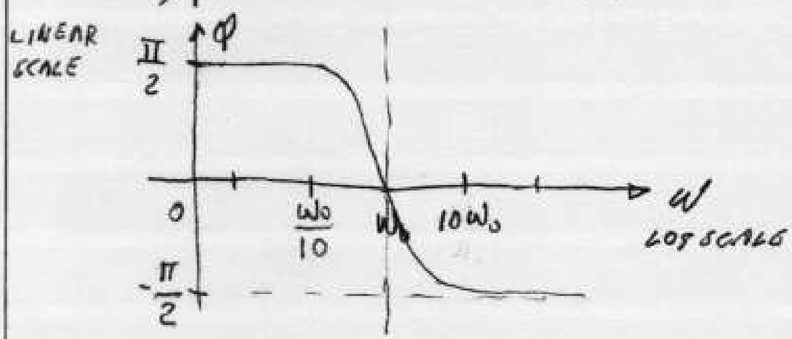
FOR L.F. $|\bar{A}| = \frac{2\alpha}{\omega_0^2}\omega$ (WHICH EQUALS $\frac{2\alpha}{\omega_0} = \frac{1}{Q}$ AT $\omega = \omega_0$), $\phi = \pi/2$

FOR H.F. $|\bar{A}| = \frac{2\alpha}{\omega}$ (WHICH EQUALS $\frac{2\alpha}{\omega_0} = 1/Q$ AT $\omega = \omega_0$) $\phi = -\pi/2$

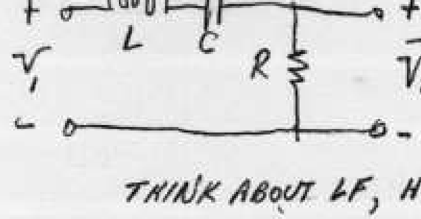
FOR SPECIFICITY <u>ASSUME</u> $Q = 10$



A NARROW-BAND FILTER



CONSIDER A <u>VARIATION</u>:



AGAIN A VOLTAGE DIVIDER:

$$\bar{A}(s) = \frac{R}{R + Ls + \frac{1}{Cs}} = \frac{RCs}{RCs + LCs^2 + 1}$$

THINK ABOUT LF, HF ASYMPTOTES

$$\bar{A}(s) = \frac{2\alpha s}{s^2 + 2\alpha s + \omega_0^2} \qquad \left.\begin{matrix} 2\alpha = \frac{R}{L} \\ \omega_0^2 = 1/LC \end{matrix}\right\}$$ NOTE <u>DIFFERENT FORM OF α</u> FOR SERIES RLC.

INTERPRETATION INCLUDING ASYMPTOTES, BANDWIDTH, BODE PLOT IDENTICAL TO PARALLEL RLC. A <u>NARROW BAND FILTER</u>.

AT RESONANCE ($\omega = \omega_0$) THE IMPEDANCES OF L AND C <u>ADD TO ZERO</u>

$$Z_C + Z_L = \frac{1}{j\omega_0 C} + j\omega_0 L = -j\sqrt{\frac{L}{C}} + j\sqrt{\frac{L}{C}} \equiv 0 \quad \text{AND } \bar{V}_2 = \bar{V}$$

SO THAT $v_2(t) = v_1(t)$

THERE ARE EQUAL AND OPPOSITE VOLTAGES ACROSS L AND C

$$\left.\begin{matrix} \bar{V}_C = \frac{1}{j\omega_0 C}(\bar{I}) = -j\sqrt{\frac{L}{C}}\,\bar{I} \\ \bar{V}_L = j\omega_0 L(\bar{I}) = +j\sqrt{\frac{L}{C}}\,\bar{I} \end{matrix}\right\} \quad \bar{V}_L + \bar{V}_C \equiv 0 \text{ AT } \omega_0$$

NOTE THAT $\bar{V}_R = \bar{V}_2 = \bar{I}R$ SO THAT $\frac{|\bar{V}_C|}{|\bar{V}_R|} = \frac{|\bar{V}_L|}{\bar{V}_R} = \sqrt{\frac{L}{C}}\frac{1}{R} = \sqrt{\frac{1}{LC}}\left(\frac{L}{R}\right) = Q$

THE VOLTAGE ACROSS L OR C AT RESONANCE IS Q TIMES THE OUTPUT VOLTAGE

ANOTHER INTERPRETATION OF Q.