MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits
Fall 2002

Final Exam

Name: ______________________ Recitation Section: __________

Recitation Instructor: __________ Teaching Assistant: __________

Enter all your work and your answers directly in the spaces provided on the printed pages. Make sure that your name is on all sheets. Use the backs of the printed pages as scratch paper, but we will only give full credit to answers that you neatly transfer to the spaces on the printed pages. Answers must be derived or explained, not just simply written down. The quiz is closed book, but **calculators and both sides of one $8\frac{1}{2}'' \times 11''$ page of notes are allowed.**

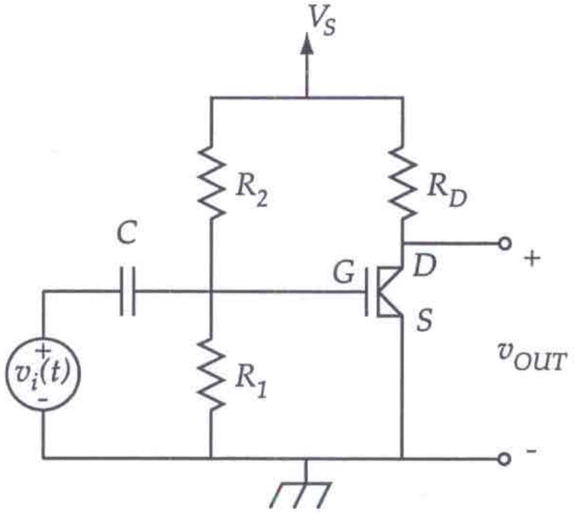
This quiz contains 16 pages including the cover sheet. Make sure that your quiz contains all 16 pages and that you hand in all pages except the last page.

Problem	Points	Grade	Grader
1	20		
2	20		
3	16		
4	20		
5	24		
Total	100		

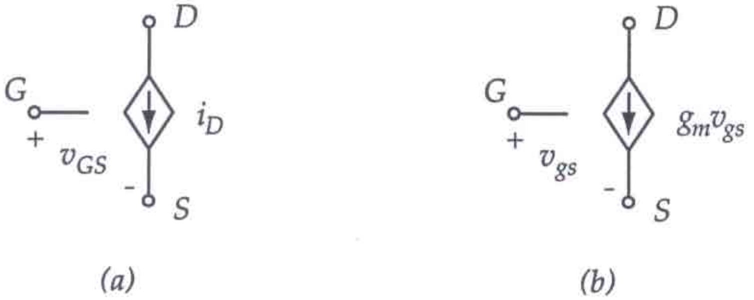
Problem 1: (*20 points*) The circuit below shows a single-stage amplifier designed around a "MOXFET", a hypothetical transistor similar to a MOSFET, except that

$$i_D = \frac{K}{3}(v_{GS} - V_T)^3$$

For the purposes of this problem, assume that the MOXFET gain parameter $K = 0.625mA/V^3$ and that its threshold voltage is $V_T = 2.2V$.



In the circuit above, $V_S = 10V$, $R_1 = 30k\Omega$, $R_2 = 70k\Omega$, $R_D = 37.5k\Omega$ and $C = 0.15\mu F$. The saturation-region models for the MOXFET are shown below, where (a) is the large-signal (bias) model, and (b) is the small-signal (incremental) model.



(A) The amplifier output voltage will be of the form $v_{OUT} = V_{OUT} + v_{out}(t)$. Determine V_{OUT} (a numerical value is required).

$V_{OUT} =$ ____________ V

(B) Find an expression for the transconductance g_m in terms of the symbolic MOXFET parameters V_T and K and the operating point V_{GS}. (Do **not** evaluate your answer numerically.)

$g_m =$ ____________

(C) For an input signal of the form $v_i(t) = V_I \cos \omega t$, the complex amplitude of the output signal $v_{out}(t)$ will be of the form

$$\hat{V}_{out} = G_0 \left[\frac{j\omega\tau}{1 + j\omega\tau} \right] V_I$$

In terms of the transconductance g_m and the circuit parameters, find expressions for the high-frequency gain G_0 and the time constant τ. (Do **not** evaluate your answers numerically.)

