Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Spring 2004

Quiz #3

28 April 2004

Name: SOLUTIONS

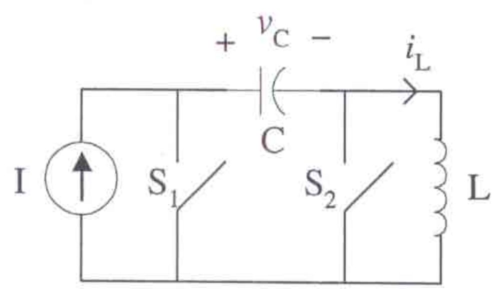
Instructor:	Kassakian	Kassakian	Wilson	Berggren	Berggren
Time:	10	11	12	1	2

- Please put your name in the space provided above, and circle the name of your recitation instructor together with the time of your recitation.
- Do your work for each question within the boundaries of the question. When finished, write your answer to each question in the corresponding answer box that follows the question.
- This is a closed-book quiz, but calculators are allowed.
- Graded quizzes will be returned in tutorial on Monday May 3, and also in Recitation on Wednesday May 5. If you do not attend tutorial and/or recitation on those days, then it is your responsibility to get your quiz from your recitation instructor. You will have until Monday May 17 to request a quiz grading review, regardless of whether or not you attend tutorial on Monday May 3 and take back your quiz. If you wish to have your quiz grade reviewed, you must return your quiz to your recitation instructor, within the two week period, together with a written explanation of why you think a grading mistake was made. This is the only way in which a quiz grade will be reviewed.
- Good luck!

Problem 1	Problem 2	Problem 3	Total Grade

Problem 1 – 20%

This problem focuses on the circuit shown below, which contains a constant current source, two switches, a capacitor and an inductor. Prior to $t = 0$, the two switches are closed, and the capacitor and inductor are both at rest.



(1A) (5%) At $t = 0$, switch S1 opens. Determine the capacitor voltage v_C and the inductor current i_L at $t = T$.

$v_C(T)$:	$\frac{I\ T}{C}$	$i_L(T)$:	0

Capacitor charges.

Inductor remains shorted.

$$V_c = \frac{Q}{C} = \frac{IT}{C}$$

(1B) (5%) At $t = T$, switch S2 opens and then switch S1 closes immediately thereafter. Determine the time at which the capacitor voltage v_C next goes to zero.

t: $T + \dfrac{\pi\sqrt{LC}}{2}$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\text{Period} = 2\pi\sqrt{LC}$$

$$\frac{1}{4}\ \text{Period} = \frac{\pi}{2}\sqrt{LC}$$

(1C) (5%) Determine the inductor current i_L when the capacitor voltage v_C goes to zero as described in Part B.

i_L: $-\dfrac{IT}{\sqrt{LC}}$

Energy Conservation $\Rightarrow$ $\underbrace{i_L}_{\text{Peak}} = \pm\sqrt{\frac{C}{L}}\,\underbrace{V_C}_{\text{Peak}}$

$\uparrow$

$\frac{IT}{C}$

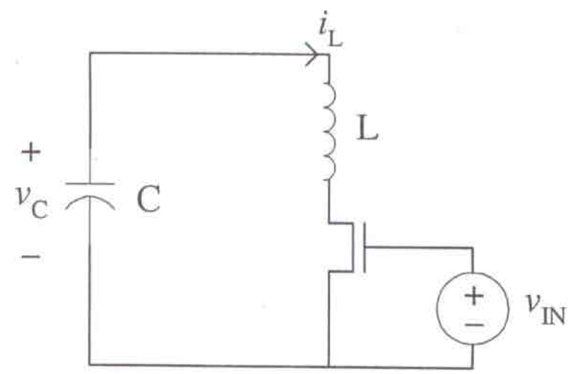
(1D) (5%) When the capacitor voltage v_C goes to zero as described in Part B, switch S2 closes and then switch S1 opens immediately thereafter. After a further period of duration T, switch S2 opens and then switch S1 closes immediately thereafter. Determine the inductor current i_L when the capacitor voltage v_C next goes to zero.

i_L:	$-\sqrt{2}\,\dfrac{IT}{\sqrt{LC}}$

Two cycles $\Rightarrow$ double energy storage.

Problem 2 – 35%

The circuit shown below contains a capacitor, an inductor and a MOSFET. Its purpose is to deliver a pulse of current to the inductor. Model the MOSFET as a switch having on-state resistance R_{ON}. Prior to $t = 0$, $v_C = V$, $i_L = 0$, and the MOSFET is held off by v_{IN}. For $t \geq 0$ the MOSFET is turned on by v_{IN}.



(2A) (15%) Derive an expression for the inductor current i_L for $t \geq 0$. In doing so, assume that C satisfies $C < 4L/R_{ON}^2$.

