OPERATIONAL AMPLIFIER STABILITY

INVERTING CONNECTION:

\[ \frac{v_o - v_i}{R_1} = \frac{v_i - v_o}{R_2} \quad \text{(Current Mirror)} \]

\[ v_o = \frac{R_2}{R_1 + R_2} v_i + \frac{R_1}{R_1 + R_2} v_o \quad \text{(Superposition)} \]

Either equation together with \( v_o = A(v_i - v_i) \) and \( v_i = 0 \) yields

\[ \frac{v_o}{v_i} = -\frac{R_2}{R_1} \quad \text{if} \quad R_1 > R_2 \]

Any disturbance is suppressed

What happens if the inputs are interchanged?

IDENTICAL CONFIGURATION EXCEPT \( v_i \) AND \( v_o \) ARE INTERCHANGED

SAME ANALYSIS YIELDS:

\[ \frac{v_o}{v_i} = \frac{A}{1 + \frac{R_1}{R_1 + R_2}} = -\frac{R_2}{R_1} \quad \text{if} \quad R_1 > R_2 \]

Any disturbance is amplified

THE EQUILIBRIUM OF THE SECOND CIRCUIT, WITH POSITIVE FEEDBACK, IS UNSTABLE AND CANNOT BE SUSTAINED IN THE REAL WORLD.

CONSIDER: INVERTED PENDULUM

ROLLER COASTER

ELECTRIC BLANKET
INTRODUCE INTERNAL DYNAMICS:

\[ A = \frac{S_a}{0.5 + 5a} \]

\( s = j\omega \)

AND ASSUME THE VOLTAGE VARIANCES HAS COMPLEX AMPLITUDES.

SUBSTITUTE \( A(s) \) FOR \( A \) IN EXPRESSIONS PREVIOUSLY DEVELOPED

NEGATIVE FEEDBACK:

\[ \frac{V_o}{V_c} = -A_0 \left( \frac{S_a}{R_2 + (R_1 + 2R_0)S_a + S_a} \right) = -A_0 \left( \frac{S_a}{R_2 + (R_1 + 2R_0)S_a + S_a} \right) \]

\[ \frac{V_o}{V_c} = A_0 \left( \frac{S_a}{S + S_a + S_a + S_a} \right) \]

NOTE THAT TIME CONSTANT OF RESPONSE OF THE AMPLIFIER IS NOW NOT \( \frac{1}{S_a} \) BUT \( \frac{S_a}{S + S_a + S_a + S_a} \)

WHICH IS SMALLER BY A FACTOR OF \( \frac{1}{A_0(R_1 + R_2)} \)

THE BANDWIDTH OF THE AMPLIFIER IS LARGER BY THE SAME FACTOR

POSITIVE FEEDBACK: ANALYSIS IS IDENTICAL

\[ \frac{V_o}{V_c} = A_0 \left( \frac{S_a}{S + S_a + S_a} \right) \]

ROOT IS NOW IN THE RIGHT HALF PLANE

THE CHARACTERISTIC EQUATION IS \( S = S_a + A_0 \frac{R_2}{R_1 + R_2} \)

OR \( S + S_a \frac{R_2}{R_1 + R_2} \)

THE TRANSIENT RESPONSE IS \( e^{S_a} \)

WHICH IS A GROWING EXPONENTIAL INSTABLE!
An op-amp can be used as a comparator by exploiting the limits (+V_s, the supply voltages) on the output voltage.

\[ V_+ = 0 \text{,} \quad V_0 = \begin{cases} V_s & \text{if } V_+ < V_{\text{ref}}, \quad V_0 = -V_s \\ V_s & \text{if } V_+ > V_{\text{ref}}, \quad V_0 = +V_s \end{cases} \]

Slew rate is very high (~20 V/\mu s).

Consider the effect of positive feedback around the comparator:

\[ V_+ = \frac{R_2}{R_1 + R_2} + \frac{R_1}{R_1 + R_2} V_0 \text{ (superposition)} \]

\[ \begin{cases} V_+ > V_{\text{ref}}, \quad V_0 = V_s \\ V_+ < V_{\text{ref}}, \quad V_0 = -V_s \end{cases} \]

Two stable states.

Assume circuit is in state \( V_0 = -V_s \). What is the constraint on \( V_+ \) to stay there?

\[ V_+ < V_{\text{ref}} \text{ or: } \left( V_s \frac{R_2}{R_1 + R_2} - V_s \frac{R_1}{R_1 + R_2} \right) < V_{\text{ref}} \text{, equivalently: } V_+ < V_{\text{ref}} \frac{R_2}{R_1 + R_2} + V_s \frac{R_1}{R_2} \]

Assume circuit is in the state \( V_0 = +V_s \). The corresponding condition to stay there is:

\[ V_+ > V_{\text{ref}} \text{ or: } V_+ > V_{\text{ref}} \frac{R_1 + R_2}{R_2} - V_s \frac{R_1}{R_2} \]
In summary:

**IN +Vs state:** \[ V_i > V_{ref} \left( \frac{R_1 + R_2}{R_2} \right) - \frac{V_s}{R_2} \]

**IN -Vs state:** \[ V_i < V_{ref} \left( \frac{R_1 + R_2}{R_2} \right) + \frac{V_s}{R_2} \]

*Graphically:*

![Graph](image)

**Hysteresis** by injection

This circuit, with bistability produced by positive feedback, can easily be made into a signal generator or oscillator.

Initially the switch is closed.

*Assume circuit is IN + state i.e. \( V_0 = +V_s \) (with switch closed circuit looks like Schmitt trigger with \( V_{ref} = 0 \)).

Let switch open at \( t = 0 \) \( V_C \) increases toward \( +V_s \).

When it reaches \( \frac{V_s}{2} \), \( V_C > V_+ \) and state changes to \( V_0 = -V_s \).

\( V_C \) now decreases toward \( -V_s \), with the state changing again when it reaches \( -\frac{V_s}{2} \) making \( V_C < V_+ \).

And the cycle continues.
To determine the period focus on interval marked \( T/2 \) and let \( t' = 0 \) at start of this interval.

By inspection:

\[
V_c(t') = -V_s + \frac{3}{2} V_s e^{-t'/T}
\]

This interval ends when

\[
V_c(t') = -V_s/2 \quad \text{or when}
\]

\[
e^{-t'/T} = \frac{1}{3}
\]

At this time

\[
t' = T/2
\]

\[
T = 2\pi \ln 3
\]

The clock in Lab #4 is different in detail, but relies on positive feedback around an amplifier, a fast inverter, which saturates at both ends of the transfer characteristic.

The analysis proceeds as in the circuit above.