

Massachusetts Institute of Technology  
Department of Electrical Engineering and Computer Science

6.002 – Circuits and Electronics  
Spring 2003

Handout S03-047 - Lab #3: Second-Order Networks

**Introduction**

The purpose of this lab is to give you experience with second-order networks, and to illustrate that real network elements do not always behave in an ideal manner. All exercises in this lab focus on the behavior of the network and network elements shown in Figure 1. You should complete the pre-lab exercises in your lab notebook before coming to lab. Then, carry out the in-lab exercises on your assigned lab day between April 14 and April 18. After completing the in-lab exercises, have a TA or LA check your work and sign your lab notebook. Finally, complete the post-lab exercises in your lab notebook, and turn it in to your TA on or before April 28.

**Pre-Lab Exercises**

- (3-1) Assume that the network is initially at rest. At  $t = 0$ , the input voltage  $v_{IN}(t)$  steps from 0 V to  $V_{TI}$ . Given this input, determine the transient response of  $v_{OUT}(t)$ . Note that  $v_{OUT}(t)$  takes the form  $v_{OUT}(t) = V_{TO} \sin(\omega_T t + \phi_T) e^{-\alpha_T t}$ . Hint: see Homework Problem 8.3, Spring 2002 at <http://web.mit.edu/6.002/www/spring02/hw8sol.pdf>.
- (3-2) Let  $L = 47$  mH,  $C = 0.0047$   $\mu$ F,  $R = 220$   $\Omega$  and  $V_{TI} = 10$  V. Under these conditions, graph the transient response of  $v_{OUT}(t)$  for  $0 \leq t \leq 0.3$  ms; graphing the peaks and zero crossings of the response and a few points in between each peak and zero crossing should be sufficient. On separate graphs, repeat this exercise for  $R = 560$   $\Omega$  and  $R = 1000$   $\Omega$ . To save time, you may wish to use MatLab on Athena as discussed below.
- (3-3) For all three values of  $R$ , compute the voltage  $V_{TP}$  at the first peak of the transient response, the frequency  $\omega_T$  at which the transient response oscillates, and the rate  $\alpha_T$  at which the transient response decays. Note that peaks of the transient response occur at times such that  $\tan(\omega_T t + \phi_T) = \omega_T / \alpha_T$ ; you should verify this.

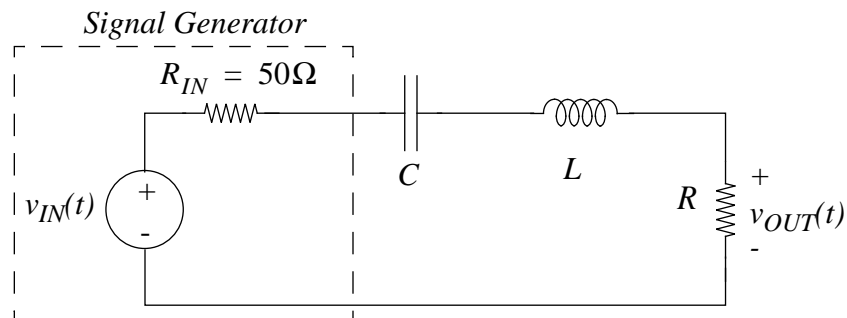


Figure 1: second-order network.

- (3-4) Assume that the network is in sinusoidal steady state. Determine the response of  $v_{\text{OUT}}(t)$  to the input  $v_{\text{IN}}(t) = V_{\text{SI}} \cos(\omega_S t)$ . Note that  $v_{\text{OUT}}(t)$  will take the form  $v_{\text{OUT}}(t) = V_{\text{SO}}(\omega_S) \cos(\omega_S t + \phi_S(\omega_S))$ . Hint: see Homework Problem 9.1, Spring 2002 at <http://web.mit.edu/6.002/www/spring02/hw9sol.pdf>.
- (3-5) Let  $L = 47 \text{ mH}$ ,  $C = 0.0047 \text{ } \mu\text{F}$  and  $R = 220 \text{ } \Omega$ . On separate graphs, graph  $\log |H_S(\omega_S)|$  and  $\phi_S(\omega_S)$  versus  $\log(\omega_S/(2\pi \times 10 \text{ kHz}))$  for  $2\pi \times 1 \text{ kHz} \leq \omega_S \leq 2\pi \times 100 \text{ kHz}$  where  $H_S(\omega_S) \equiv V_{\text{SO}}(\omega_S)/V_{\text{SI}}$ . Ten to fifteen points per graph should be sufficient to clearly outline  $H_S$  if you space the points more closely near the peak of  $H_S$ . Again on separate graphs, repeat this exercise for  $R = 560 \text{ } \Omega$  and  $R = 1000 \text{ } \Omega$ . You may find it easiest to use log-log graph paper for the graph of  $H_S$  and linear-log graph paper for the graph of  $\phi_S$ . Alternatively, to save time, you may wish to use MatLab on Athena as discussed below.
- (3-6) For all three values of  $R$  compute the peak value  $H_{\text{SP}}$  of  $H_S$ , the frequency  $\omega_{\text{SP}}$  at which the peak occurs, and  $Q$ . Note that  $Q$  is defined as  $Q \equiv \omega_{\text{SP}}/2\alpha_T$ , and that  $H_S(\omega_S)$  will have fallen from its peak value of  $H_{\text{SP}}$  by a factor of  $\sqrt{2}$  at  $\omega_S \approx \omega_{\text{SP}} \pm \alpha_T$ .

