Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

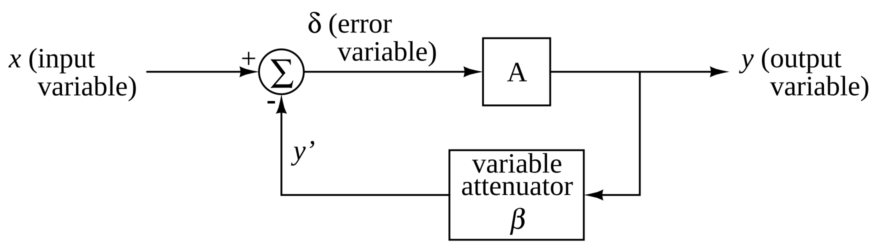
6.002 – Circuits and Electronics
Spring 2003

Handout S03-052 - Homework #10

Issued: Wed. Apr 16
Due: Fri. May 2

Please note that HW#10 is a two week assignment focusing on operational amplifiers. It will receive double weight in the assignment of homework grades.

Problem 10.1: Most circuits which employ operational amplifiers rely on negative (error-correcting) feedback. This problem explores several of the consequences of such feedback in an abstract form. Understanding a simple abstract example illustrating some of the consequences of feedback may make the operational amplifier easier to understand.



The element A is an amplifying element which has very high gain. However, the gain varies greatly from unit to unit and is strongly temperature dependent.

The element B is a variable attenuator. Its output y' equals βy where $\beta < 1$.

Σ is a summing element. Its output δ is the difference between the input x and the fed-back variable y', that is, $\delta = x - y'$.

The summing element compares the input with a fraction β of the output. Thus, in the ideal situation, the output would be an amplified replica of the input and the "error variable" would be vanishingly small.

(A) Assume A is a constant - a large positive number. Show that:

$$T = \frac{y}{x} = \frac{\mathrm{A}}{1 + \mathrm{A}\beta}$$ This is the transfer ratio of the system

Note that if $\mathrm{A}\beta >> 1$

$$T = \frac{\mathrm{A}}{1 + \mathrm{A}\beta} \approx \frac{1}{\beta}$$

The effect of negative feedback is to yield a closed loop amplification which is determined by the variable attenuator alone and is only weakly dependent on the gain of the unreliable amplifier A.

(B) Assume that β is precisely known, but that A may vary over a range of 10:1 because of manufacturing variations, temperature, etc.

$$10^5 < \mathrm{A} < 10^6 \qquad \beta = 10^{-2}$$

Evaluate T at the extremes of A and determine ΔT, the overall variation in T. What is the corresponding fractional change $\frac{\Delta T}{T}$?

A better way of determining the sensitivity of T to changes in A is to:

Evaluate $\frac{dT}{d\mathrm{A}}$ and form $\frac{dT}{T} = f(\mathrm{A})\frac{d\mathrm{A}}{\mathrm{A}}$

Show that $f(\mathrm{A}) = \frac{1}{1+\mathrm{A}\beta}$

What is $\frac{dT}{T}$ for the midrange of the numbers above?

The effect of negative feedback is to reduce the consequences of variations in A by a factor of $\frac{1}{1+\mathrm{A}\beta} \approx \frac{1}{\mathrm{A}\beta}$. A$\beta$ is the gain around the closed loop or the closed loop amplification.

(C) Assume that A is a noisy amplifier. That is

$$y = \mathrm{A}\delta + N(t)$$

where A is the nominal gain and $N(t)$ is an undesired noise signal.

Determine the noise signal seen at the system output and show that the noise at the output is approximately $\frac{N(t)}{\mathrm{A}\beta}$ where Aβ is the loop gain.

Hint: Think superposition.

For the numbers in Part B), what is the peak-to-peak value of the noise at the output if $N(t) = 5$V peak-to-peak?

