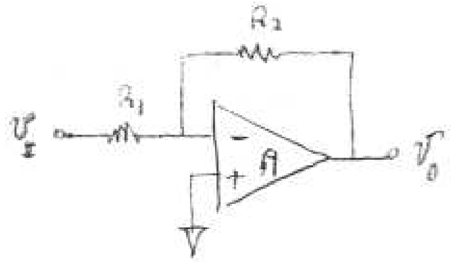
NOTES FOR 6.002 LECTURE # 21, TUESDAY, APRIL 29, 2003

OPERATIONAL AMPLIFIER STABILITY READ 16.6 - 16.8

INVERTING CONNECTION:



$$\frac{v_I - v_-}{R_1} = \frac{v_- - v_0}{R_2} \quad \text{(CURRENT MIRROR)}$$

OR

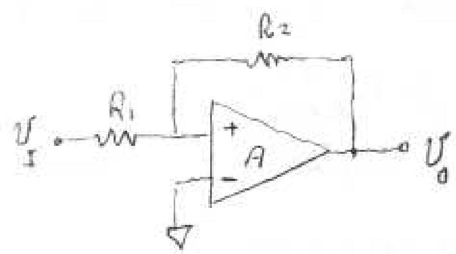
$$v_- = \frac{R_2}{R_1+R_2} v_I + \frac{R_1}{R_1+R_2} v_0 \quad \text{(SUPERPOSITION)}$$

EITHER EQUATION TOGETHER WITH $v_0 = A(v_+ - v_-)$ AND $v_+ = 0$ YIELDS

$$\frac{v_0}{v_I} = -\frac{A\frac{R_2}{R_1+R_2}}{1 + \underbrace{A\frac{R_1}{R_1+R_2}}_{\text{LOOP GAIN}}} \cong -R_2/R_1 \quad \text{IF } A\frac{R_1}{R_1+R_2} \gg 1$$

ANY DISTURBANCE IS SUPPRESED

WHAT HAPPENS IF THE INPUTS ARE INTERCHANGED?



IDENTICAL CONFIGURATION EXCEPT v_+ AND v_- ARE INTERCHANGED

SAME ANALYSIS YIELDS:

$$\frac{v_0}{v_I} = \frac{A\frac{R_2}{R_1+R_2}}{1 - A\frac{R_1}{R_1+R_2}} \cong -\frac{R_2}{R_1} \quad \text{IF } A\frac{R_1}{R_1+R_2} \gg 1$$

ANY DISTURBANCE IS AMPLIFIED

THE EQUILIBRIUM OF THE SECOND CIRCUIT, WITH POSITIVE FEEDBACK, IS UNSTABLE AND CANNOT BE SUSTAINED IN THE REAL WORLD.

CONSIDER: INVERTED PENDULUM

ROLLER COASTER

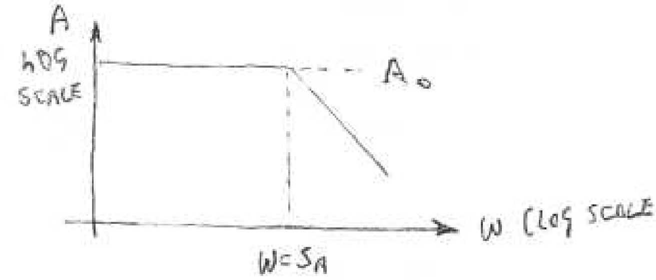
ELECTRIC BLANKET

INTRODUCE INTERNAL DYNAMICS:

LET $A = A_0 \frac{S_A}{S + S_A}$ $\quad S = j\omega$

AND ASSUME THE VOLTAGE VARIABLES ARE <u>COMPLEX</u> <u>AMPLITUDES</u>.

