6.002 – Circuits and Electronics
Spring 2003

Handout S03-045 - Homework #9

Issued: Wed. Apr 9
Due: Fri. Apr 18

Problem 9.1:
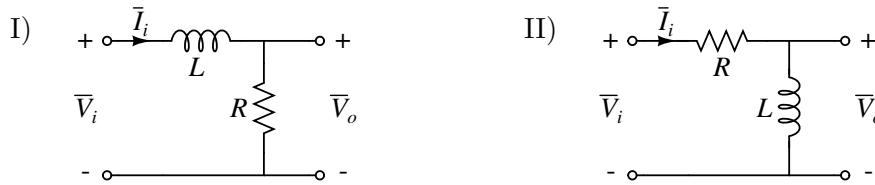
$\overline{V}_i$, $\overline{V}_o$, and $\overline{I}$ are complex amplitudes

(A) For each of these circuits determine the voltage transfer ratio $\overline{A} = \frac{\overline{V}_o}{\overline{V}_i}$ and the input impedance $Z_i = \frac{\overline{V}_i}{\overline{I}_i}$.

Express your results as ratios of polynomials in $s = j\omega$.

(B) Sketch and dimension Bode plots for each of the voltage transfer ratios. Use logarithmic coordinates for $|\overline{A}|$ and ω and a linear coordinate for $\angle \overline{A}$.

Hint: For help quick tutorial on using Matlab, please see the end of the problem set.

Problem 9.2:
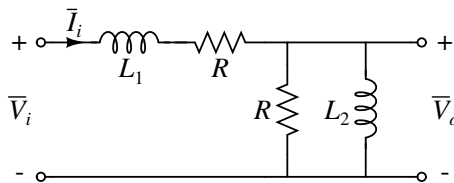
$$R = 1\text{k}\Omega = 10^3 \Omega$$
$$L_1 = 1 \text{ mH} = 10^{-3} \text{ H}$$
$$L_2 = 0.1 \text{ H}$$

(A) Derive an expression (in terms of R, L_1, L_2 and $s = j\omega$) for the voltage transfer ratio $\overline{A}(s) = \frac{\overline{V}_o}{\overline{V}_i}$.

Express your result as a ratio of polynomials in s and show that it can be expressed as

$$\overline{A}(s) = \frac{\frac{R}{L_1}s}{s^2 + \left(\frac{2R}{L_1} + \frac{R}{L_2}\right)s + \frac{R}{L_1} \cdot \frac{R}{L_2}}$$

(B) Use the given numerical values for R, L_1, and L_2 and express $\overline{A}(s)$ in the following form by factoring the denominator quadratic, and determining the values of s_1 and s_2.

$$\overline{A}(s) = \frac{10^6 s}{(s+s_1)(s+s_2)} \qquad \begin{aligned} s_1 &= \quad ? \\ s_2 &= \quad ? \end{aligned}$$

(C) Sketch and dimension the Bode plot for this filter. Use the coordinates specified in Problem 9.1.

(D) There is a range of frequencies $s = j\omega$ for which $\overline{A}(s)$ is approximately independent of s. Determine this range. Express the limits in both rad/sec and Hz.

(E) Determine the magnitudes of the impedances of L_1 and L_2 for a frequency in the middle of the range found in Part C).

Compare the values of ωL_1 and ωL_2 to the value of R and explain why $\overline{A}(s)$ is approximately independent of ω in this range.

(F) This circuit is driven by $v_I(t) = 10\cos(\omega t)$ where $\omega = 10^5$ rad/sec (about 16 kHz). Determine the magnitude and phase of $v_O(t)$. That is, determine V and ϕ where $v_O(t) = V\cos(\omega t + \phi)$.

Problem 9.3: The circuit below includes an ideal diode.
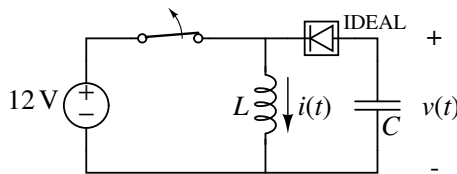
$L = 0.1$ H, $C = 1\ \mu\text{F} = 10^{-6}$ F

The switch closes at $t = 0$
before which $i(t) = v(t) = 0$.

At $t = T = 1$ second the
switch opens

(A) What is the state of the diode when the switch is closed? That is, is it conducting or non-conducting?

(B) Derive an expression for $i(t)$ during the interval $0 < t < T$ in which the switch is closed.

(C) What is the numerical value of $i(t)$ at $t = T$?

(D) How much energy is stored in L at $t = T$?

(E) What is the state of the diode immediately after the switch opens?

(F) Derive an expression for $v(t)$ valid for $t > T$ until the diode changes state again.

(G) At what time does the diode change state?

(H) At this time, what is the value of $v(t)$?

Matlab and Linear System Simulation/Modeling

There are two ways presented below to generate Bode plots using Matlab. Note: while the second way is much easier, you will not have Matlab on the final. You may want to at least try the first way to ensure that you get the same results as using `bode()`, to be sure you understand what's going on.

Start matlab. Run `add matlab; matlab &` at an Athena prompt. Matlab processes everything in vectors. First, make a vector over which to evaluate the frequency, omega (note: Matlab does angles in radians, not degrees). Let's say we want to examine the frequency range 1 Hz to 1 MHz. We'd do something like `omega = 2*pi*logspace(0,6,10000)`. This generates 10000 points on a log scale between $2\pi 10^0$ and $2\pi 10^6$. Now, define the complex vector `s = j*omega`.

Given a transfer function, say

$$H = \frac{\frac{s}{\tau_1}}{\left(\frac{s}{\tau_2} + 1\right)\left(\frac{s}{\tau_3} + 1\right)}$$

we can evaluate it (as a complex number!) like this:

```
% note:  tau1, tau2, and tau3 have already been defined
H = (s/tau1)./(s/tau2+1)./(s/tau3+1)
```

Note that divisision is written as `./`. The extra period means that you want to divide the first element of the vector `s/tau1` by the first element of the vector `s/tau2`. We don't need it for the time constants because they are scalars.

`H` is now a vector containing the complex value of the transfer function evaluated at all the frequencies in `omega`. To find the magnitude, do `mag = abs(H)`. To find the angle, do `ang = angle(H)`.

Now, plot your data. You want to plot the magnitude and frequency on log scales, and the angle on a linear scale. You can do this by:

```
figure(1)
loglog(omega,mag)
figure(2)
semilogx(omega,ang)
```

And there are your plots! Remember: the frequency axis is in radians. If you want to convert it back to degrees, instead you can type `loglog(omega/2/pi,mag)`, etc.

Using the systems toolbox

Matlab can evaluate transfer functions directly. Assume we have the τ time constants defined still. Make $s = j\omega$ by typing `s = tf('s')`. Now, make your transfer function by typing `H = s/tau1/(s/tau2+1)/(s/tau3+1)`. Now, make your bode plot by typing `bode(H)`. You're done!