Exercise 1.1

(i) The three resistors form a series connection. Therefore, by directly applying equation (2.59) in the text, we obtain:

\[ R_{eq} := R_1 + R_2 + R_3 \]

(ii) The three resistors form a parallel connection. This time, by applying (2.108) in the text, we obtain:

\[
\frac{1}{R_{eq}} := \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}
\]

\[
R_{eq} := \frac{R_1 \cdot R_2 \cdot R_3}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3}
\]

Note the pleasing symmetry in the solution.

(iii) Split up the problem into two parts. First, apply the result from (ii) to \( R_2, R_3, \) and \( R_4 \). Call this solution \( R_p \). Second, apply the result from (i) to \( R_1, R_p, \) and \( R_5 \):

\[ R_p := \frac{R_2 \cdot R_3 \cdot R_4}{R_2 \cdot R_3 + R_2 \cdot R_4 + R_3 \cdot R_4} \]

\[ R_{eq} := R_1 + R_p + R_5 \]

Exercise 1.2

(i) 0.6\( \Omega \) is equivalent to \( 3/5 \Omega \). Because a series connection cannot achieve this, we try to apply a parallel connection first:

\[ \frac{3}{5} := \frac{A \cdot B}{A + B} \]

By setting \( B = 1 \Omega \), we obtain \( A = 1.5 \Omega \). 1.5\( \Omega \) can easily be achieved using 3 additional 1\( \Omega \) resistors, as the following diagram shows:

(ii) 1 \( 2/3 \Omega \) can be achieved using a 1\( \Omega \) and a \( 2/3 \Omega \) resistor. Using the same method from (i),
we see immediately that the 2/3Ω resistor can be synthesized from a 2Ω and a 1Ω resistor. Therefore, the solution looks like the following:

![Circuit Diagram]

**Problem 1.1**

(i) Using KVL equations around loop L1 and L2, we obtain the following equations:

\[
\begin{align*}
\text{Loop L1:} & \quad 2V - V_1 - 1V = 0 \\
\text{Loop L2:} & \quad V_2 - 1V - 2V = 0
\end{align*}
\]

Therefore, the unknown voltages are: \[V_1 := 1V \quad V_2 := 3V\]

(ii) The bottom portion of the circuit is exactly the same as (i); therefore, we can immediately obtain \(V_1\) and \(V_3\). Using KVL around loop L1, we obtain the following equation:

\[
\begin{align*}
- V_2 + \quad - 1V + \\
+ 1V \quad + \quad + 2V \quad + \quad + V_1 + \quad + 1V \quad + \quad + V_2 + \quad + 2V \quad + \quad + V_3 + \\
\text{Loop L1:} & \quad - V_2 + V_1 + 1V = 0
\end{align*}
\]

Therefore, the unknown voltages are: \[V_1 := 1V \quad V_2 := 2V \quad V_3 := 3V\]

We can verify the calculation by perform KVL on the exterior loop consisting of \(V_2\), \(V_3\), and the 1V drop. This gives \(3V - 2V - 1V = 0\), which confirms the solution.

(iii) Applying KCL at node N1, we obtain the following equation:
Node N1: \( 1A + 2A = I_1 \)

Therefore, the unknown current is: \( I_1 := 3A \)

(iv) First we apply KCL at N3 since that is the only node with 1 unknown variable. Upon solving \( I_3 \), \( I_2 \) can be found next by KCL at N2. Finally, \( I_1 \) is obtained by KCL at N1.

Node N1: \( I_1 = I_2 + 2A \)
Node N2: \( I_2 + I_3 + 2A = 0 \)
Node N3: \( 2A = I_3 + 1A \)

Therefore, the unknown currents are: \( I_1 := -1A \) \( I_2 := -3A \) \( I_3 := 1A \)

In particular, notice that some of the currents as defined in the diagram are negative.

Problem 1.2

(i) There are a total of 3 nodes in the circuit, as indicated by the diagram below. The KCL equations for each node are as follows:

Node N1: \( I_1 + I_2 + I_3 = 0 \)
Node N2: \( I_3 = I_4 \)
Node N3: \( I_1 + I_2 + I_4 = 0 \)

Only 2 of the KCL equations are independent. This can be seen by substituting \( I_3 \) in place of \( I_4 \) since they are the same as indicated by the N2 equation, into N3 and...
observing that the N1 and N3 equations become identical.

(ii) There are a total of 3 loops in the circuit, as indicated by the diagram below. The KVL equations for each node are as follows:

\[
\text{Loop L1: } V_1 - V_2 = 0
\]
\[
\text{Loop L2: } V_1 - V_3 - V_4 = 0
\]
\[
\text{Loop L3: } V_2 - V_3 - V_4 = 0
\]

Again, only 2 of the KVL equations are independent. This can be seen by substituting \( V_1 \) in place of \( V_2 \), since they are the same as indicated by the L1 equation, into L3 and observing that the L2 and L3 equations become identical.

(iii) The key point to notice in this part is that the voltage across an ideal current source and the current across an ideal voltage source cannot be defined on their own; instead the rest of the circuit dictates their values.

