Massachusetts Institute of Technology
Department of **Electrical Engineering**

6.002 Problem Set #9 Solutions

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Problem 9.1:

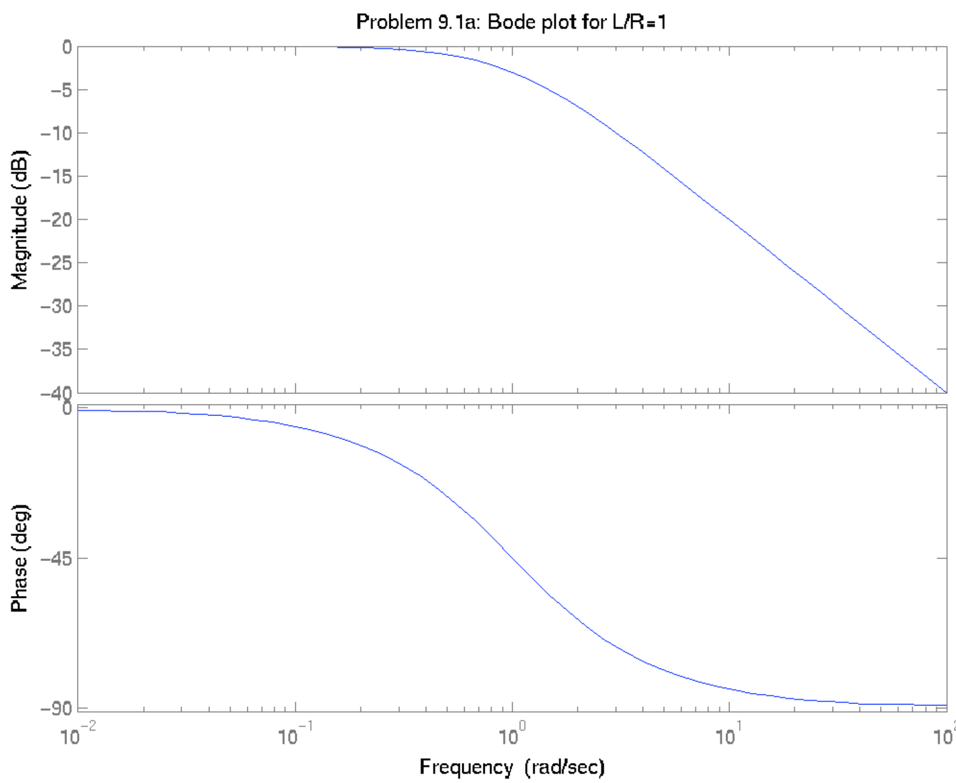
A) I:

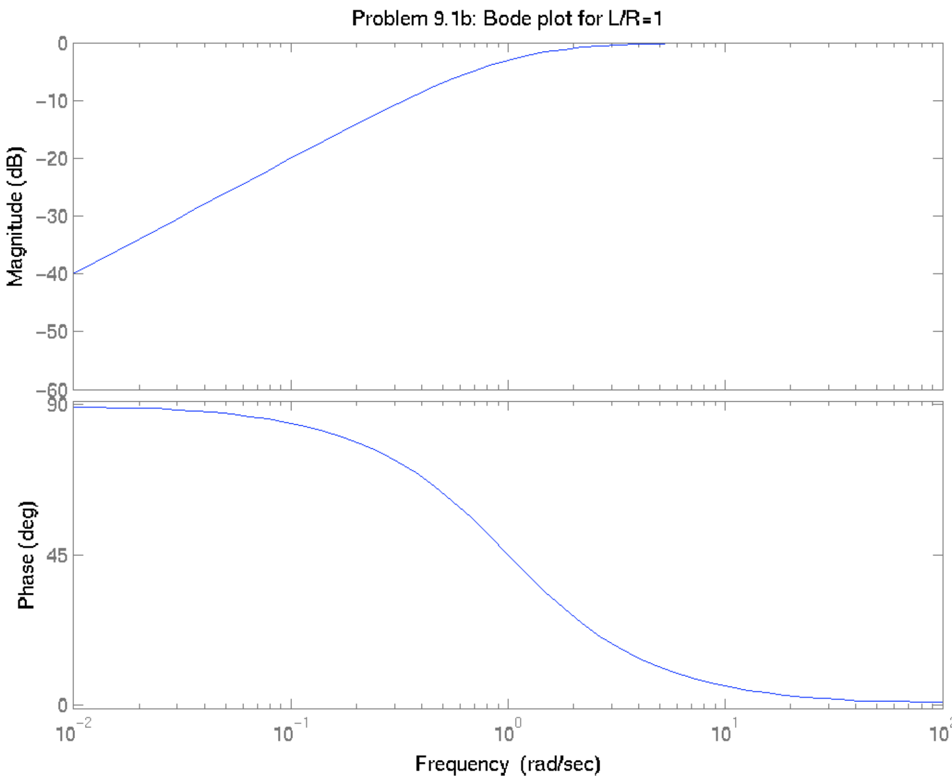
$$\overline{A}(s) = \frac{R}{R+Ls} = \frac{1}{1+\frac{L}{R}s};\ Z_i(s) = R + Ls;\ Z_o(s) = R$$

