Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

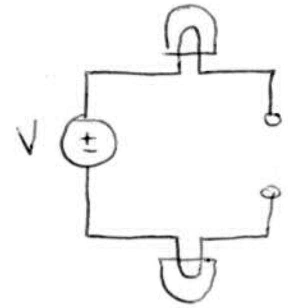
6.002 Circuits & Electronics

Spring 2004

Problem Set # 2 solutions.

Exercise 2.1

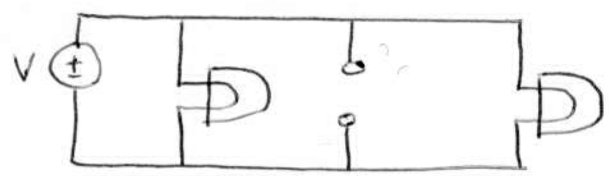
a).



After burn-out, no current flows through the circuit because there is now an open circuit.

Therefore the other bulbs go out.

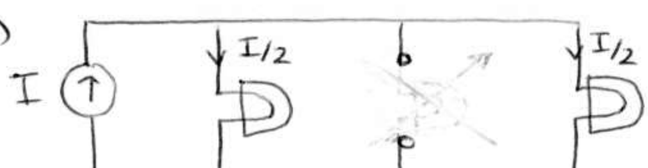
b)



Since the bulbs are in parallel and are being driven by a voltage source, after burn-out there is no change in the voltage delivered to each bulb (it is still V volts).

Therefore the other two bulbs exhibit no change in intensity.

c)



Each bulb now receives $I/2$ rather than $I/3$. Since the current through the other two bulbs increases, they get brighter.

Exercise 2.2

Since we have 3 unknowns (e_1, e_2 and e_3), we need 3 independent equations.

(1) $I = G_1(e_2 - e_1) + G_3(e_2 - e_1)$

(2) $G_1(e_2 - e_1) = G_2 e_1$

(3) $G_3(e_2 - e_3) + G_4(V - e_3) = G_5 e_3$

Now we assemble these equations into matrix form.

$$\begin{bmatrix} -G_1 & G_1 + G_3 & -G_3 \\ G_1 + G_2 & -G_1 & 0 \\ 0 & -G_3 & G_3 + G_4 + G_5 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ G_4 V \end{bmatrix}$$

Problem 2.1

(A) $$\frac{V_1 - e}{R_1} + \frac{V_2 - e}{R_2} = \frac{e}{R_3}$$ Using the node method.

$\therefore\ R_2 R_3 V_1 - R_2 R_3 e + R_1 R_3 V_2 - R_1 R_3 e = R_1 R_2 e$

$R_2 R_3 V_1 + R_1 R_3 V_2 = e\,(R_1 R_2 + R_1 R_3 + R_2 R_3)$

$$\boxed{e = \frac{R_3\,(R_2 V_1 + R_1 V_2)}{R_1 R_2 + R_1 R_3 + R_2 R_3}}$$

Problem 2.1 B)

First, we 'switch off' voltage source V_2 by making it a short circuit.

e_1 = node voltage e due to V_1 alone

$$= \frac{(R_2 \| R_3)}{R_1 + (R_2 \| R_3)} V_1 = \frac{R_2 R_3 V_1}{R_2 R_3 + R_1 R_2 + R_1 R_3}$$

Now we 'switch off' V_1 by making it a short circuit.

e_2 = node voltage e due to V_2 alone

$$= \frac{(R_1 \| R_3)}{R_2 + (R_1 \| R_3)} V_2 = \frac{R_1 R_3 V_2}{R_1 R_3 + R_2 R_1 + R_2 R_3}$$

