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NOTES FOR 6.002 LECTURE # 17, APRIL 10, 2003

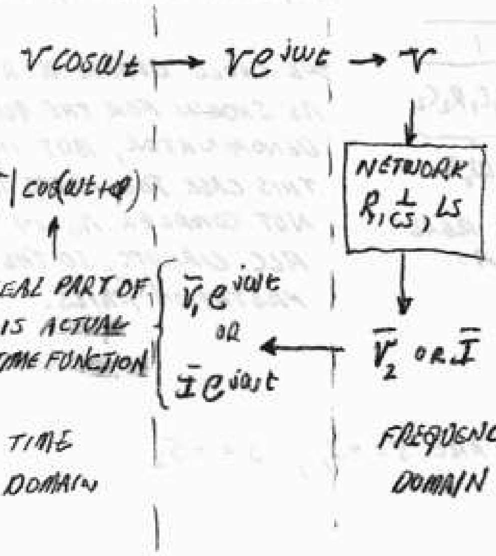
READ: CH 14

LAB # 3 NEXT WEEK

LAB # 4 THREE WEEKS LATER (W/O MAY 5)! TERM ENDS ON MAY 15

MORE SINUSOIDAL STEADY STATE:

ARGUMENT ONCE AGAIN:



1) ACTUAL EXCITATION IS SINUSOIDAL E.G. $V\cos\omega t$

2) EXCITE INSTEAD WITH $Ve^{j\omega t}$, THE REAL PART OF WHICH IS $V\cos\omega t$.

3) DROP $e^{j\omega t}$, WHICH APPEARS EVERYWHERE (IN ALL CURRENTS AND VOLTAGES. FOCUS ON COMPLEX AMPLITUDES

4) USE COMPLEX IMPEDANCES OR ADMITTANCES TO REPRESENT R, L, C.

5) SOLVE FOR THE DESIRED COMPLEX AMPLITUDE USING THE ALGEBRA OF COMPLEX NUMBERS

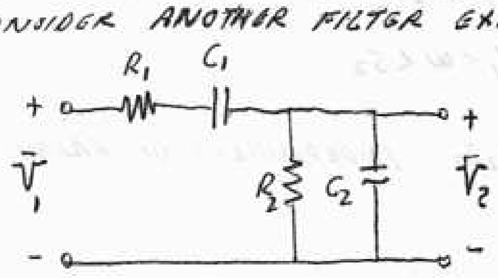
6) MULTIPLY THE DESIRED COMPLEX AMPLITUDE (IN POLAR FORM) BY $e^{j\omega t}$ AND TAKE THE REAL PART, WHICH IS THE DESIRED ACTUAL TIME FUNCTION.

WHY IS FREQUENCY RESPONSE IMPORTANT

1) ANY PERIODIC FUNCTION OF TIME CAN BE REPRESENTED AS A SUM OF SINUSOIDS, HARMONICALLY RELATED: FOURIER SERIES.

2) APERIODIC FUNCTIONS OF TIME CAN BE REPRESENTED AS AN INTEGRAL OF A FREQUENCY SPECTRUM: FOURIER INTEGRAL

CONSIDER ANOTHER FILTER EXAMPLE:



FOR APPROPRIATE ELEMENT VALUES THERE WILL BE A REGION OF ω IN WHICH:

$\bar{A} = \frac{\bar{V}_2}{\bar{V}_1}$ IS APPROX. CONSTANT.

AGAIN A VOLTAGE DIVIDER: $\bar{A} = \dfrac{\bar{V}_2}{\bar{V}_1} = \dfrac{R_2 \| 1/C_2 s}{R_2 \| 1/C_2 s + R_1 + \frac{1}{C_1 s}}$. $R_2 \| \dfrac{1}{C_2 s} = \dfrac{R_2/C_2 s}{R_2 + 1/C_2 s}$

$$\bar{A} = \frac{R_2/C_2 s}{R_2/C_2 s + (R_2 + 1/C_2 s)(R_1 + 1/C_1 s)} = \frac{R_2}{R_2 + (R_2 C_2 s + 1)(R_1 + 1/C_1 s)}$$

$$\bar{A} = \frac{R_2 C_1 s}{R_2 C_1 s + (R_2 C_2 s + 1)(R_1 C_1 s + 1)} = \frac{R_2 C_1 s}{R_1 R_2 C_1 C_2 s^2 + (R_2 C_1 + R_2 C_2 + R_1 C_1) s + 1}$$

FINALLY:

$$\bar{A}(s) = \frac{s/R_1 C_2}{s^2 + \underbrace{\left(\frac{1}{R_1 C_2} + \frac{1}{R_1 C_1} + \frac{1}{R_2 C_2}\right)}_{2\alpha} s + \underbrace{\frac{1}{R_1 C_1 R_2 C_2}}_{\omega_0^2}}$$

NOTE DIMENSIONAL CHECK

WE COULD DEFINE α AND ω_0^2 AS SHOWN FOR THE QUADRATIC DENOMINATOR, BUT IN THIS CASE THE ROOTS ARE REAL, NOT COMPLEX AS IN TUESDAY'S RLC CIRCUITS, SO THE INTERPRETATION FAILS.