A LOW PASS FILTER



SUBSTITUTE $A(s)$ FOR A IN EXPRESSIONS PREVIOUSLY DEVELOPED

<u>NEGATIVE FEEDBACK</u>:

$$\frac{\bar{V}_o}{\bar{V}_i} = \frac{-A_0\left(\frac{R_2}{R_1+R_2}\right)\left(\frac{S_a}{S+S_a}\right)}{1 + A_0\left(\frac{R_1}{R_1+R_2}\right)\left(\frac{S_a}{S+S_a}\right)} = -A_0\left(\frac{R_2}{R_1+R_2}\right)\frac{S_a}{S + S_a + \underbrace{A_0\left(\frac{R_1}{R_1+R_2}\right)}_{\text{LOOP GAIN}} S_a}$$

$$\frac{\bar{V}_o}{\bar{V}_i} \cong -A_0 \frac{R_2}{R_1+R_2}\left(\frac{S_a}{S + S_a A_0 \frac{R_1}{R_1+R_2}}\right)$$

NOTE THAT TIME CONSTANT OF RESPONSE OF THE AMPLIFIER IS NOW NOT $\frac{1}{S_a}$ BUT $\frac{1}{S_a} \times \frac{1}{A_0 \frac{R_1}{R_1+R_2}}$

WHICH IS <u>SMALLER</u> BY A FACTOR OF $1/A_0\left(\frac{R_1}{R_1+R_2}\right)$

THE BANDWIDTH OF THE AMPLIFIER IS <u>LARGER</u> BY THE SAME FACTOR

<u>POSITIVE FEEDBACK</u>: ANALYSIS IS IDENTICAL

$$\frac{\bar{V}_o}{\bar{V}_i} \cong A_0 \frac{R_2}{R_1+R_2}\left(\frac{S_a}{S - S_a A_0 \frac{R_2}{R_1+R_2}}\right)$$

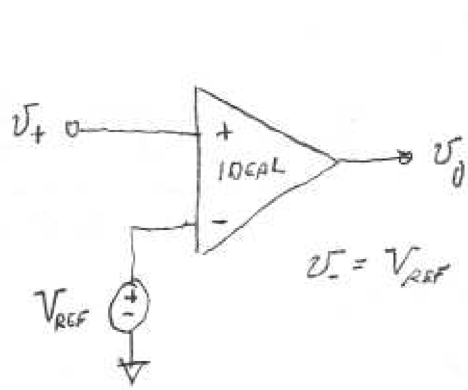
ROOT IS NOW IN THE RIGHT HALF PLANE

THE CHARACTERISTIC EQUATION IS $S - S_a A_0 \frac{R_2}{R_1+R_2}$

OR $S = + S_a A_0 \frac{R_2}{R_1+R_2}$ THE TRANSIENT RESPONSE IS e^{st}

WHICH IS A <u>GROWING</u> EXPONENTIAL <u>UNSTABLE</u>!

AN OP-AMP CAN BE USED AS A <u>COMPARATOR</u> BY EXPLOITING THE LIMITS ($\pm V_S$, THE SUPPLY VOLTAGES) ON THE OUTPUT VOLTAGE.

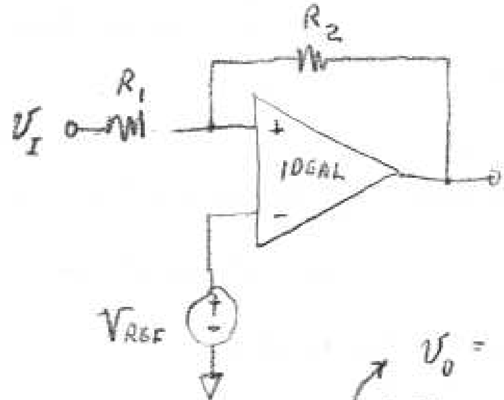


IF $v_+ < V_{REF}$, $v_0 = -V_S$

IF $v_+ > V_{REF}$, $v_0 = +V_S$

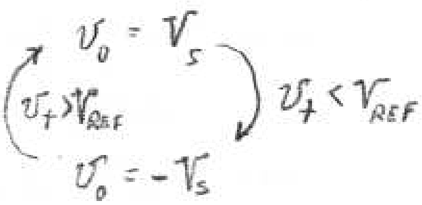
SLEW RATE IS VERY HIGH ($\sim 20\ V/\mu sec$)

CONSIDER THE EFFECT OF <u>POSITIVE</u> FEEDBACK AROUND THE COMPARATOR:



$$v_+ = v_I \frac{R_2}{R_1+R_2} + v_0 \frac{R_1}{R_1+R_2} \quad \text{(SUPERPOSITION)}$$