You are strongly encouraged, although not required, to use MatLab to plot the graphs outlined above. To use MatLab, you must first type “add matlab” at the Athena prompt, and then invoke MatLab by typing the command “matlab” at the Athena prompt. The MatLab commands, “step” and “bode” can then be used to plot the desired graphs. You can learn how to use these commands by typing “help step” and “help bode” at the MatLab prompt. The “step” command accepts an optional time-vector argument to specify the time range over which the step response is to be plotted. To define an appropriate time vector  $T$ , type “ $T = \text{linspace}(0, 3e-4, 100);$ ” at the MatLab prompt. The vector  $T$  will then contain 100 evenly spaced points between 0 ms and 0.3 ms. The “bode” command accepts an optional frequency-vector argument to specify the frequency range over which the frequency response is to be plotted. To define an appropriate frequency vector  $W$ , type “ $W = 2*\text{pi}*\text{logspace}(3, 5, 100);$ ” at the MatLab prompt. The vector  $W$  will then contain 100 logarithmically spaced points between  $10^3$  and  $10^5$  Hz. Additionally, note that the “bode” command in MatLab uses the frequency variable  $s$ , where  $s = j\omega$ . Finally, figures plotted by MatLab may be printed on an Athena printer using the MatLab “print” command. To learn more, type “help print” at the MatLab prompt.

### In-lab Exercises

The in-lab exercises involve measuring both the step response and sinusoidal response of the network shown in Figure 1 for three values of  $R$ . You should feel free to measure these responses for only one value of  $R$ , and then share your measurements with two other partners who have measured the network responses for the other two values of  $R$ . However, should you take this team approach, all team members must use the same inductor and capacitor. You should also indicate in your lab notebook which responses you measured, and which responses you have taken from another team member. Finally, you are advised to look at all responses on the oscilloscope before you leave the lab so that you see for yourself how they vary as  $R$  varies.

Real network elements do not always behave the way we model them in 6.002. For example, a real inductor might be better modeled as an ideal inductor in series with a resistor as shown in Figure 2. The resistor is a parasitic element, meaning that it is undesired, but unavoidable. The resistor accounts for the resistance of the wire used to wind the inductor. Yet more complex models could account for core losses and the capacitance between winding turns. For this reason, the model

shown in Figure 2 is not the only possible model. In a similar way, a real capacitor might be better modeled as an ideal capacitor in parallel with a parasitic conductance which models leakage through the dielectric of the capacitor. This is also shown in Figure 2.

In the exercises which follow, the network in Figure 1 will be exposed to inputs that vary at high enough frequencies that you can ignore the parasitic parallel conductance of the capacitor. Therefore, we need only be concerned with the parasitic series resistance of the inductor.

- (3-1) Take a 47 mH inductor, a 0.0047  $\mu\text{F}$  capacitor, a 220  $\Omega$  resistor, a 560  $\Omega$  resistor and a 1000  $\Omega$  resistor from your lab kit to the instrument desk and use the GenRad impedance meter to measure these elements and determine the parasitic resistance and conductance of the inductor and capacitor, respectively. To measure the inductor, set the meter for 1 kHz, the series model, and the appropriate element type and value range. The meter will directly read the inductor value. It will also read  $Q$  from which you can determine  $R_P$  from  $Q = \omega L/R_P$ , where  $\omega = 2\pi \times 1 \text{ kHz}$ . To measure the capacitor, set the meter for 1 kHz, the parallel model, and the appropriate element type and value range. The meter will now directly read the capacitor value. It will also read  $D$  from which you can determine  $G_P$  from  $D = G_P/\omega C$ , where  $\omega = 2\pi \times 1 \text{ kHz}$ .
- (3-2) Construct the second-order network shown in Figure 1 using the measured inductor, capacitor and 220  $\Omega$  resistor.
- (3-3) Set the signal generator to produce a 10 V peak-to-peak square wave at 50 Hz with a 5 V offset so that its open-circuit output voltage steps between 0 V and 10 V. With the oscilloscope, measure the transient response of the resistor voltage  $v_{\text{OUT}}(t)$  to the positive going step, and compare the response to that plotted during the pre-lab exercises. Measuring data at times which correspond to the points graphed during Pre-Lab Exercise 3-2 is sufficient. Also measure the voltage  $V_{\text{TP}}$ , the oscillation frequency  $\omega_T$ , the rate of decay  $\alpha_T$ . To measure  $\alpha_T$  it is easiest to measure the time  $\tau_T$  over which the transient response decays by  $1/e$  and then compute  $\alpha_T = 1/\tau_T$ .
- (3-4) Set the signal generator to produce an open-circuit 10 V peak-to-peak sinusoidal voltage with zero offset; the open-circuit voltage is  $v_{\text{IN}}(t)$ . Connect the SYNC output of the generator to one oscilloscope channel, and trigger the oscilloscope off that channel. Use the other oscilloscope channel to measure the resistor voltage  $v_{\text{OUT}}(t)$  over the frequency range of 1 kHz to 100 kHz. In particular, measure the peak-to-peak amplitude of  $v_{\text{OUT}}$ , and the difference in time between neighboring zero crossings of the SYNC output and  $v_{\text{OUT}}$ ; note that the SYNC output has the same zero crossings as  $v_{\text{IN}}$ . From this data you will determine  $H_S$  and  $\phi_S$  during the post-lab exercises. Measuring data at frequencies which correspond to the points graphed during Pre-Lab Exercise 3-5 is sufficient. Also measure