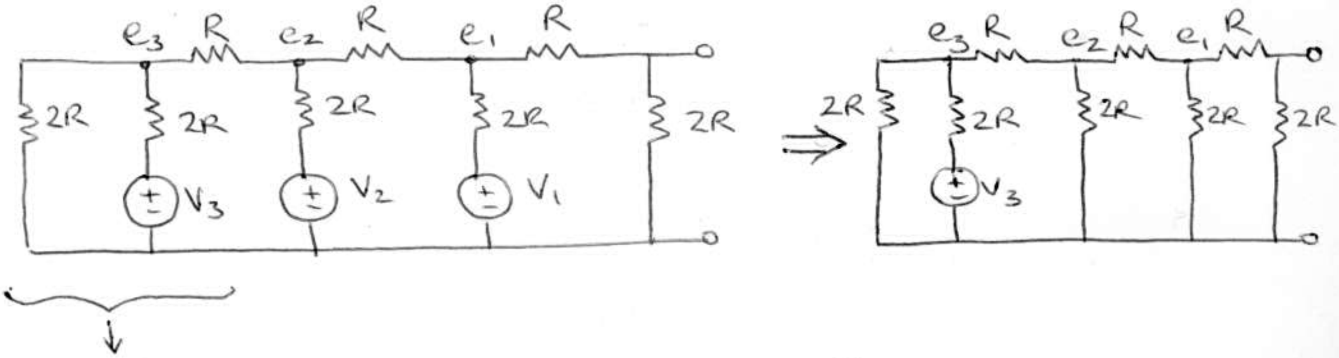
$\therefore \; e = e_1 + e_2$

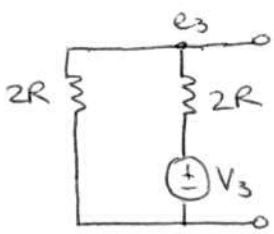
$$\boxed{e = \frac{R_3 (R_2 V_1 + R_1 V_2)}{R_1 R_3 + R_1 R_2 + R_2 R_3}}$$

Problem 2.1 c)

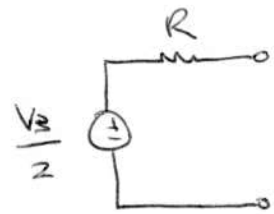
To solve this problem we use superposition and Thevenin equivalent circuits.

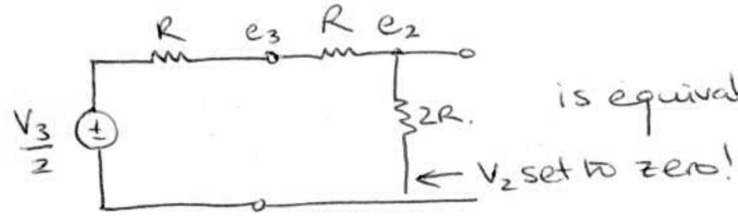
i). Turn on V_3, turn off V_1 and V_2





is equivalent to





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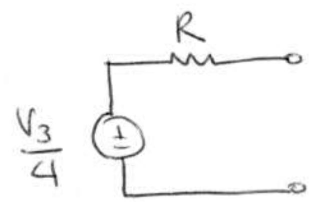

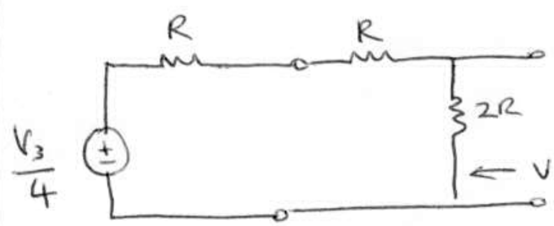

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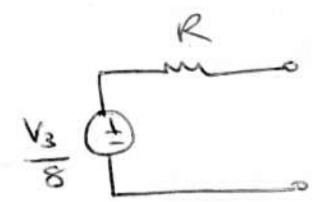
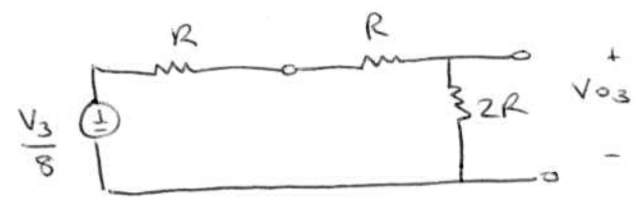


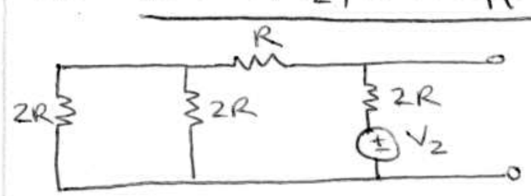
$\therefore$



$$V_{o3} = \frac{V_3}{16}$$

V_o due to voltage source V_3 alone.

ii) Turn on V_2, turn off V_1 and V_3.