IF $v_+ > V_{REF}$, $v_0 = V_S$
IF $v_+ < V_{REF}$ $v_0 = -V_S$
} TWO STABLE STATES



ASSUME CIRCUIT IS IN $v_0 = -V_S$. WHAT IS THE CONSTRAINT ON v_I TO STAY THERE?

$$v_+ < V_{REF} \quad \text{OR:} \quad \left(v_I \frac{R_2}{R_1+R_2} - V_S \frac{R_1}{R_1+R_2} \right) < V_{REF}$$

$$\text{EQUIVALENTLY:} \quad v_I < V_{REF}\left(\frac{R_1+R_2}{R_2}\right) + V_S \frac{R_1}{R_2}$$

ASSUME CIRCUIT IS IN THE STATE $v_0 = +V_S$. THE CORRESPONDING CONDITION TO STAY THERE IS:

$$v_+ > V_{REF} \quad \text{OR:} \quad v_I > V_{REF}\left(\frac{R_1+R_2}{R_2}\right) - V_S \frac{R_1}{R_2}$$

CONDITION FOR STAYING IN STATE:

IN SUMMARY:

IN $+V_S$ STATE: $v_I > V_{REF}\left(\frac{R_1+R_2}{R_2}\right) - V_S\frac{R_1}{R_2}$

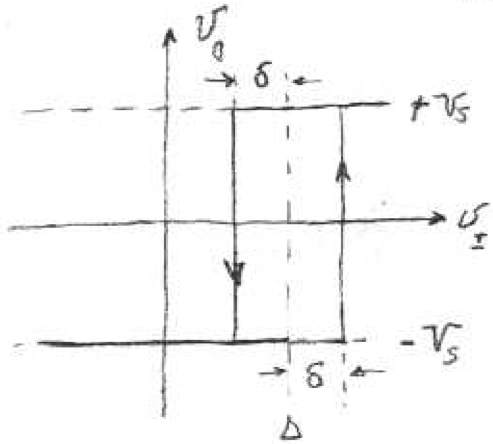
$\uparrow v_I$ INCREASE $\qquad \downarrow v_I$ DECREASES

IN $-V_S$ STATE: $v_I < V_{REF}\left(\frac{R_1+R_2}{R_2}\right) + V_S\frac{R_1}{R_2}$

$\underbrace{\qquad}_{\Delta}$ $\underbrace{\quad}_{\pm\delta}$

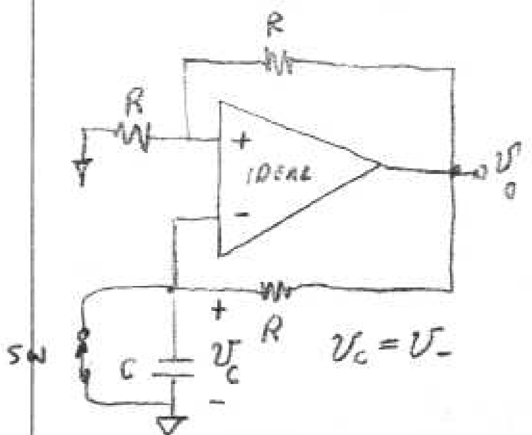
GRAPHICALLY:

HYSTERESIS BY INTENTION



A MEMBER OF A FAMILY OF CIRCUITS KNOWN AS "SCHMITT TRIGGERS"

THIS CIRCUIT, WITH BISTABILITY PRODUCED BY POSITIVE FEEDBACK, CAN EASILY BE MADE INTO A SIGNAL GENERATOR OR OSCILLATOR:



INITIALLY THE SWITCH IS CLOSED

ASSUME CIRCUIT IS IN + STATE I.E. $v_O = +V_S$

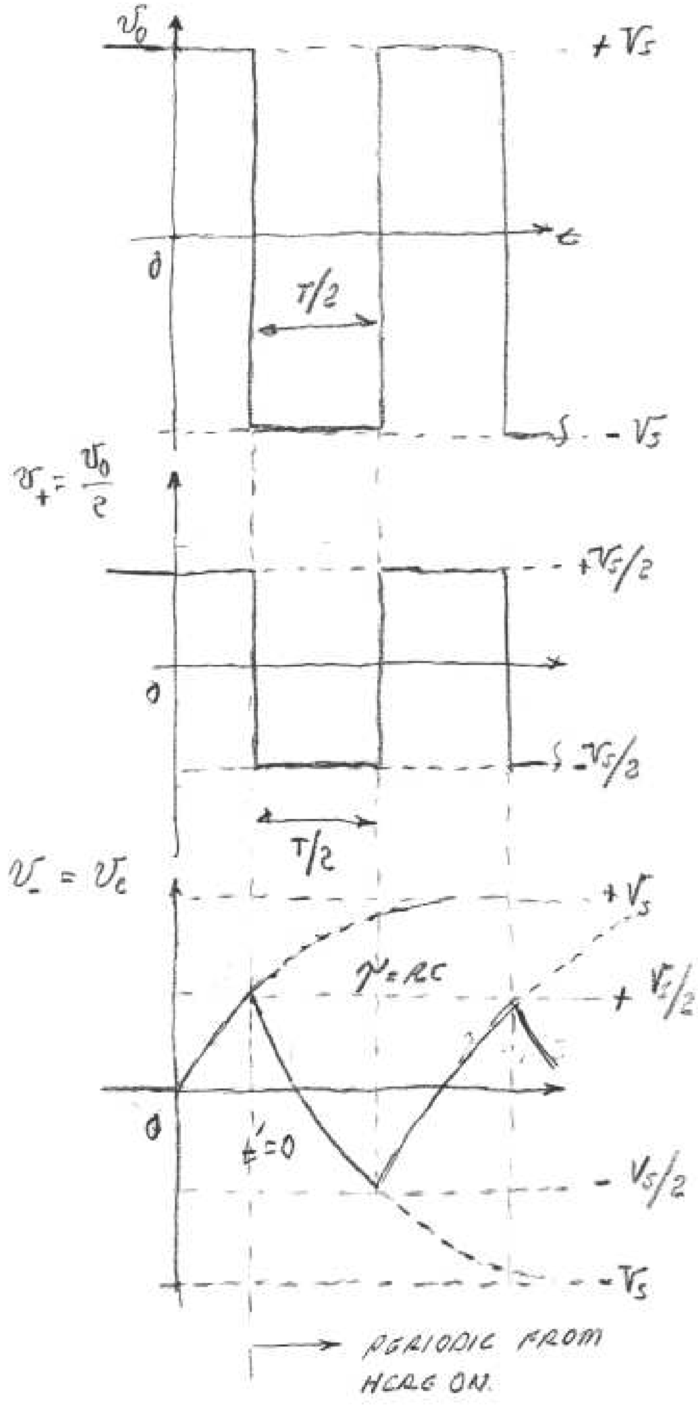
(WITH SWITCH CLOSED CIRCUIT LOOKS LIKE SCHMITT TRIGGER WITH $V_{REF} = 0$)

LET SWITCH OPEN AT $t=0$ v_C INCREASES TOWARD $+V_S$.

WHEN IT REACHES $\frac{V_S}{2}$ $v_C > v_+$ AND STATE CHANGES TO $v_O = -V_S$

v_C NOW DECREASE TOWARD $-V_S$, WITH THE STATE CHANGING AGAIN WHEN IT REACHES $-\frac{V_S}{2}$ MAKING $v_C < v_+$

AND THE CYCLE CONTINUES



TO DETERMINE THE PERIOD FOCUS ON INTERVAL MARKED T/2 AND LET $t' = 0$ AT START OF THIS INTERVAL. BY INSPECTION:

$$v_c(t') = -V_s + \frac{3}{2} V_s e^{-t'/\tau}$$

THIS INTERVAL ENDS WHEN

$$v_c(t') = -V_s/2 \quad \text{OR WHEN}$$

$$e^{-t'/\tau} = \frac{1}{3}$$

AT THIS TIME $t' = T/2$

$$\frac{T}{2} = \tau \ln 3$$

$$T = 2\tau \ln 3$$

THE CLOCK IN LAB #4 IS DIFFERENT IN DETAIL, BUT RELIES ON POSITIVE FEEDBACK AROUND AN AMPLIFIER, A FET INVERTER, WHICH SATURATES AT BOTH ENDS OF THE TRANSFER CHARACTERISTIC.

THE ANALYSIS PROCEEDS AS IN THE CIRCUIT ABOVE.