(503-062)

PROBLEM 10.1

A) First, we convert to a system of 3 equations:

$$y = A\delta \qquad \delta = x - y' \qquad y' = \beta y$$

Next, simplify to a single equation:

$$y = A[x - y'] = A[x - \beta y]$$

$$y = Ax - A\beta y$$

Solving for y we get:

$$y = \frac{Ax}{1+A\beta} \Rightarrow \boxed{\frac{y}{x} = T = \frac{A}{1+A\beta}}$$

B) $T = \frac{A}{1+A\beta} \Rightarrow$ (in table form) $\Rightarrow$

A	T
10^5	99.90
5×10^5	99.98
10^6	99.99

from the table,

$$\boxed{\Delta T = 0.09}$$

assuming $A = 500{,}000$ (the midrange value)

$$\boxed{\frac{\Delta T}{T} = \frac{0.09}{99.98} \approx 9\times10^{-4}}$$

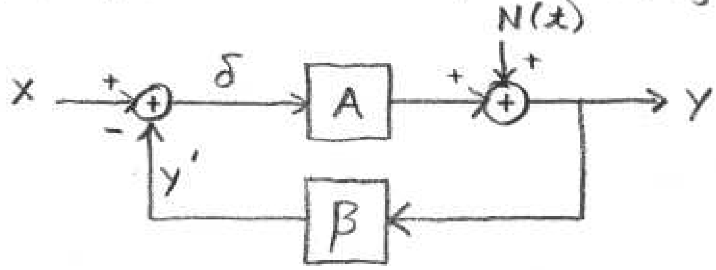
$$dT = \frac{dA(1+A\beta) - A(\beta dA)}{(1+A\beta)^2} \Rightarrow \boxed{\frac{dT}{dA} = \frac{1}{(1+A\beta)^2}}$$

$$\frac{dT}{T} = \left[\frac{1}{(1+A\beta)^2}\right]\left[\frac{1+A\beta}{A}\right]dA = \left[\frac{1}{1+A\beta}\right]\frac{dA}{A} = f(A)\frac{dA}{A}$$

$$\boxed{f(A) = \frac{1}{1+A\beta}}$$

$$\frac{dT}{T} = f(A)\frac{dA}{A} = \left(\frac{1}{1+A\beta}\right)\left(\frac{dA}{A}\right)\Bigg|_{A=5\times10^5} = \boxed{(1.9996\times10^{-4})\left(\frac{dA}{A}\right)}$$

c) First, we draw a system diagram:



Then we write a system of equations and solve for y as a function of N(t).

Because we are only interested in y as a function of N(t), we can ignore the x input (superposition): $x=0$

$$\delta = 0 - y' = -y'$$
$$y' = \beta y$$
$$y = A\delta + N(t)$$

$$y = A(-y') + N(t) = A(-\beta y) + N(t)$$

$$y[1+A\beta] = N(t)$$

$$\boxed{y = \frac{N(t)}{1+A\beta}} \quad \text{or} \quad \boxed{\frac{y}{N(t)} = \frac{1}{1+A\beta}}$$

from Part B: $A = 5\times10^5$, $\beta = .01$

$$\boxed{y = \frac{5V}{1+A\beta} = 9.998\times10^{-4}}$$

D) We can substitute $A(s) = \frac{A_o s_1}{s+s_1}$ directly into the expression for T from Part A:

$$T = \frac{A(s)}{1+A(s)\beta} = \frac{\frac{A_o s_1}{s+s_1}}{1+\frac{\beta A_o s_1}{s+s_1}} = \frac{\frac{A_o s_1}{s+s_1}}{\frac{s+s_1+\beta A_o s_1}{s+s_1}} = \frac{A_o s_1}{s+(1+A_o\beta)s_1}$$

$$\boxed{T \approx \frac{A_o s_1}{s + A_o\beta s_1}}$$

This is the transfer function of a lowpass filter with a single pole at $\omega = A_o\beta s_1$.

PROBLEM 10.2

These are all ideal opamps, therefore we make the following assumptions:

- $i_{IN+} = i_{IN-} = 0$
- $v_+ = v_-$
- v_o is whatever it has to be to guarantee the other assumptions

A) $v_+ = 0$ implies that $v_- = 0$

All of the current, I, flowing through R_1 must flow through R_2 ... the input current, i_{IN-}, is zero.