$i_L(t \geq 0)$: $\frac{V}{L\omega} e^{-\alpha t} \sin(\omega t)$ $\qquad \alpha \equiv R_{ON}/2L$, $\quad \omega \equiv \sqrt{1/LC - (R_{ON}/2L)^2}$

$$R_{ON}\, i_L + L\frac{di_L}{dt} + \frac{1}{C}\int_{-\infty}^{t} i_L\, dt = 0$$

$$\frac{d^2 i_L}{dt^2} + \frac{R_{ON}}{L}\frac{di_L}{dt} + \frac{1}{LC} i_L = 0$$

$$i_L \sim e^{st} \Rightarrow s = -\underbrace{\frac{R_{ON}}{2L}}_{\alpha} \pm j\underbrace{\sqrt{\frac{1}{LC} - \left(\frac{R_{ON}}{2L}\right)^2}}_{\omega}$$

$$\Rightarrow i_L = A_s e^{-\alpha t}\sin(\omega t) + A_c e^{-\alpha t}\cos(\omega t)$$

$$i_L(0) = 0 \Rightarrow A_c = 0$$

$$\frac{di_L}{dt}(0) = \frac{V}{L} \Rightarrow A_s = \frac{V}{L\omega}$$

Initial Conditions

(2B) (10%) It is desired that the inductor current i_L be a single pulse of minimal time duration with no reversal in sign. Determine the value of C that achieves this objective; C need not satisfy $C < 4L/R_{ON}^2$.

C: $\dfrac{4L}{R_{ON}^2}$

No sign reversal $\Rightarrow$ 2 real roots for s.

Minimal duration $\Rightarrow$ Both real roots for s are equal, otherwise the slow one can be made faster

Requirement $\Rightarrow$ Critical damping $\Rightarrow$ $\omega = 0$

$$\Rightarrow \quad C = \frac{4L}{R_{ON}^2}$$

(2C) (10%) Determine the peak, or maximum positive value, of the inductor current i_L given the design of Part B.

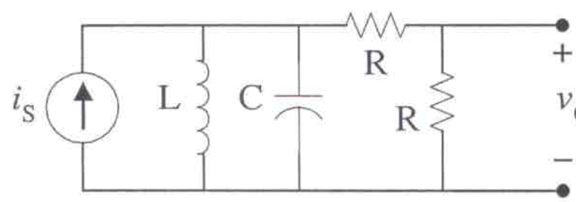
Peak i_L:	$\dfrac{2V}{e R_{ON}}$

$\lim \omega \to 0 \Rightarrow i_L(t) = \frac{Vt}{L} e^{-\alpha t}$ ... From Part A.

$\frac{di_L}{dt} = 0 \Rightarrow t = 1/\alpha \Rightarrow \text{Peak } i_L = \frac{2V}{e R_{ON}}$

Problem 3 – 45%

This problem focuses on the network shown below. It contains a current source, a capacitor, an inductor and two identical resistors. It also has a single port. Except for the last part of this problem, the network operates in the sinusoidal steady state with the sourced current taking the form $i_S(t) = I\cos(\omega t)$. For the last part of this problem, the network is at rest until the sourced current takes a step of the form $i_S(t) = Iu_{-1}(t)$.



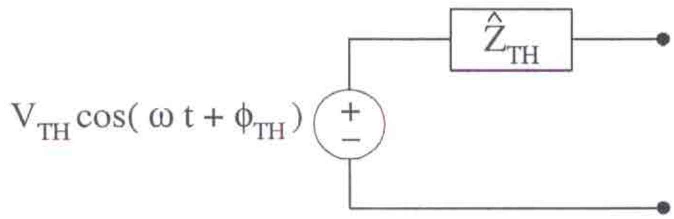
(3A) (10%) The output voltage v_O at the port takes the form $v_O(t) = V\cos(\omega t + \phi)$ in sinusoidal steady state. Determine V and ϕ.

V: $\dfrac{\omega L I/2}{\sqrt{(1-\omega^2 LC)^2 + (\omega L/2R)^2}}$ $\qquad\phi$: $\operatorname{Tan}^{-1}\left(\dfrac{1-\omega^2 LC}{\omega L/2R}\right)$

$$\hat{V} = \frac{1}{\frac{1}{j\omega L} + j\omega C + \frac{1}{2R}} \, I \cdot \frac{1}{2}$$

$$= \frac{j\omega L/2R}{1-\omega^2 LC + j\omega L/2R} \cdot R I$$

(3B) (10%) The sinusoidal-steady-state Thevenin equivalent of the network, as viewed from its port is shown below. Determine the parameters V_{TH}, ϕ_{TH} and $\hat{Z}_{\mathrm{TH}}$ that define the equivalent. Note that $\hat{Z}_{\mathrm{TH}}$ is the complex Thevenin-equivalent impedance.

