Exercise 8.1: Using one 1-nF capacitor and two resistors, construct a two-port network that has the following response to a 1-V step input; assume that the capacitor voltage is zero prior to the step. Provide a diagram of the network, and specify the values of the two resistors.

\[
v_{\text{IN}}(t) = 1\,\text{V}\,u(t)
\]

\[
v_{\text{OUT}}(t) = 0.5\,\text{V}\,(1-e^{-\frac{t}{1\,\mu\text{s}}})\,u(t)
\]

Exercise 8.2: Repeat Exercise 8.1 given that the allowable components are now one 1-mH inductor and two resistors; assume that the inductor current is zero prior to the step.

Problem 8.1: The network shown below includes a switch with three positions: A, B and C. Prior to \( t = 0 \), the switch is in Position B, and the inductor current \( i(t) \) and capacitor voltage \( v(t) \) are both zero.

(A) At \( t = 0 \) the switch moves to Position A, and it remains there until \( t = T_1 \). Find \( i(t) \) and \( v(t) \) for \( 0 \leq t \leq T_1 \).

(B) At \( t = T_1 \) the switch moves to Position C, and it remains there until \( i(t) \) goes to zero, at which time the switch moves back to Position B. Define the time at which \( i(t) \) goes to zero as \( t = T_2 \). Determine \( T_2 \), and find both \( i(t) \) and \( v(t) \) for \( T_1 \leq t \leq T_2 \).

(C) The switch remains in Position B until \( t = T_3 \). Find both \( i(t) \) and \( v(t) \) for \( T_2 \leq t \leq T_3 \).

(D) At \( t = T_3 \) the switch moves again to Position A, and it remains there until \( t = T_4 \). Find \( i(t) \) and \( v(t) \) for \( T_3 \leq t \leq T_4 \).
(E) Finally, at \( t = T_4 \) the switch moves to Position C, and it remains there until \( i(t) \) again goes to zero, at which time the switch moves back to Position B. Define the time at which \( i(t) \) again goes to zero as \( T_5 \). Determine \( T_5 \), and find both \( i(t) \) and \( v(t) \) for \( T_4 \leq t \leq T_5 \).

(F) Sketch and clearly label \( i(t) \) and \( v(t) \) for \( 0 \leq t \leq T_5 \).

Problem 8.2: This problem is a continuation of Problem 8.1. It explores the use of energy conservation to analyze the operation of the network described therein.

(A) Determine the energy stored in the inductor at \( t = T_1 \).

(B) The energy stored in the inductor at \( t = T_1 \) is fully transferred to the capacitor at \( t = T_2 \). Use this fact to determine \( v(T_2) \). This answer should match your answer to Part B of Problem 8.1 when the latter is evaluated at \( t = T_2 \).

(C) Determine the energy stored in the inductor at \( t = T_4 \).

(D) Use energy conservation to determine the energy stored in the capacitor at \( t = T_5 \), and then determine \( v(T_5) \). This answer should match your answer to Part E of Problem 8.1 when the latter is evaluated at \( t = T_5 \).

(E) Now let the switch move repetitively through the cycle of Positions B to A to C to B. Assume that in each cycle the switch remains in Position A for the duration \( T \). Further, assume that switch always moves from Position C to Position B when \( i(t) \) reaches zero. Assuming that \( v \) and \( i \) are initially zero, determine \( i \) at the end of the \( n \)th switching cycle in terms of \( n, C, L, T \) and \( V \).
Problem 8.3: In the network shown below, the inductor and capacitor have zero current and voltage, respectively, prior to \( t = 0 \). At \( t = 0 \), a step in voltage from 0 to \( V_0 \) is applied by the voltage source as shown.

(A) Find \( v_C, v_L, v_R \), \( i \) and \( \frac{di}{dt} \) just after the step at \( t = 0 \).

(B) Argue that \( i = 0 \) at \( t = \infty \) so that \( i(t) \) has no constant component.

(C) Find a second-order differential equation which describes the behavior of \( i(t) \) for \( t \geq 0 \).

(D) Following (B) the current \( i(t) \) takes the form \( i(t) = I \sin(\omega t + \phi)e^{-\alpha t} \). Find \( I \), \( \omega \), \( \phi \) and \( \alpha \).

(E) Suppose that the input is a voltage impulse with area \( \Lambda_0 \) where \( \Lambda_0 = \tau V_0 \), \( V_0 \) is the amplitude of the voltage step shown below, and \( \tau \) is a given time constant. Find the response of the network shown below to the impulse. Hint: before solving this problem directly, consider the relation between step and impulse responses.

Save a copy of your answers to this problem. They will be useful during the pre-lab exercises for Lab #3.