- **Voltage Source:** \( V_S = V_4 \)
- **Current Source:** \( I_S = -I_1 \)
- \( R_1: \quad R_1 = V_2 / I_2 \)
- \( R_2: \quad R_2 = V_3 / I_3 \)

(iv) The 8 equations include N2, either N1 or N3, L1, either L2 or L3, and the 4 equations from part (iii). From the voltage and current source equations, we immediately have:

\[
I_1 := -3 \text{A} \quad V_4 := 3 \text{V}
\]

Next we substitute the \( R_1 \) and \( R_2 \) constitutive laws into N1; together with L3, we now have to a system of 2 equations and 2 unknowns (we have already solved \( I_1 \)):

\[
\text{Equation 1: } I_1 + \frac{V_2}{R_1} + \frac{V_3}{R_2} := 0
\]
\[
\text{Equation 2: } V_2 - V_3 := V_4
\]

By substituting \( V_2 \) from equation 2 into equation 1 and simplifying, we obtain:

\[
V_3 := \left( -I_1 - \frac{V_4}{R_1} \right) \cdot \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}
\]
Therefore, substituting in numbers gives us \( V_3 := 2V \) \( V_2 := 5V \)

Finally, by substituting \( V_2 \) and \( V_3 \) into the \( R_1 \) and \( R_2 \) constitutive laws, we obtain the remaining unknown values:
\[
\begin{align*}
I_2 & := 2.5A \\
I_3 & := 0.5A 
\end{align*}
\]

The following table summarizes all the unknown currents and voltages:

<table>
<thead>
<tr>
<th>Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current (A)</td>
<td>-3.0</td>
<td>2.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Voltage (V)</td>
<td>5.0</td>
<td>5.0</td>
<td>2.0</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Using equation (1.9) from the text and noting the defined direction of the currents and the polarity of the voltages in the circuit, we obtain the following:
\[
\begin{align*}
P_{\text{current}} & = I_1V_1 = -15W \quad \text{(source)} \\
P_{R1} & = I_2V_2 = 12.5W \quad \text{(sink)} \\
P_{R2} & = I_3V_3 = 1W \quad \text{(sink)} \\
P_{\text{voltage}} & = I_4V_4 = 1.5W \quad \text{(sink)} 
\end{align*}
\]

Since the branch powers add up to 0, we can be confident that the calculations were done correctly. Note that as stated on page 45 of the text, an element is considered sourcing power if the power is negative under the associated variables convention while it is considered sinking power if the power is positive. All the current and voltage definitions in the circuit conform to the convention, with the current entering on the positive terminal of the elements.

**Problem 1.3**

The resistor network in this problem is also known as a *bridge circuit*. Often times, we use this circuit in the lab to determine 1 unknown resistor value within \( R_1 \) to \( R_4 \) using the remaining 3. Page 246 of the text shows how this circuit can be used (note that because \( R_3 \) and \( R_4 \) are switched compared to the book, the bridge branch is not balanced).

All node voltages \( e_1 \) through \( e_3 \) are defined positively with respect to the ground node. The current directions have been chosen arbitrarily; wrong direction will be indicated by a final current value that is negative. Now, applying KCL at the 3 nodes, we have:
In order to facilitate computation, we turn the 3 equations with 3 unknown into a matrix and perform row reduction on the result:

\[
\begin{pmatrix}
\frac{4}{3R} & \frac{-1}{R} & \frac{-1}{3R} \\
\frac{-1}{3R} & \frac{7}{R} & \frac{-1}{3R} \\
\frac{1}{R} & \frac{1}{3R} & \frac{-7}{3R}
\end{pmatrix}
\begin{pmatrix}
e_1 \\
e_2 \\
e_3
\end{pmatrix}
= 
\begin{pmatrix}
I_1 = e_1 - e_2 \\
I_2 = e_1 - e_3 \\
I_3 = e_2 - e_3
\end{pmatrix}
\]

Now, the node voltages by inspection are:

\[
\begin{pmatrix}
e_1 \\
e_2 \\
e_3
\end{pmatrix}
= 
\begin{pmatrix}
\frac{5}{3}I_3 \\
\frac{2}{3}I_3 \\
\frac{3}{3}I_3
\end{pmatrix}
\]

Applying the constitutive law for resistors, it is now trivial to obtain the branch currents:

\[
\begin{align*}
I_1 &:= \frac{e_1 - e_2}{3R} \\
I_2 &:= \frac{e_1 - e_3}{R} \\
I_3 &:= \frac{e_2}{R} \\
I_4 &:= \frac{e_3}{3R} \\
I_5 &:= \frac{e_2 - e_3}{R}
\end{align*}
\]
Problem 1.4

Using the first experiment, as diagramed below, we observe that because $R_1$ is floating, the full 1A from the current source at port 2 must flow through $R_2$ and $R_3$. This also means that there is no voltage drop across $R_1$. By defining a convenient reference node, as shown in the diagram, we can solve for $R_3$:

Next we apply the result of the second experiment, as diagramed below. By the KCL equation at node N1, we can find the current, and hence the voltage, of $R_3$. The value of $R_1$ and $R_2$ follows:

\begin{align*}
1A & := \frac{2V - 0V}{R_3} \\
R_3 & := 2\Omega
\end{align*}

\begin{align*}
1A & := \frac{V_3}{2\Omega} \\
V_3 & := 2V
\end{align*}

\begin{align*}
3A & := \frac{11V - 2V}{R_1} \\
R_1 & := 3\Omega
\end{align*}

\begin{align*}
2A & := \frac{2V - 0V}{R_2} \\
R_2 & := 1\Omega
\end{align*}