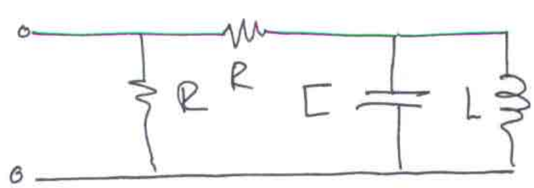


V_{TH}: V From Part A

ϕ_{TH}: ϕ From Part A

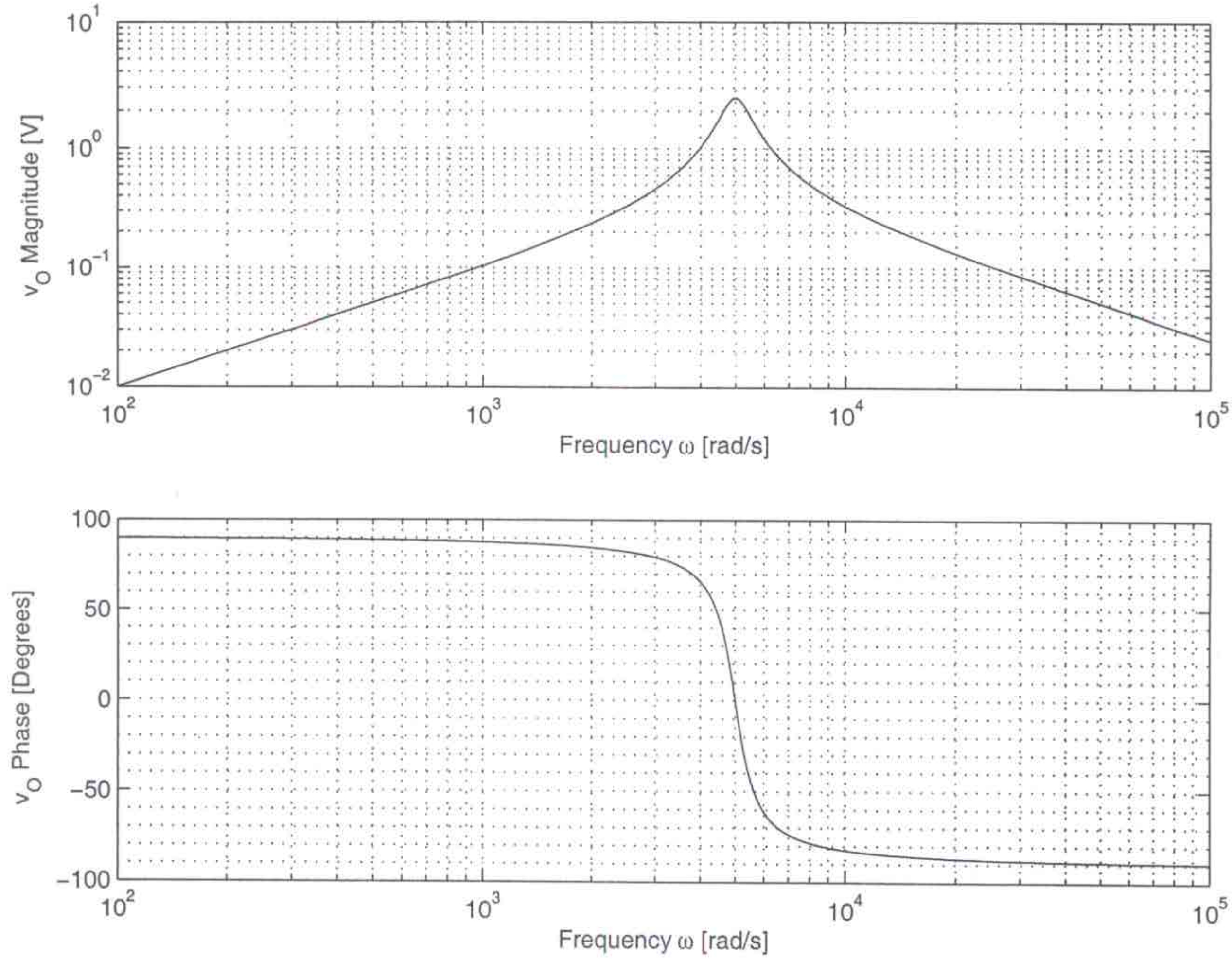
$\hat{Z}_{\mathrm{TH}}$: $\dfrac{R\left(R + \dfrac{j\omega L}{1-\omega^2 LC}\right)}{2R + \dfrac{j\omega L}{1-\omega^2 LC}}$

$Z_{TH} \Rightarrow$



$$R \parallel \left(R + \frac{j\omega L}{1-\omega^2 LC}\right)$$

The figures shown below present a numerical evaluation of the magnitude V and the phase ϕ of v_O as functions of frequency ω for the specific case of $I = 100$ mA. Use this numerical evaluation to answer Parts C and D of this problem.