II:

$$\overline{A}(s) = \frac{Ls}{R+Ls} = \frac{\frac{L}{R}s}{1+\frac{L}{R}s};\ Z_i(s) = R + Ls;\ Z_o(s) = Ls$$

B)





Problem 9.2:

A)

$$\overline{A}(s) = \frac{R\|L_2 s}{R\|L_2 s + (R + L_1 s)} = \frac{\dfrac{RL_2 s}{R + L_2 s}}{\dfrac{RL_2 s}{R + L_2 s} + (R + L_1 s)}$$

$$= \frac{RL_2 s}{RL_2 s + (R + L_1 s)(R + L_2 s)} = \frac{RL_2 s}{RL_2 s + R^2 + Rs(L_1 + L_2) + L_1 L_2 s^2}$$

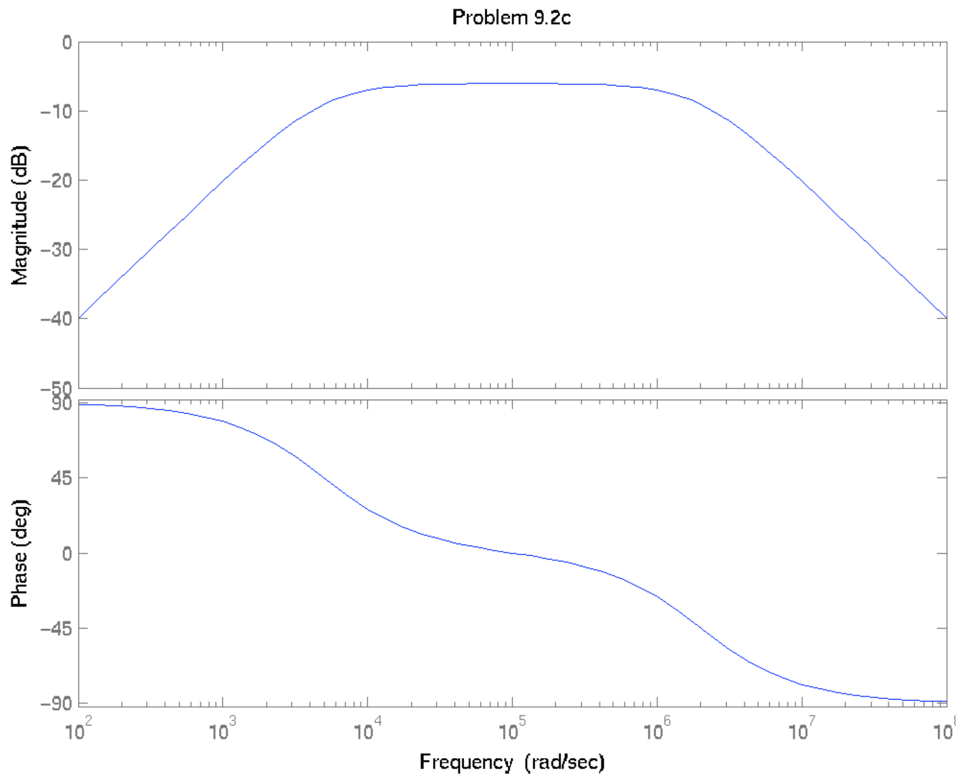
$$= \frac{\dfrac{R}{L_1} s}{s^2 + \left(\dfrac{2R}{L_1} + \dfrac{R}{L_2}\right)s + \dfrac{R^2}{L_1 L_2}}$$

