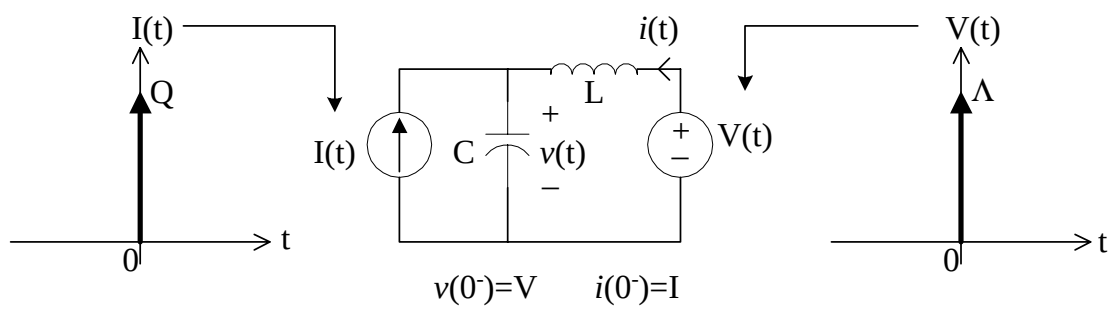
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Spring 2004

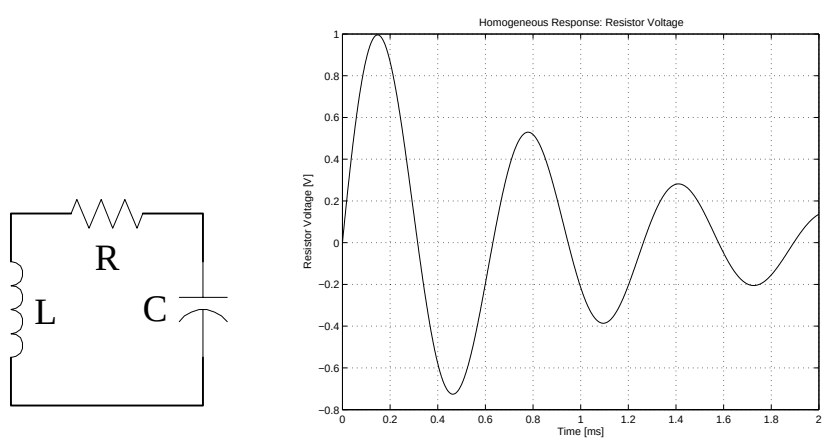
Problem Set #9

Issued 4/7/04 – Due 4/14/04

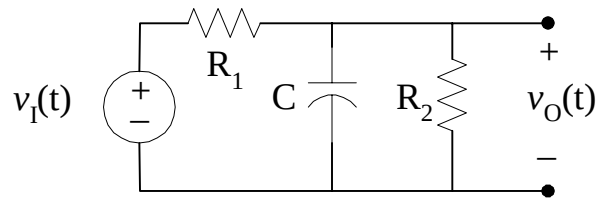
Exercise 9.1: Determine $i(t)$ and $v(t)$ for $t \geq 0^+$ in the network shown below. Note that the inductor and capacitor both have nonzero states at $t = 0$. Hint: use superposition to establish initial conditions at $t = 0^+$.



Exercise 9.2: Shown below is a series resistor-inductor-capacitor network. Also shown below is a graph of the resistor voltage as a function of time; the voltage part of the homogeneous response of the network to some initial conditions. From the voltage graph, determine the resistance of the resistor and the inductance of the inductor given that the capacitance of the capacitor is 1 μF. *Note: the last page of this problem set contains a larger graph of the resistor voltage. It can be turned in with your problem set solutions.*



Problem 9.1: The network shown below is driven in steady state by the sinusoidal input voltage $v_{\mathrm{I}}(t) = V_{\mathrm{I}} \cos(\omega t)$. The output of the network is the voltage $v_{\mathrm{O}}(t)$, which takes the form $v_{\mathrm{O}}(t) = V_{\mathrm{O}} \cos(\omega t + \phi)$. Find V_{O} and ϕ as functions of ω as follows.



(A) Using the Taylor Series expansions for e^x, $\cos(x)$ and $\sin(x)$, show that $e^{jx} = \cos(x) + j\sin(x)$. Following this, recognize that $\cos(x) = \Re\{e^{jx}\}$.