(3C) (15%) Determine the numerical values of R, L and C. Remember to include units.

R:	$25\,\Omega$	L:	$2\,mH$	C:	$20\,\mu F$

Low Frequency Assymptote @ $\omega = 10^2\ \frac{rad}{s}$

$$\overset{10^{-2}}{|V_o|} \approx \frac{\overset{10^{2}}{\omega} L \overset{10^{-1}}{I}}{2} \quad \Rightarrow \quad L = 2\,mH$$

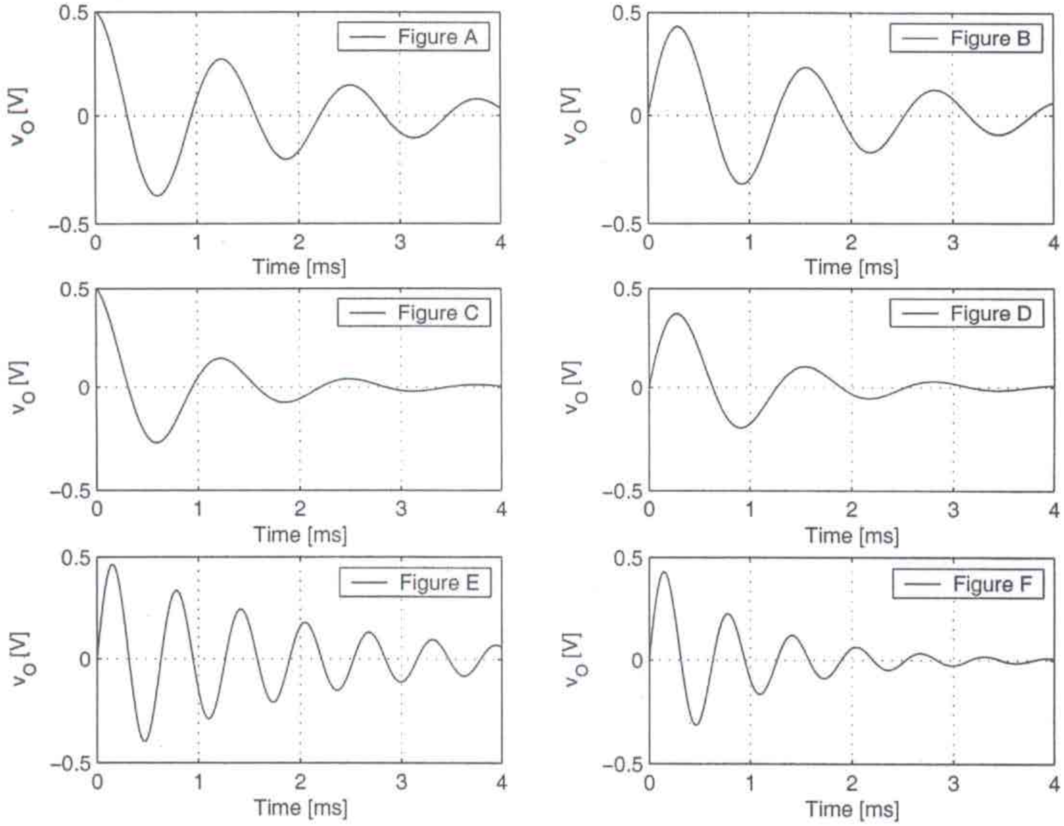
High Frequency Assymptote @ $\omega = 5 \cdot 10^4\ \frac{rad}{s}$

$$\overset{5\cdot 10^{-2}}{|V_o|^2} \approx \frac{\overset{10^{-1}}{I}}{2\underset{5\cdot 10^{4}}{\omega} C} \quad \Rightarrow \quad C = 20\,\mu F$$

Peak @ $\omega = 5 \cdot 10^3\ \frac{rad}{s} = \frac{1}{\sqrt{LC}}$

$$\overset{2.5}{|V_o|} = R\overset{10^{-1}}{I} \quad \Rightarrow \quad R = 25\,\Omega$$

(3D) (10%) Assuming the network is at initially rest, which of the following figures corresponds to v_O for $t \geq 0$ given that $i_S(t) = 100$ mA $u_{-1}(t)$?



Circle One: A B C D E F

Initial Voltage = 0 ... Rules out A & C.

Period = $\frac{2\pi}{5000}$ s > 1 ms ... Rules out E & F.

$\frac{1}{e}$ Decay Time = 4RC = 2 ms ... Rules out C & D.

Elimination $\Rightarrow$ B.