B) Using R=1 kΩ, L_1=1 mH, L_2=0.1 H

$$\overline{A}(s) = \frac{10^6 s}{s^2 + (2x10^6 + 2x10^4)s + (10^6 * 10^4)} = \frac{10^6 s}{(s + 2.005x10^6)(s + 0.005x10^6)}$$

$s_1 = -2.005x10^6$, $s_s = -0.005x10^6$

C)



D) From the above graph, the magnitude is almost flat for roughly 10^4 rad/s < ω < 10^6 rad/s
or 1.6 kHz < f < 160 kHz

E) For $\omega = 10^5$ rad/s,

$$\left|Z_{L_1}\right| = \omega L_1 = 100\Omega << R$$

$$\left|Z_{L_2}\right| = \omega L_2 = 10k\Omega >> R$$

$$\overline{A}(j10^5) = \frac{10^6\left(j10^5\right)}{\left(j10^5 + 2.005x10^6\right)\left(j10^5 + 0.005x10^6\right)}$$

$$\approx \frac{10^6\left(j10^5\right)}{\left(2.005x10^6\right)\left(j10^5\right)} = 0.5$$

From the above we can see that A(jω) is a constant at this frequency
To check with the Bode plot, we convert 0.5 to dB via:
$dB = 20\log\left|A(j\omega)\right|$ to obtain –6 dB.

F) Reading the magnitude and phase from the Bode plot, we have

$$\left|A\left(j10^5\right)\right| \approx 0.5, \quad < A\left(j10^5\right) \approx 0^\circ$$

$$v_O(t) = 10 * 0.5\cos(\omega t \angle 0) = 5\cos\omega t$$

Problem 9.3:
A) The diode is **nonconducting** because the voltage source wants current to flow in the wrong direction for the diode.

B) $$v_L(t) = L\frac{di_L(t)}{dt}$$
$$i_L(t) = \int \frac{v_L(t)}{L}dt = \frac{12V}{0.1H}t$$
(the constant of integration is zero due to the initial condition)

C) $i_L(T = 1\sec) = \frac{12V}{0.1H}(1\sec) = 120A$

D) $E_L = \frac{1}{2}LI_L^2 = \frac{1}{2}(0.1H)(120A)^2 = 3600 Joules$

E) After the switch opens, the inductor will force current through the diode so that there isn't an instantaneous change in the inductor current.

F) After the switch opens, the inductor forces current through diode and we have an LC circuit. The solutions we know are sinusoidal. Therefore, we can write:
$v(t) = -V_C \sin\left(\frac{t-T}{\sqrt{LC}}\right)$. The minus sign comes from the direction of the current that charges the capacitor. To find V_C, we can use conservation of energy:
$\frac{1}{2}LI_L^2 = \frac{1}{2}CV_C^2 \Rightarrow V_C = I_L\sqrt{\frac{L}{C}}$.
$v(t) = -I_L\sqrt{\frac{L}{C}}\sin\left(\frac{t-T}{\sqrt{LC}}\right)$, t > T

G) The diode will stop conducting when the v(t) reaches a maximum (the inductor current goes to zero). This occurs when $\frac{t-T}{\sqrt{LC}} = \frac{\pi}{2}$. Plugging in numbers we find that the diode stops conducting at 1.0005s.

H) We know that the capacitor voltage will be at a peak, so we only need to compute the amplitude of the expression derived in part F).
$v(1.0005) = -I_L\sqrt{\frac{L}{C}} = -37.9V$.