(B) Show that $A + Bj = \sqrt{A^2 + B^2} e^{j \arctan(B/A)}$. Thus, the magnitude and phase of $A + Bj$ are $\sqrt{A^2 + B^2}$ and $\arctan(B/A)$, respectively.

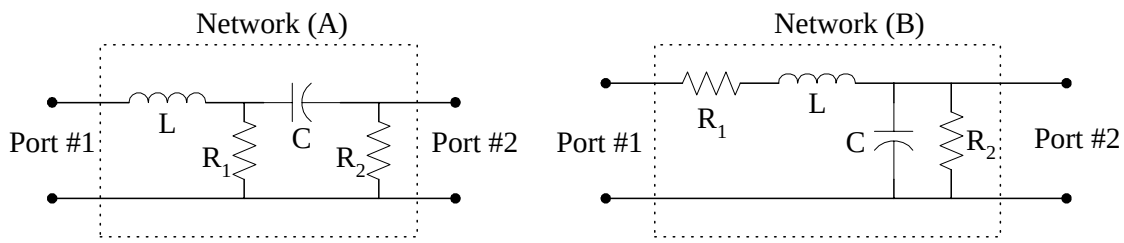
(C) Find a differential equation that can be solved for $v_{\mathrm{O}}(t)$ given $v_{\mathrm{I}}(t)$.

(D) Following Part A, let $v_{\mathrm{I}}(t) = V_{\mathrm{I}} e^{j\omega t}$. Also, let $v_{\mathrm{O}}(t) = \hat{V}_{\mathrm{O}} e^{j\omega t}$ where $\hat{V}_{\mathrm{O}}$ is a complex function of the circuit parameters, ω and V_{I}. With these substitutions, use the differential equation to find $\hat{V}_{\mathrm{O}}$.

(E) Following Parts A and B, first express v_{O} from Part (D) in the form $v_{\mathrm{O}}(t) = |\hat{V}_{\mathrm{O}}| e^{j(\omega t + \angle \hat{V}_{\mathrm{O}})}$, and determine $|\hat{V}_{\mathrm{O}}|$ and $\angle \hat{V}_{\mathrm{O}}$ as functions of the circuit parameters, ω and V_{I}. Then, find V_{O} and ϕ for the original cosine input, again both as functions of the circuit parameters, ω and V_{I}.

(F) Sketch and clearly label $V_{\mathrm{O}}/V_{\mathrm{I}}$ and ϕ as functions of ω. Identify the low-frequency and high-frequency asymptotes on the sketch.

Problem 9.2: This problem concerns the sinusoidal-steady-state behavior of the networks shown below, both of which have two ports.



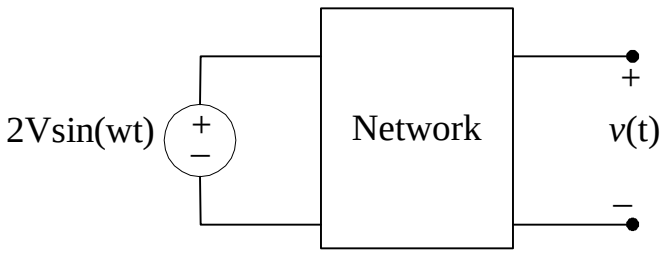
(A) Determine the impedance of each network as viewed into Port #1 under the assumption that Port #2 is open.

(B) Assume that Port #1 of each network is driven in sinusoidal steady state by the voltage $V_1 \cos(\omega t)$, and that Port #2 is open. Determine the current into the positive terminal of each network at Port #1. Express the current in the form $I_1 \cos(\omega t + \phi_1)$ where I_1 is an amplitude and ϕ_1 is a phase angle.

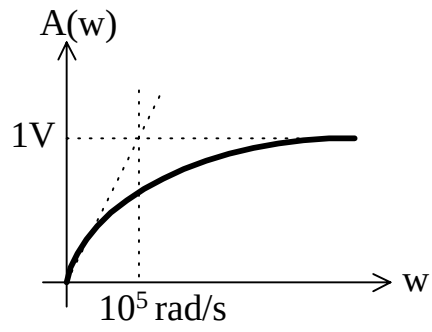
(C) Assume that Port #1 of each network is again driven in sinusoidal steady state by the voltage $V_1 \cos(\omega t)$, and that Port #2 is again open. Determine the voltage which appears at Port #2. Express the voltage in the form $V_2 \cos(\omega t + \phi_2)$ where V_2 is an amplitude and ϕ_2 is a phase angle.