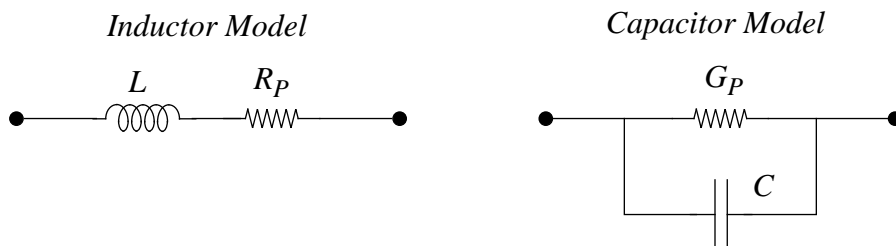


Figure 2: improved models for a real inductor and a real capacitor.

the peak voltage ratio  $H_{SP}$  of  $H_S$ , the frequency  $\omega_{SP}$  at which the peak occurs, and  $Q$ .  $Q$  is most easily measured by first measuring the difference between the two frequencies at which the ratio  $H_S$  falls from its peak value of  $H_{SP}$  by a factor of  $\sqrt{2}$ ; this frequency difference should be  $2\alpha_T$ ;  $Q$  is then given by  $Q = \omega_{SP}/2\alpha_T$ .

It is important to note that  $\phi_S(\omega_S)$  is the phase shift of  $v_{OUT}$  relative to  $v_{IN}$ . The most convenient manner in which to measure  $\phi_S$  is to measure the time delay from zero crossings of  $v_{IN}$  to zero crossings of  $v_{OUT}$ . This time delay may then be converted to a phase shift. Unfortunately,  $v_{IN}$  is internal to the signal generator, and hence not measurable. However, the SYNC output of the generator and  $v_{IN}$  have the same zero crossings, which is why the SYNC output is used in this exercise as a reference for measuring phase shift.

- (3-5) Repeat In-Lab Exercises 3-2 through 3-4 with the measured 560  $\Omega$  resistor and the 1000  $\Omega$  resistor.

### Post-lab Exercises

Complete these exercises in your lab notebook using the responses which you calculated during the pre-lab exercises, and the data which you measured during the in-lab exercises. The primary goal of the post-lab exercises is to explain any discrepancies which may exist between the responses you calculated during the pre-lab exercises and the responses you measured during the in-lab exercises.

- (3-1) Plot the recorded transient response data on the corresponding graph prepared during the pre-lab exercises. How well does the measured data match the theoretical graph? That is, how do they differ?
- (3-2) Consider again the transient response. In chart form for the three cases of  $R$ , compare your calculated and measured voltage  $V_{TP}$ , frequency  $\omega_T$  and decay rate  $\alpha_T$ . How do the measured and calculated parameters compare? Next, utilize the calibrated values of the network elements, including the parasitic resistance of the inductor, to re-calculate the parameters. Include the new parameters in the chart. Do the new parameters compare more favorably with the measured parameters? Do the new parameters explain the differences observed in Post-Lab Exercise 3-1?
- (3-3) First, convert the amplitudes of  $v_{OUT}$  measured in sinusoidal steady state to measurements of  $H_S(\omega_S)$  by dividing the measured amplitudes by the 10 V amplitude of  $v_{IN}$ . Second, convert the zero-crossing time differences measured in sinusoidal steady state to measurements of  $\phi_S(\omega_S)$  by dividing the time differences by the corresponding waveform period, and then multiplying by 360 degrees. Finally, plot the frequency response data on the corresponding graph prepared during the pre-lab exercises. How well does the measured data match the theoretical graph? That is, how do they differ?
- (3-4) Consider again the frequency response. In chart form for the three cases of  $R$ , compare your calculated and measured peak voltage ratio  $H_{SP}$ , frequency  $\omega_{SP}$  and  $Q$ . How do the measured and calculated parameters compare? Next, utilize the calibrated values of the network elements, including the parasitic resistance of the inductor, to re-calculate the parameters. Include the new parameters in the chart. Do the new parameters compare more favorably with the measured parameters? Do the new parameters explain the differences observed in Post-Lab Exercise 3-3?