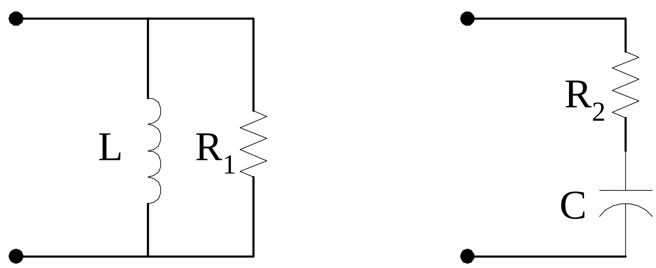
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Spring 2004

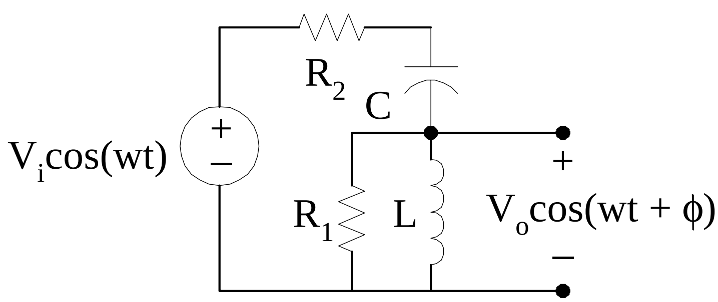
Problem Set #10

Issued 4/14/04 – Due 4/21/04

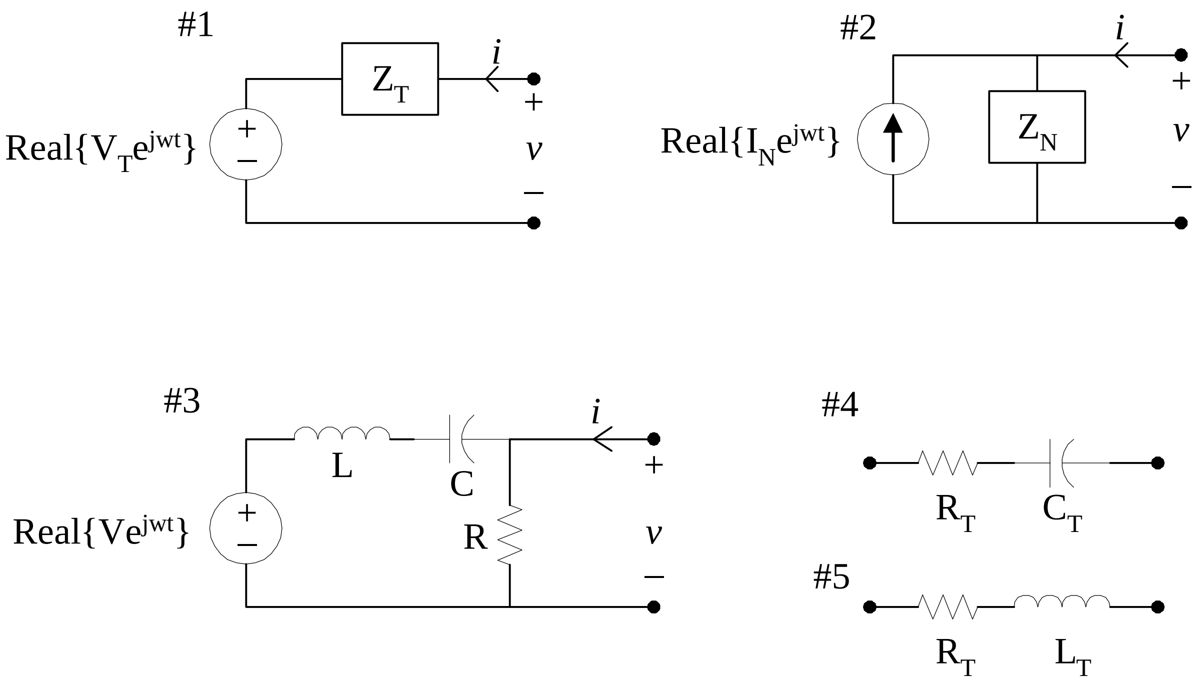
Exercise 10.1: Determine the impedance of each network shown below. Also, identify the asymptotic dependence of the impedances on frequency for very low frequencies and for very high frequencies, and explain the dependences physically.