Therefore: $I = \dfrac{v_1 - v_-}{R_1} = \dfrac{v_- - v_o}{R_2}$ $(v_- = 0)$

$$\frac{v_1}{R_1} = -\frac{v_o}{R_2}$$

$$\boxed{v_o = \left(-\frac{R_2}{R_1}\right) v_1}$$

B) We can find v_+ using the voltage divider relationship:

$v_+ = v_2 \left[\dfrac{R_2}{R_1 + R_2}\right]$ (and we know that $v_- = v_+$)

Now we use the relationship from Part A:

$$\frac{v_1 - v_-}{R_1} = \frac{v_- - v_o}{R_2}$$

solving for v_o and collecting terms gives

$$v_o = \left[\frac{R_1 + R_2}{R_1}\right] v_- - \frac{R_2}{R_1} v_1$$

substituting for v_-, we get

$$v_o = \left[\frac{R_1 + R_2}{R_1}\right]\left[\frac{R_2}{R_1 + R_2}\right] v_2 - \frac{R_2}{R_1} v_1 \quad \text{or} \quad \boxed{v_o = \frac{R_2}{R_1}\left[v_2 - v_1\right]}$$

c) Again, we can solve directly for v_+ (and therefore v_-).

v_+ is halfway between v_1 and v_2:

$$v_+ = \frac{v_1 + v_2}{2} = v_-$$

To find v_o, we write

$$\frac{v_3 - v_-}{R} = \frac{v_- - v_o}{R} \Rightarrow v_o = -(v_3 - 2v_-)$$
$$= 2v_- - v_3$$

substituting for v_- gives

$$v_o = 2\left(\frac{v_1 + v_2}{2}\right) - v_3 \Rightarrow \boxed{v_o = v_1 + v_2 - v_3}$$

PROBLEM 10.3

For all of these opamp problems, we first solve directly for v_+ or v_- and assume the other one is the same.

Then we use the fact that the input currents are zero to write an expression relating v_o to the input voltages and v_+ and v_-.

Finally we substitute for v_+ and v_- and solve for v_o.

$$v_+ = 0 = v_-$$

$$i_D = \frac{K}{2}(v_{GS} - V_T)^2 = \frac{K}{2}(v_1 - \cancel{v_-}^{\,0} - V_T)^2 = \frac{\cancel{v_-}^{\,0} - v_o}{R}$$

$$v_o = -\frac{KR}{2}[v_1 - V_T]^2 = -.25[v_1 - 1]^2$$

$$v_o = -\left(\frac{v_1 - 1}{2}\right)^2$$

This expression is only valid if $v_1 > 1V$. If $v_1 < 1V$, there is no current flowing and v_o must be zero.

$$\boxed{v_o = \begin{cases} -\left(\frac{v_1 - 1}{2}\right)^2 & v_1 > 1V \\ 0 & v_1 < 1V \end{cases}}$$

PROBLEM 10.4

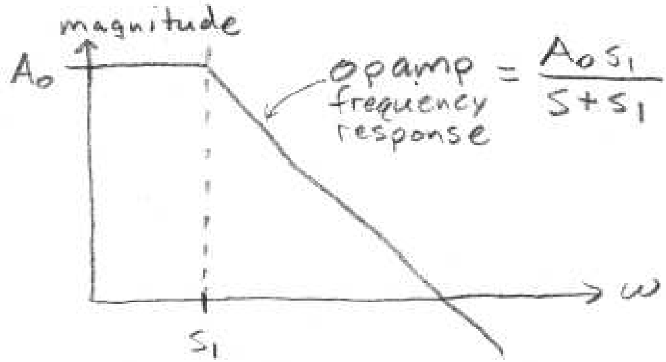
A) This is an inverting amplifier, similar to problem 10.2 part A. The only difference is that R_2 has been replaced by a complex impedance, $\frac{1}{Cs}$.