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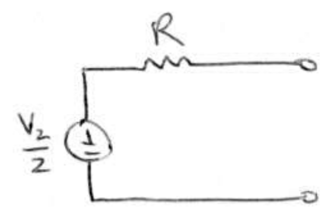
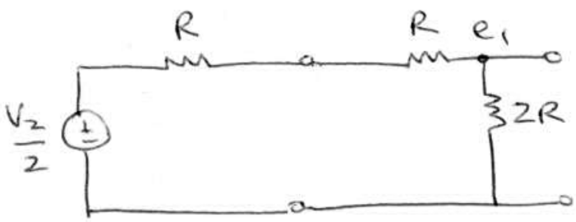
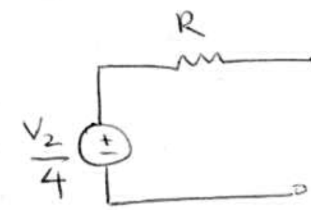


problem 2.1 c) continued



is equivalent to



$\therefore$ V_{o2} = V_o due to voltage source V_2 alone

$$V_{o2} = \frac{V_2}{8}$$

iii) Turn on V_1, turn off V_2 and V_3

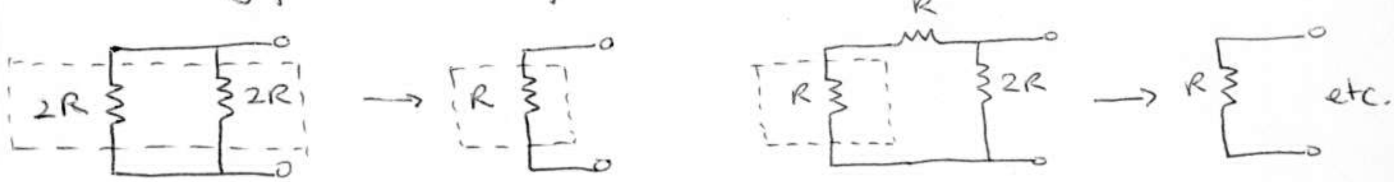
Doing the same as above, $V_{o1} = \frac{V_1}{4}$

By superposition, since this is a linear circuit,

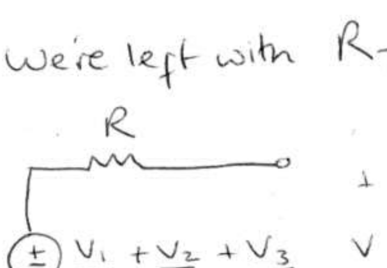
$$\boxed{V_o = \frac{V_1}{4} + \frac{V_2}{8} + \frac{V_3}{16}}$$

Since V_o is the open-terminal voltage, $V_o = V_{TH}$

To find the Thevenin equivalent resistance, we set all voltage sources to zero. Then we decompose the resistors starting from the left:



We're left with $R_{TH} = R$!



This is useful as a D/A converter, where V_1, V_2 and V_3 are the bit values of a 3 bit number $B_1 B_2 B_3$. (since the weighting of V_1 = twice that of V_2 etc.).

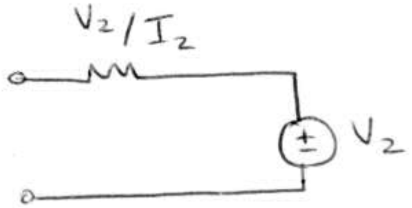
Problem 2.2

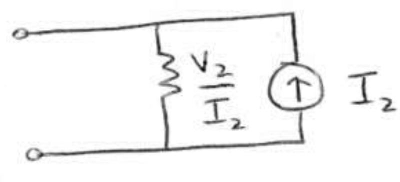
A). Network #2

$R_{eq} = \frac{V_2}{I_2}$ = slope of i-v relationship

When $i = 0$, $v = V_2$ $\therefore V_{TH} = V_2$

$v = 0$, $i_2 = -I_2$ $\therefore$ i leaving positive terminal of $N_2 = I_2$.



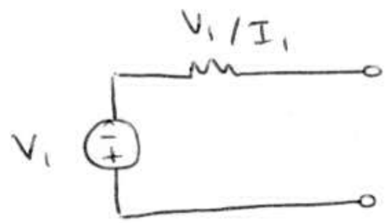


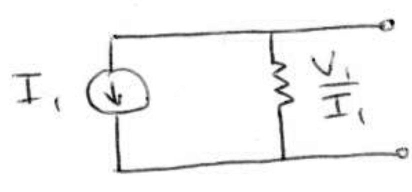
Network #1

$R_{eq} = \frac{V_1}{I_1}$ when $i = 0$, $v = -V_1$ $\therefore V_{TH} = -V_1$.