Note that the results of this problem are useful when completing the pre-lab exercises to Lab #3.

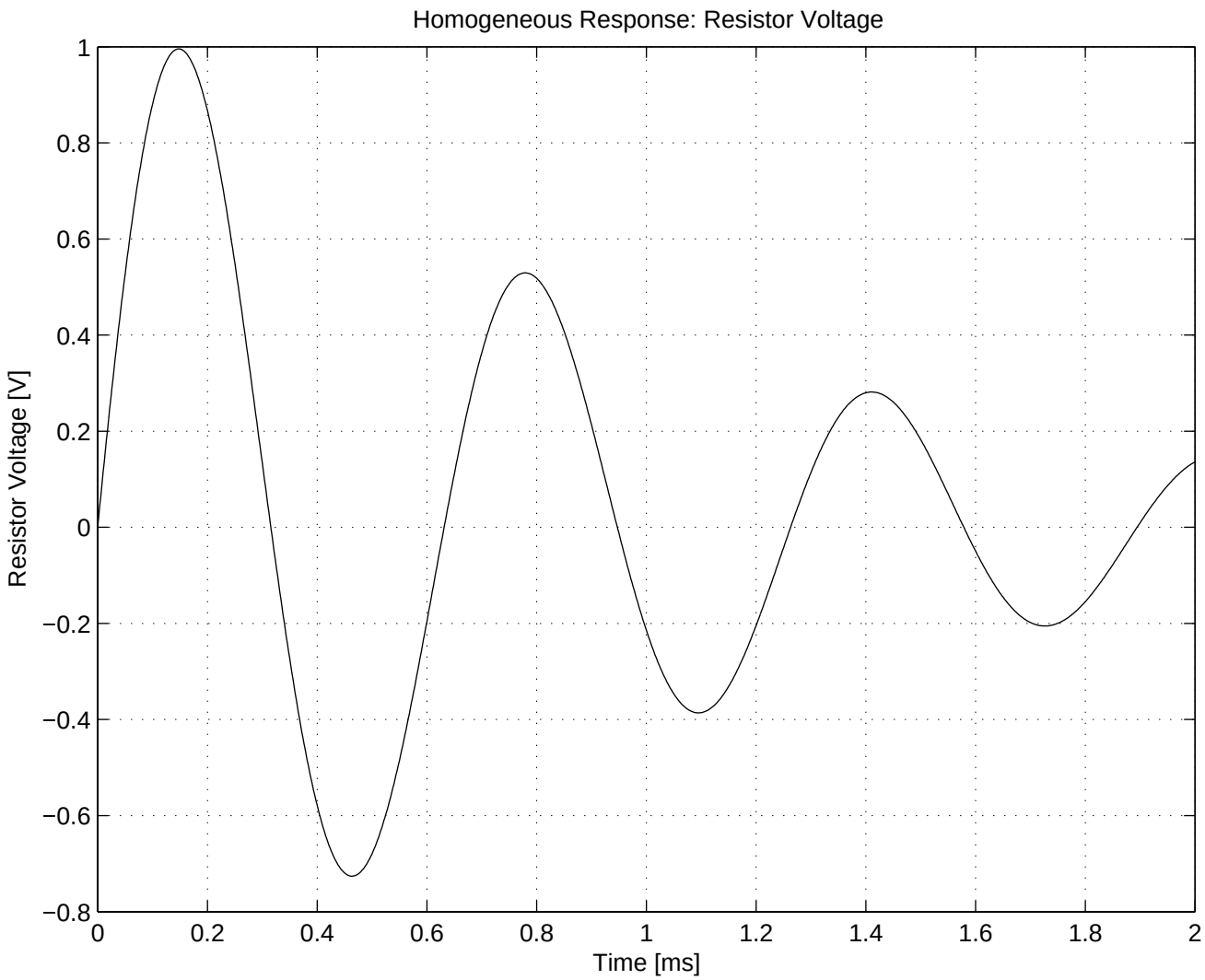
Problem 9.3: Using a 1-mH inductor and two resistors, design a two-port network that has the following input-output relation in the sinusoidal steady state. Note that the relation is defined for the case of an unloaded, or open-circuited, output port.



$v(t) = A(w)\sin(wt+\phi(w))$



Problem 9.4: Repeat Problem 9.3 assuming that the allowable components include only a 1-μF capacitor and two resistors.


Homogeneous Response: Resistor Voltage
Resistor Voltage [V]
Time [ms]
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2