Therefore, $\frac{\bar{V}_o}{\bar{V}_i} = \bar{A}(s) = -\frac{1/Cs}{R}$

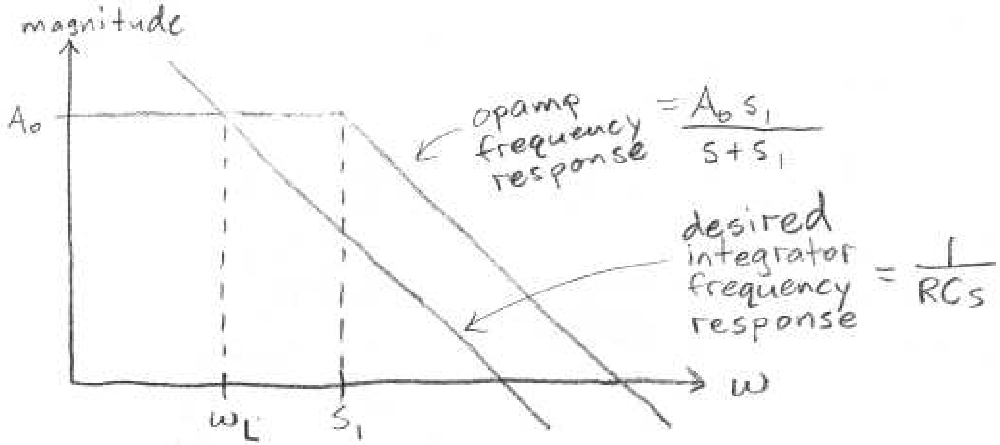
$\boxed{\bar{A}(s) = -\frac{1}{RCs}}$ which is an integrator. (think of Laplace)

Because the opamp is ideal, this expression holds for all frequencies: $\boxed{0 \le \omega \le \infty}$

But in the real world, the opamp has a finite gain, A_o, at low frequencies as shown below:



At some frequency $\omega = \omega_L$, the integrator runs out of "headroom". It needs more gain than the opamp can provide. So the $\bar{A}(s) = \frac{1}{RCs}$ is only valid for $s \gg \omega_L$.



B) This is a non-inverting amplifier topology. We can write $\bar{A}(s)$ directly, in a way similar to Part A:

$$\bar{A}(s) = \frac{\bar{V}_o}{\bar{V}_i} = 1 + \frac{R}{Ls \| \frac{1}{Cs}} \quad , \text{where } Ls \| \frac{1}{Cs} = \frac{L/C}{Ls + \frac{1}{Cs}} = \frac{Ls}{LCs^2+1}$$

$$\bar{A}(s) = 1 + \frac{R}{\frac{Ls}{LCs^2+1}} = 1 + \frac{RLCs^2+R}{Ls} = \frac{Ls+RLCs^2+R}{Ls}$$

$$\boxed{\bar{A}(s) = \frac{s^2 + \frac{1}{RC}s + \frac{1}{LC}}{\frac{1}{RC}s}}$$

assuming R is large enough, $A(s) \approx RC\left(\frac{s^2+LC}{s}\right)$

see attached Bode Plot.

PROBLEM 10.5

We know that the leftmost opamp subcircuit is an inverter with a gain of 1. Therefore, its output is just $-v_1$. This is the input to the second subcircuit.

We know that this signal must be negative, because it's given that $v_1 > 0$.

The second subcircuit is an inverter with a nonlinear element in the feedback path. i_D must flow through R_2, therefore we can write:

$$i_D = \frac{K}{2}(v_{GS} - V_T)^2 = \frac{v_- - (-v_1)}{R_2}$$

substituting $(v_2 = v_{GS})$ and $(V_T = 0)$ and $(v_- = 0)$ we get

$$\frac{K}{2}(V_2)^2 = \frac{v_1}{R_2}$$

solving for v_2 and substituting, we get

$$v_2 = \sqrt{\left(\frac{2}{KR_2}\right)v_1} = \sqrt{\frac{2v_1}{(.0001)(1000)}} = \boxed{\sqrt{20v_1} = v_2}$$

PROBLEM 10.6

For Parts A and B, we solve in similar fashion to Problems 10.5 and 10.3...