$v = 0$, $i = I_1$ $\therefore$ $i_{1sc} = I_1$ (where i_{1sc} is directed out of the positive terminal of the open circuit)

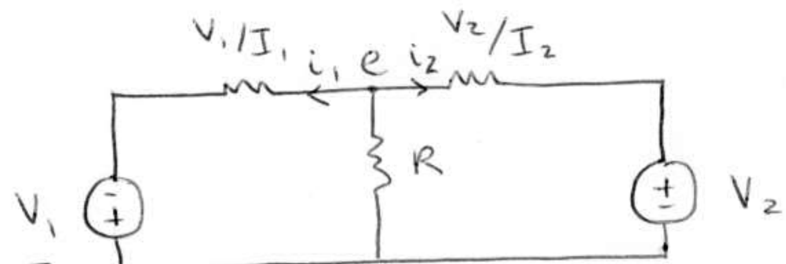
$\therefore I_{Norton} = -I_1$.





Note the orientations of the voltage & current sources!

B).



note orientation!

USE SUPERPOSITION : i) Turn on V_1, switch of V_2

$$e_1 = \frac{R \parallel \frac{V_2}{I_2}}{\frac{V_1}{I_1} + R \parallel \frac{V_2}{I_2}} \cdot -V_1 = \frac{-R V_1 V_2 I_1}{V_1 V_2 + R(V_1 I_2 + V_2 I_1)}$$

Problem 2.2 B) continued

ii) Turn on V_2, switch off V_1

$$e_2 = \frac{\left(\frac{V_1}{I_1} \parallel R\right) \cdot V_2}{\frac{V_2}{I_2} + \left(\frac{V_1}{I_1} \parallel R\right)} = \frac{\frac{RV_1/I_1}{V_1 + RI_1} \cdot V_2}{\frac{V_2}{I_2} + \frac{RV_1/I_1}{V_1 + RI_1}}$$

$e_1 + e_2$

$\downarrow$

$$\therefore \boxed{e = \frac{RV_1V_2(I_2 - I_1)}{V_1V_2 + R(V_1I_2 + V_2I_1)}}$$

$$i_1 = \frac{e + V_1}{V_1/I_1} \quad \text{(by the node method)}$$

$$\therefore \boxed{i_1 = \frac{I_1[RI_2(V_1 + V_2) + V_1V_2]}{V_1V_2 + R(V_1I_2 + V_2I_1)}}$$

$$i_2 = \frac{e - V_2}{V_2/I_2}$$

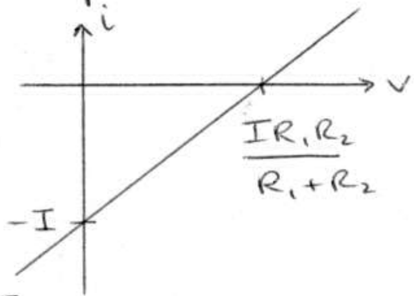
$$\therefore \boxed{i_2 = \frac{-I_2(RI_1(V_1 + V_2) + V_1V_2)}{V_1V_2 + R(V_1I_2 + V_2I_1)}}$$

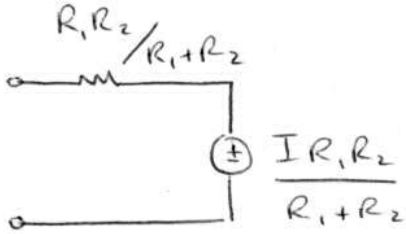
Problem 2.3 A) $V_{TH} = I \cdot (R_1 \| R_2)$

$$= \frac{I R_1 R_2}{R_1 + R_2}$$

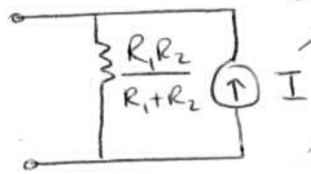
To calculate R_{TH}, we set $I = 0$ (an open circuit)

$$\therefore R_{TH} = \frac{R_1 R_2}{R_1 + R_2} \qquad -I_{sc} = I.$$





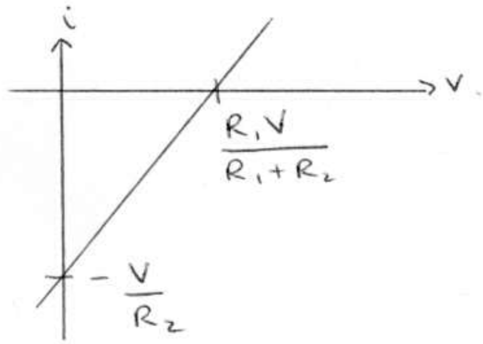
Thevenin



Norton

(B) $V_{TH} = \frac{R_1 V}{R_1 + R_2} \qquad R_{TH} = (R_1 \| R_2) \qquad I_{SC} = -I_N = \frac{-V}{R_2}$

$$\therefore I_N = \frac{V}{R_2}$$



(C) First find the thevenin equivalent resistance $\boxed{R_{TH} = \frac{R_1 R_2}{R_1 + R_2}}$