Exercise 10.2: Assume that the network shown below is operating in sinusoidal steady state. Determine the amplitude V_o and phase ϕ of the voltage across the parallel inductor and resistor. Hint: see Exercise 10.1.



Problem 10.1: This problem explores the Thevenin and Norton equivalence of networks operating in sinusoidal steady state. All networks considered here are comprised of linear resistors, capacitors and inductors, and voltage and current sources all operating at the same frequency ω. Therefore, all branch currents and voltages operate at the frequency ω.



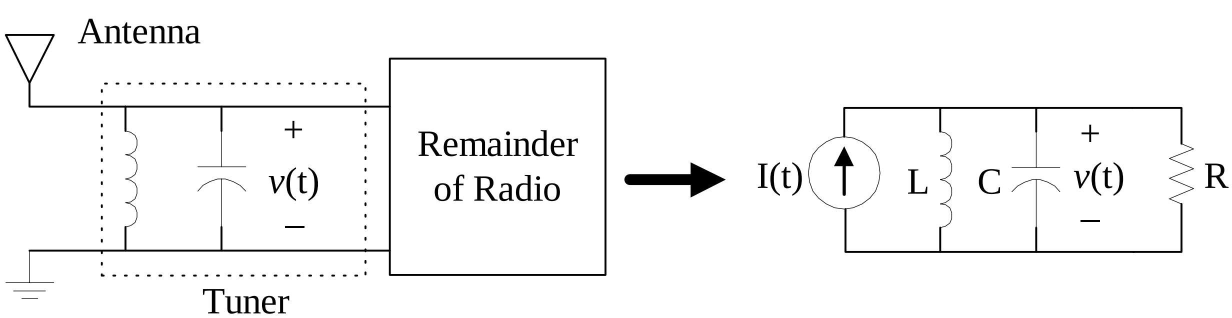
(A) Determine the relations between V_{T}, Z_{T}, I_{N} and Z_{N} which must exist for the i-v relations at the terminals of Networks #1 and #2 to be identical when operating in sinusoidal steady state.

(B) Review the arguments for networks involving only linear resistors and sources, and then briefly explain why Networks #1 and #2 may serve as the Thevenin and Norton equivalents, respectively, of an arbitrary linear network operating in sinusoidal steady state.

(C) Determine V_{T} and Z_{T} in the Thevenin equivalent of Network #3.

(D) Suppose Z_{T} in Part C is implemented with Networks #4 and #5. Determine R_{T} and C_{T} in Network #4, and R_{T} and L_{T} in Network #5, in terms of R, L, C and ω. Under what circumstances is Network #4 preferred over Network #5, and vice versa?

Problem 10.2: This problem examines the very simple tuner for an AM radio shown below. Here, the tuner is the parallel inductor and capacitor. The injection of radio signals into the tuner by the antenna is modeled by a current source, while the Norton resistance of the antenna in parallel with the remainder of the radio is modeled by a resistor. (You will learn about antenna modeling in 6.013.) The AM radio band extends from 540 kHz through 1600 kHz. The information transmitted by each radio station is constrained to be within ± 5 kHz of its center frequency. (You will learn about AM radio transmission in 6.003.) To prevent frequency overlap of neighboring stations, the center frequency of each station is constrained to be a multiple of 10 kHz. Therefore, the purpose of the tuner is to pass all frequencies within 5 kHz of the center frequency of the selected station, while attenuating all other frequencies.



(A) Assume that $I(t) = I\cos(\omega t)$. Find $V(\omega)$ and $\phi(\omega)$ in $v(t)$ where $v(t) = V\cos(\omega t + \phi)$. Note that $v(t)$ is the output of the tuner, namely the signal that is passed on to the remainder of the radio.

(B) For a given combination of I, C, L and R, at what frequency is V maximized?