(D) Assume that the amplifying element has frequency limitations. That is to say, represent it as a low-pass filter:

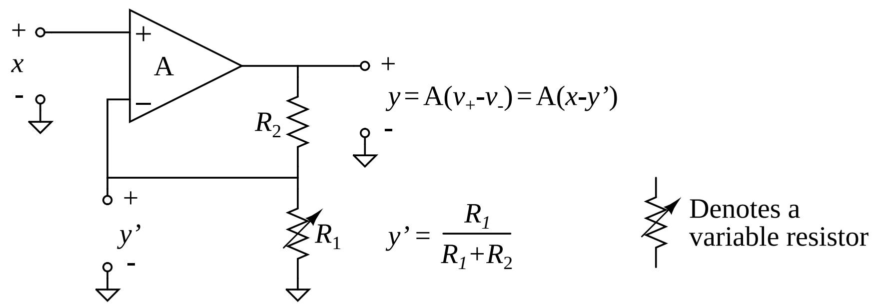
$$\overline{\mathrm{A}}(s) = \frac{\mathrm{A}_0 s_1}{s + s_1}$$

where A_0 and s_1 (the bandwidth) are constants.

For low frequencies ($s = j\omega << s_1$) $\overline{\mathrm{A}}(j\omega) \approx \mathrm{A}_0$, but for frequencies above s_1, $\overline{\mathrm{A}}(j\omega)$ decreases as $\frac{1}{\omega}$. s_1 is the bandwidth of $\overline{\mathrm{A}}(j\omega)$.

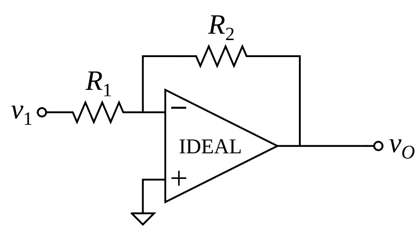
Show that the bandwidth of the closed loop transfer function $T(s)$ is approximately $\mathrm{A}_0\beta s_1$.

In the operational amplifier, the variables are voltages, the summing function and the gain function are intrinsic and the variable attenuator is provided in the external circuit.

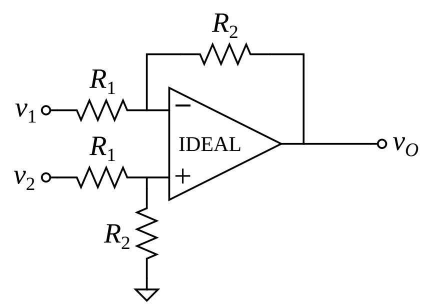


Problem 10.2: Determine the output voltages v_O of the following circuits. Assume that the operational amplifiers are ideal. That is, the gain A $\rightarrow \infty$. All voltages are defined with respect to ground.

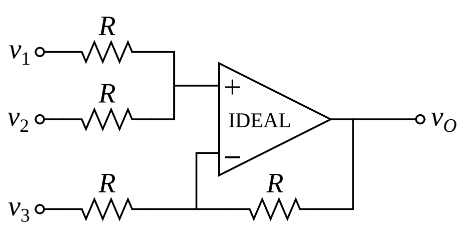
(A)



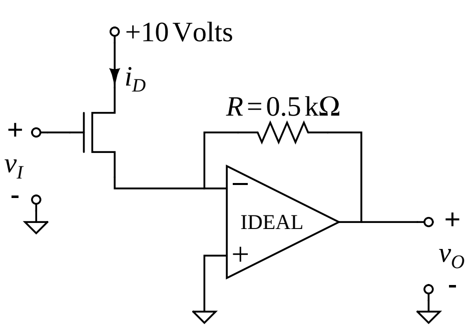
(B)



(C)



Problem 10.3:



The FET is described by the square-law model:

$$i_D = \begin{cases} \frac{K}{2}(v_{GS} - V_T)^2 & v_{GS} > V_T \\ 0 & v_{GS} < V_T \end{cases}$$

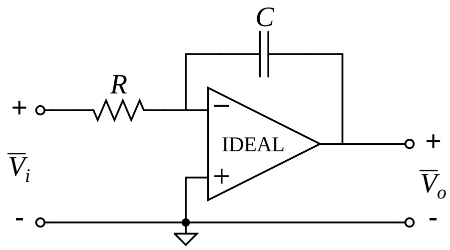
$$K = 1\frac{\mathrm{mA}}{\mathrm{V}^2} \quad V_T = 1\mathrm{V}$$

How does v_O depend on v_I? Derive $v_O = v_O(v_I)$.