A) we can write

$$\frac{v_1 - v_-^{\to 0}}{R} = i \approx I_s\left(e^{-v_0/V_{TH}}\right) \; ; \; V_{TH} = \frac{KT}{q}$$

$$\ln\left(\frac{v_1}{RI_s}\right) = -\frac{v_0}{V_{TH}} \Rightarrow \boxed{v_0 = -\frac{KT}{q} \ln\left(\frac{v_1}{RI_s}\right)}$$

current can only flow through the diode if $\boxed{v_0 \text{ is negative}}$

B) $i \approx I_s e^{-v_1/V_{TH}} = \frac{v_0}{R} \; ; \; V_{TH} = \frac{KT}{q}$

$$\boxed{v_0 = RI_s e^{-v_1 q/KT}}$$

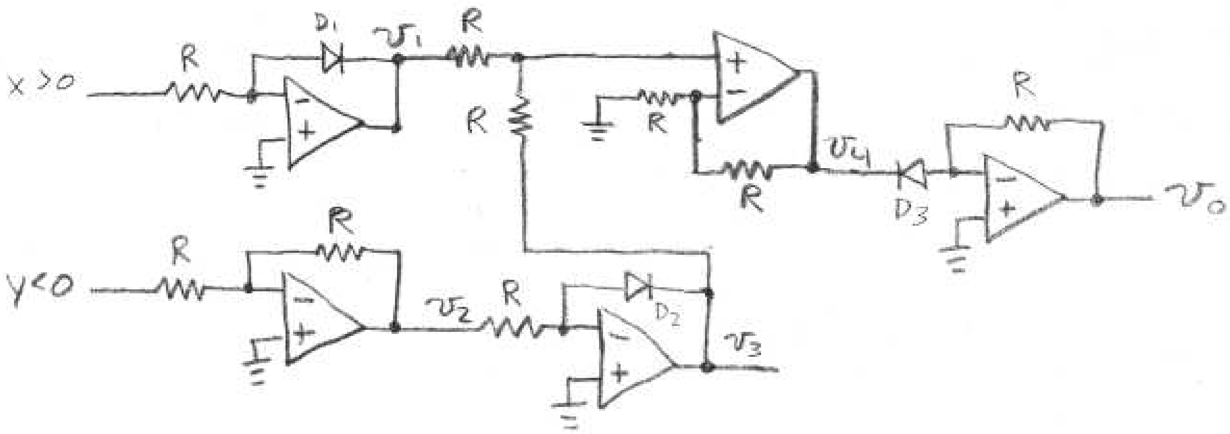
$\boxed{v_0 \text{ is positive}}$

C) Here is a straightforward circuit, using the ideas that

- $\ln(AB) = \ln A + \ln B$
- $e^{\ln X} = X$

The detailed analysis is left to the reader.

(see the next page)



$$v_1 = -\frac{kT}{q} \ln\left(\frac{x}{RI_s}\right)$$

$$v_2 = -y = |y|$$

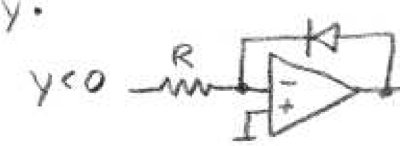
$$v_3 = -\frac{kT}{q} \ln\left(\frac{|y|}{RI_s}\right)$$

$$v_4 = v_1 + v_3 = -\frac{kT}{q}\left[\ln\left(\frac{x}{RI_s}\right) + \ln\left(\frac{|y|}{RI_s}\right)\right] = -\frac{kT}{q} \ln\left(\frac{x|y|}{(RI_s)^2}\right)$$

$$v_o = RI_s e^{-\frac{q}{kT}v_4} = RI_s e^{\ln\left(\frac{(x)|y|}{(RI_s)^2}\right)} = \frac{RI_s\, x|y|}{(RI_s)^2}$$

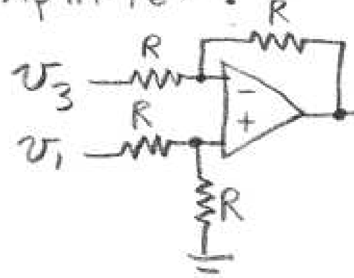
$$\boxed{v_o = \left(\frac{1}{RI_s}\right)(x)(|y|)}$$

By reversing the direction of diode D_2, the inverter can be omitted and ($y<0$) can be connected directly:



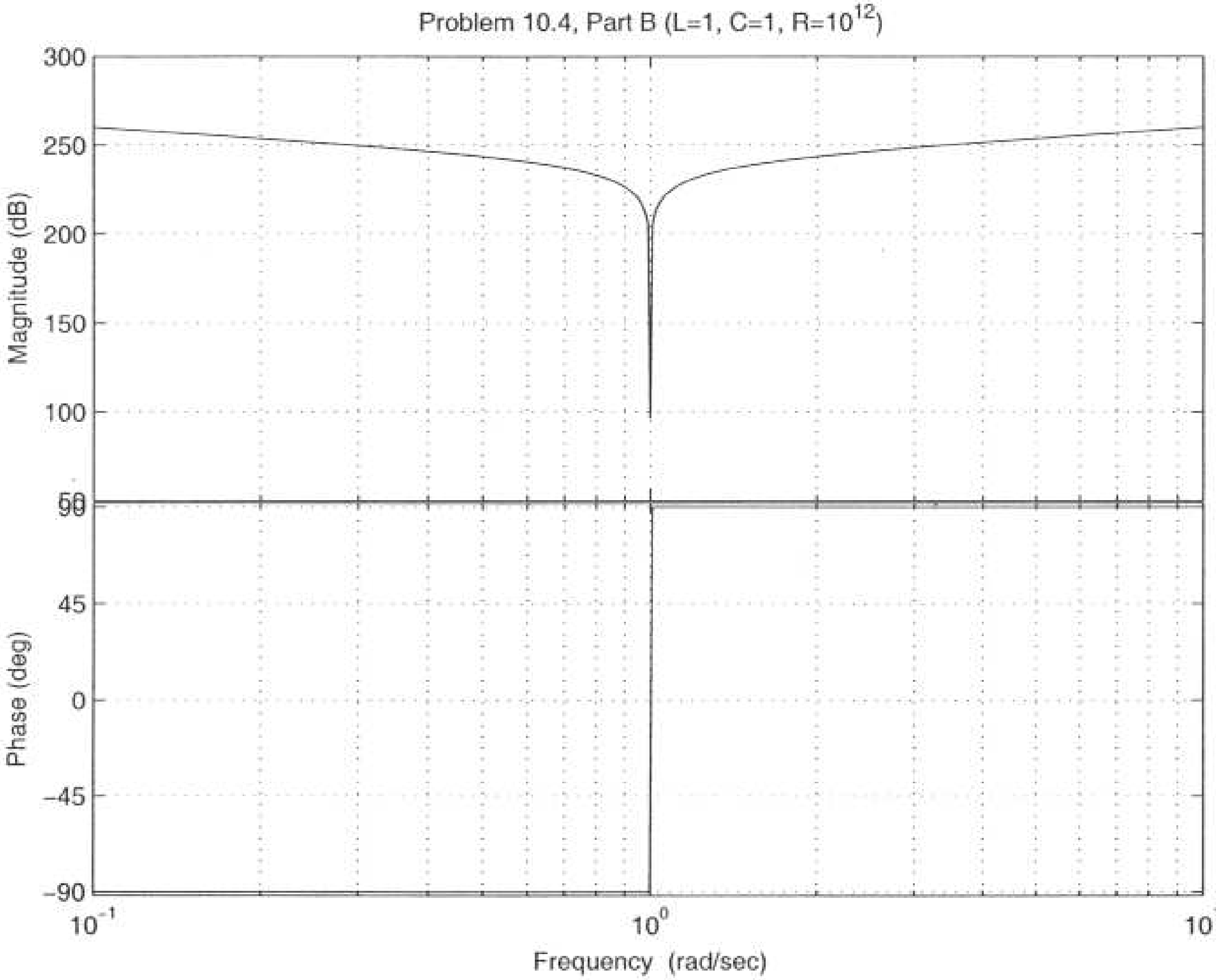
$$v_3 = \frac{kT}{q} \ln\left(\frac{|y|}{RI_s}\right)$$

Then v_4 can be computed using a difference amplifier:



$$v_4 = v_1 - v_3 = -\frac{kT}{q}\left[\ln\left(\frac{x}{RI_s}\right) + \ln\left(\frac{|y|}{RI_s}\right)\right]$$

This gives the same final result, $v_o = \left(\frac{1}{RI_s}\right)(x)(|y|)$ but with only 4 opamps.


Problem 10.4, Part B (L=1, C=1, R=10^{12})
300
250
200
150
100
50
90
45
0
−45
−90
Magnitude (dB)
Phase (deg)
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)