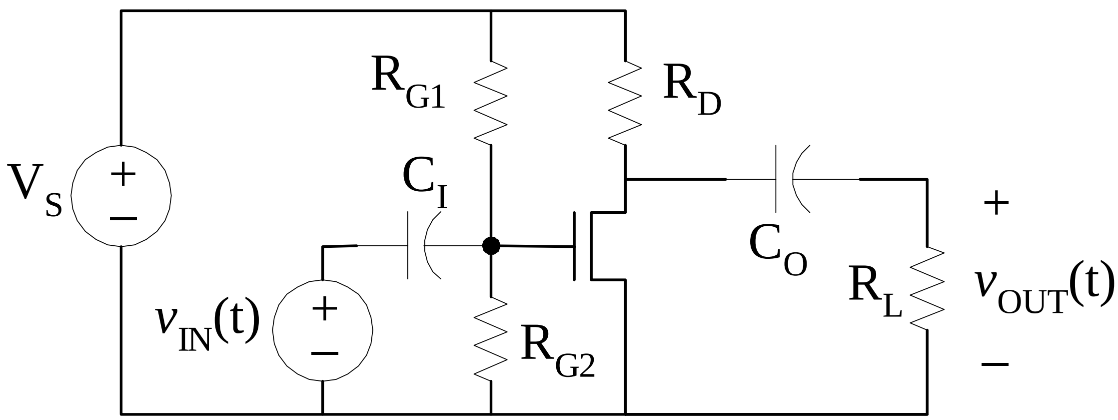
(C) Assume that $L = 365\ \mu$H. Over what range of capacitance must C vary so that the frequency of maximum V/I may be tuned over the entire AM band. Note that tuning the frequency of maximum V/I to the center frequency of a particular station tunes in that station.

(D) As a compromise between passing all frequencies within 5 kHz of a center frequency and rejecting all frequencies outside that band, let the design of R be such that $V(1\text{ MHz} \pm 5\text{ kHz})/V(1\text{ MHz}) \approx 0.25$ when the tuner is tuned to 1 MHz. Given this design criterion, determine R.

(E) Given your design for R, determine $V(1\text{ MHz} \pm 10\text{kHz})/V(1\text{ MHz})$. Also, determine Q for the tuner and its load resistor when the tuner is tuned to 1 MHz.

(F) Suppose the tuner is first tuned to another station and then quickly tuned to the station broadcasting at 1 MHz. Approximately how long will it take for $v(t)$ to depend primarily on the signal from the station broadcasting at 1 MHz. Assume that both stations broadcast signals of equal strength. Hint: consider the time-domain interpretation of Q.

Problem 10.3: This problem studies capacitive coupling as used by the amplifier shown below. The amplifier employs both input and output capacitive coupling. That is, the input voltage source v_{IN} is coupled to the amplifier through the capacitor C_{I}, and the load resistor R_{L} is coupled to the amplifier through the capacitor C_{O}. For the purposes of this problem, let $v_{\mathrm{IN}}(t) = V_{\mathrm{IN}} + v_{\mathrm{in}}(t)$ and $v_{\mathrm{OUT}}(t) = V_{\mathrm{OUT}} + v_{\mathrm{out}}(t)$, where V_{IN} and V_{OUT} are the constant large-signal bias components of $v_{\mathrm{IN}}(t)$ and $v_{\mathrm{OUT}}(t)$, respectively, and $v_{\mathrm{in}}(t)$ and $v_{\mathrm{out}}(t)$ are the time-varying small-signal components of $v_{\mathrm{IN}}(t)$ and $v_{\mathrm{OUT}}(t)$, respectively.



(A) Let $v_{\mathrm{in}}(t) = 0$. Determine V_{GS} for the MOSFET, and V_{OUT}. That is, determine the steady-state (constant) bias values of $v_{\mathrm{GS}}(t)$ and $v_{\mathrm{OUT}}(t)$ given that $v_{\mathrm{IN}}(t) = V_{\mathrm{IN}}$.

(B) In view of your answer to Part A, is it necessary, or even useful, to bias the input with a nonzero V_{IN}?

(C) Assume that the amplifier is designed so that the MOSFET is biased into operation within its saturation region. In this case, develop a small signal model for the amplifier that can be used to determine $v_{\mathrm{out}}(t)$ from $v_{\mathrm{in}}(t)$. Include the MOSFET capacitance C_{GS} as part of the model. Clearly label all component values in the model. Hint: review your solutions to Problems 4.3 and 5.2.

(D) Assume that $v_{\mathrm{in}} = V_{\mathrm{in}} \cos(\omega t)$, and correspondingly that $v_{\mathrm{out}} = V_{\mathrm{out}} \cos(\omega t + \phi)$. Using the small-signal model from Part C, determine V_{out} and ϕ. Hint: you may find it easiest to first find the small signal v_{gs} from v_{in}, next find v_{out} from v_{gs}, and finally combine the two earlier results find v_{out} from v_{in}.