Problem 10.4: In each of the circuits below, the input is a sinusoid at frequency ω sec^{-1}. The voltage variables are complex amplitudes. Assume that the op-amps are ideal and that they operate in the linear region. For each circuit, derive the voltage transfer ratio

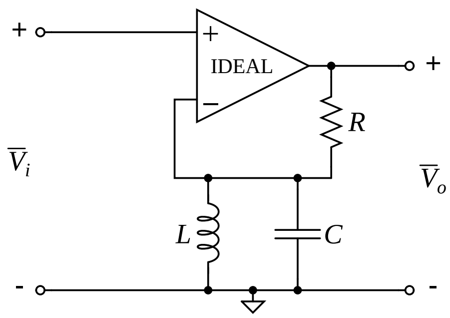
$$\overline{\mathrm{A}}(s) = \frac{\overline{V}_O}{\overline{V}_I}$$

(A)



For what range of ω can this circuit be regarded as an integrator?

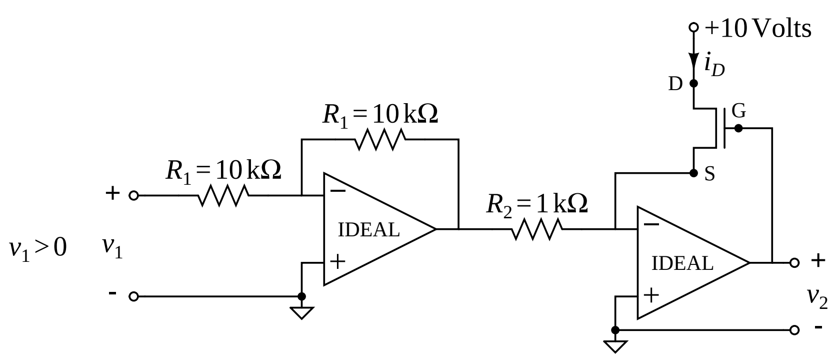
(B)



Sketch the Bode plot for $\frac{\overline{V}_O}{\overline{V}_I}$. Assume R is large enough so the the damping factor α is small.

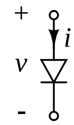
Problem 10.5: In the circuit below, the op-amps are ideal and the FET operates in the saturation region and fits the square law model:

$$i_D = \frac{K}{2}(v_{GS} - V_T)^2 \qquad \begin{cases} K &= 0.1 \text{ mA/V}^2 \\ V_T &= 0 \text{ V} \end{cases}$$



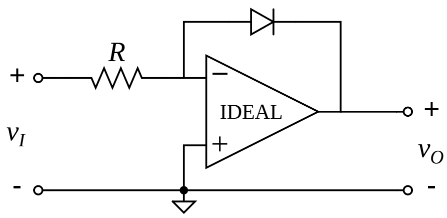
Find an expression for v_2 in terms of v_1.

Problem 10.6: The semiconductor diode in the op-amp circuits below has the usual i-v characteristic:



$i = I_S(e^{qv/(kT)}-1)$ where I_S is a constant

(A)



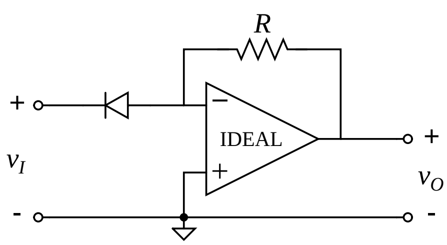
$v_i > 0$ and $v_i \gg kT/q$

so that $i \approx I_S e^{qv/(kT)}$

What is the sign of v_O?

Derive an expression for v_O as a function of v_I.

(B)



$v_i < 0$ and $|v_i| \gg kT/q$

so that $i \approx I_S e^{qv/(kT)}$

What is the sign of v_O?

Derive an expression for v_O as a function of v_I.

(C) Let x be a voltage variable which is positive and y be a voltage variable which is negative.

Design a circuit using no more than six op-amps which will produce at its output a voltage proportional to the product $(x)(|y|)$.