Now use superposition

i) I_{sc} due to $I = -I$

ii) I_{sc} due to $V = -V/R_2$

$$\Big\} \; i_{sc} = -\left(I + \frac{V}{R_2}\right)$$

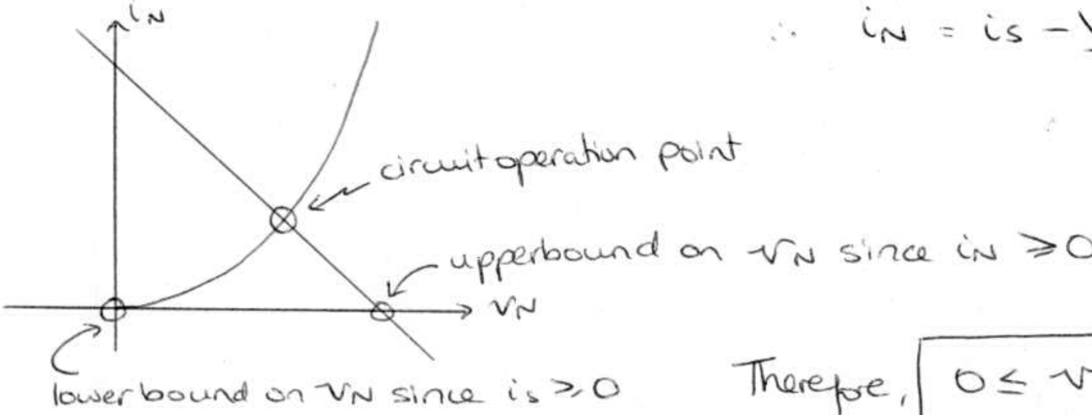
$$V_{TH} = -i_{sc} \cdot R_{TH} = \frac{I R_1 R_2}{R_1 + R_2} + \frac{V R_1}{R_1 + R_2}$$

$$\boxed{V_{TH} = \frac{R_1}{R_1 + R_2}(V + I R_2)}$$

Problem 2.4

A) The two constraints on the relationship between i_N and v_N are: $\quad i_N = \alpha v_N^2$ and $\quad i_s = \frac{v_N}{R} + i_N$

$$\therefore \quad i_N = i_s - \frac{v_N}{R}$$



Therefore, $\boxed{0 \leq v_N \leq R i_s}$

B) $\quad i_s = \frac{v_N}{R} + i_N = \frac{v_N}{R} + \alpha v_N^2$, where $i_s \geq 0$

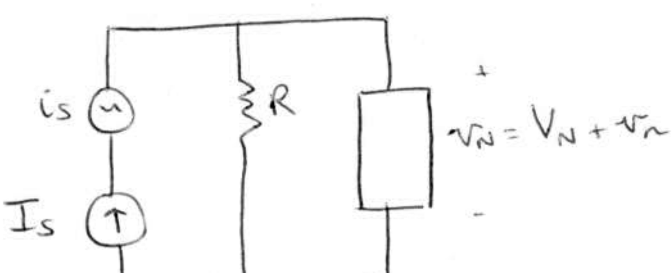
$$\therefore \quad \alpha v_N^2 + \frac{v_N}{R} - i_s = 0$$

$$v_N = \frac{-\frac{1}{R} \pm \sqrt{\frac{1}{R^2} + 4\alpha i_s}}{2\alpha} = \frac{-1}{2\alpha R} \pm \sqrt{\frac{1}{4\alpha^2 R^2} + \frac{i_s}{\alpha}}$$

$$\therefore \quad \boxed{v_N = -\frac{1}{2\alpha R} + \sqrt{\frac{1}{4\alpha^2 R^2} + \frac{i_s}{\alpha}}}$$

We are only interested in the positive solution for v_N, since i_s is assumed to be ≥ 0. Therefore v_N can never be negative.

C)



At the operating point,

$$V_N = \frac{-1}{2\alpha R} + \sqrt{\frac{1}{4\alpha^2 R^2} + \frac{I_s}{\alpha}}$$

$$v_n = \left.\frac{df(i_s)}{d i_s}\right|_{I_s} i_s \text{ by Taylor Series} = \frac{1/\alpha}{2\sqrt{\frac{1}{4\alpha^2 R^2} + \frac{I_s}{\alpha}}}\, i_s = \boxed{\frac{R i_s}{\sqrt{1 + 4\alpha R^2 I_s}}}$$