THE DENOMINATOR HAS TWO NEGATIVE REAL ROOTS. THUS $A(s)$ CAN BE WRITTEN AS

$$\bar{A}(s) = \frac{s/R_1 C_2}{(s + s_1)(s + s_2)}$$

WHERE $s_1 \ll s_2$ ROOTS ARE $s = -s_1$, $s = -s_2$

CONSIDER THE ASYMPTOTES:

LOW FREQUENCIES: $s = j\omega \ll s_1$ AND s_2

$$\bar{A}(\omega) \cong (j\omega) \frac{1/R_1 C_2}{s_1 s_2}$$

THE LINEAR DEPENDENCE ON ω TRANSLATES ON LOG-LOG COORDINATES TO A STRAIGHT LINE OF SLOPE +1 AND INTERCEPT AT $\omega = s_1$ OF $1/R_1 C_2 s_2$. THE PHASE IS j OR $+\pi/2$

HIGH FREQUENCIES: $s = j\omega \gg s_1$ AND s_2

$$\bar{A}(\omega) = \frac{1}{R_1 C_2 (j\omega)}$$

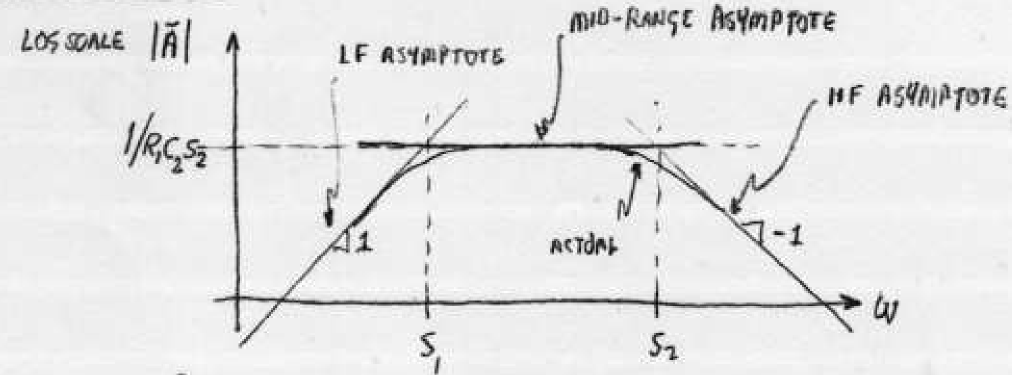
THE RECIPROCAL DEPENDENCE ON ω TRANSLATES ON LOG-LOG COORDINATES TO A STRAIGHT LINE OF SLOPE -1 AND INTERCEPT AT $\omega = s_2$ OF $1/R_1 C_2 s_2$. THE PHASE IS $-j$ OR $-\pi/2$

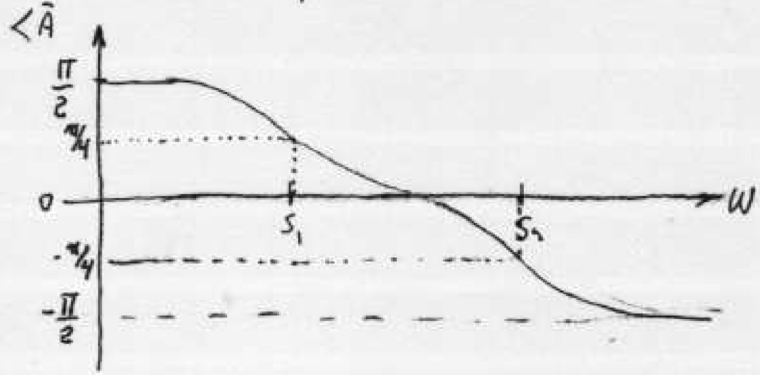
MIDDLE FREQUENCIES: $s_1 < \omega < s_2$

$$\bar{A}(\omega) = \frac{j\omega/R_1 C_2}{j\omega s_2} = 1/R_1 C_2 s_2$$

INDEPENDENT OF FREQUENCY.

BODE DIAGRAMS





LOG SCALES

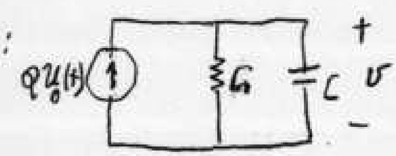
FOR THE DEMO: $R_1 = 10K$, $C_1 = 0.1\mu F$, $R_2 = 2K$, $C_2 = 0.01\mu F$

SOLUTION OF THE QUADRATIC YIELDS: $s_1 = 8.2\times10^2$ (130 Hz)

$s_2 = 6.1\times10^4$ (9.7 KHz)

REPRISE ON IMPULSES   DIFFICULTIES WITH DIMENSIONS.

CONSIDER:



DE: $Qu_0(t) = vG + C\underbrace{\frac{dv}{dt}}$

AMPERE-SEC ← ? → AMPERES

$u_0(t)$ HAS DIMENSION OF $\frac{1}{SEC}$

WHY?

IMPULSE IS DEFINED AS: $u_0(t) = \begin{cases} 0 & t<0 \\ 0 & t>0 \end{cases}$ AND $\int_{-\infty}^{+\infty} u_0(t)\,dt = 1$

THIS IS DIMENSIONLESS

$u_0(t)$ MUST HAVE DIMENSION OF $\frac{1}{TIME}$ SO THAT $u_0(t)\,dt$ CAN BE DIMENSIONLESS.