$G_0 =$ ____________

$\tau =$ ____________

(D) The small input signal is now given by $v_i(t) = (V_a \cos \omega t + V_b)u(t)$. The resulting output signal has the form

$$v_{out}(t) = (V_c \cos(\omega t + \phi) + V_d + V_e e^{-\beta t})u(t)$$

Determine V_c, ϕ, V_d, V_e and β in terms of ω, V_a, V_b, τ and G_0, where τ and G_0 are defined in part (C).

$V_c =$ ____________

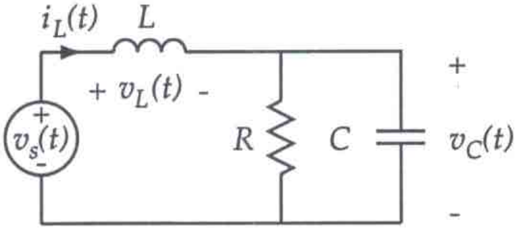
$\phi =$ ____________

$V_d =$ ____________

$V_e =$ ____________

$\beta =$ ____________

Problem 2: (*20 points*) Consider the RLC circuit shown below.



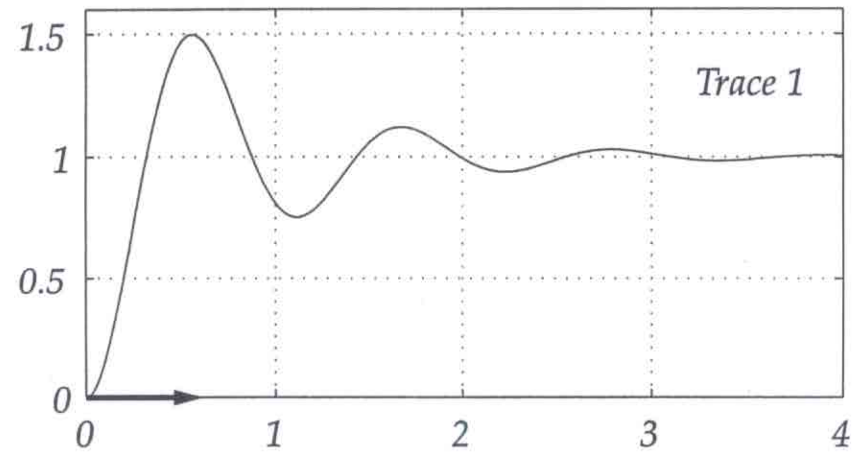
(A) The voltage source is a unit step at time $t = 0$. Three circuit variables, v_C, v_L, and i_L are presented plotted against time, as shown in the three figures on the next page. Time is measured in milliseconds on the horizontal axes. The vertical axes may represent voltage, measured in Volts, or current, measured in Amperes, as appropriate.

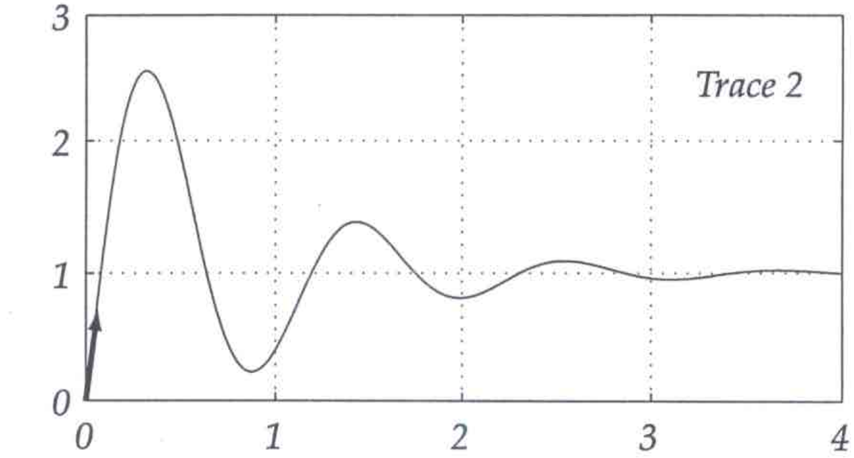
The traces are labeled 1, 2, and 3. (The label is in the upper right corner of each trace.) You are to identify the traces: which trace is which circuit variable? Circle the circuit variable that corresponds to the indicated trace below:

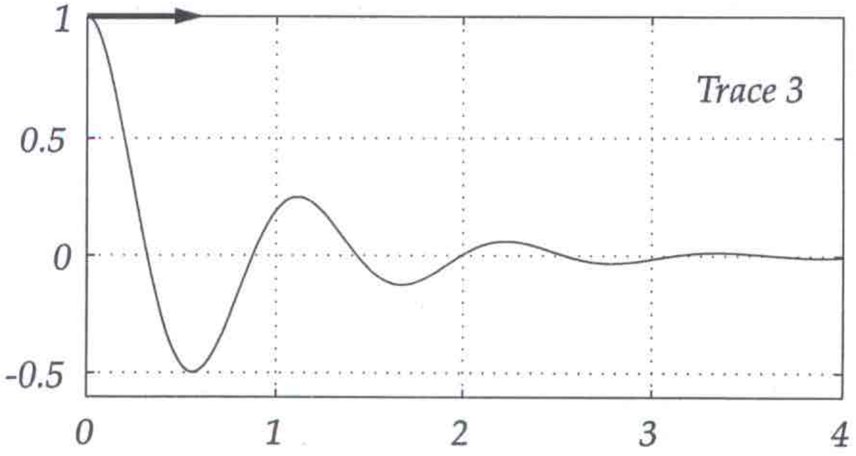
Trace 1: v_C v_L i_L

Trace 2: v_C v_L i_L

Trace 3: v_C v_L i_L







Note: Time is measured in milliseconds on the horizontal axes. The vertical axes may represent voltage, measured in Volts, or current, measured in Amperes, as appropriate. The arrows represent the slope of the curve at $t = 0^+$.

(B) Assume the inductance L is $75\mu H$.

(i) What is the approximate numerical value of the capacitance?

$C =$ ____________

(ii) What is the approximate numerical value of the resistance?

$R =$ ____________

(iii) Estimate the Q of the circuit. Circle the best answer:

$Q < 1$ $\quad 1 < Q < 10$ $\quad 10 < Q$

(C) For each question circle the correct completion.

(i) If the resistance R is increased, the Q

increases. **decreases.** **remains the same.**

(ii) If the capacitance C is increased, the Q

increases. **decreases.** **remains the same.**

(iii) If the inductance L is increased, the oscillatory period

increases. **decreases.** **remains the same.**

(iv) If the capacitance C is increased, the oscillatory period

increases. **decreases.** **remains the same.**

(D) After a long time T, v_s returns to zero and the stored energy decays.

(i) If the capacitance C is increased, the time at which the stored energy falls to half its value at time T

increases. **decreases.** **remains the same.**

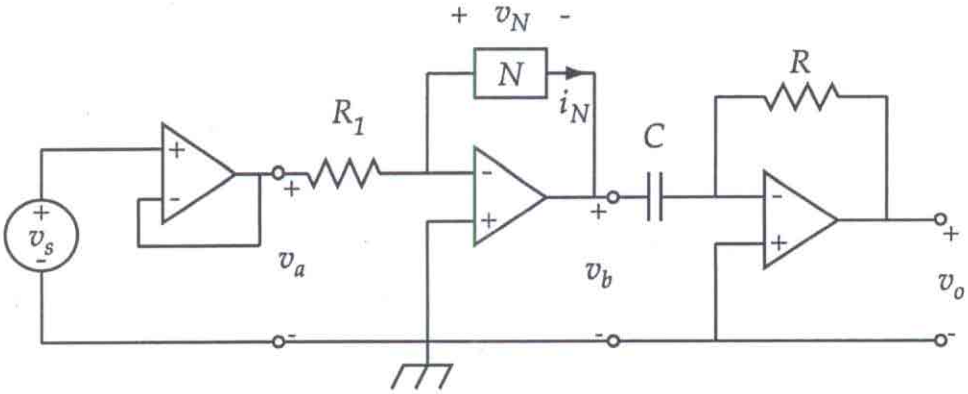
(ii) If the inductance L is increased, the time at which the stored energy falls to half its value at time T

increases. **decreases.** **remains the same.**

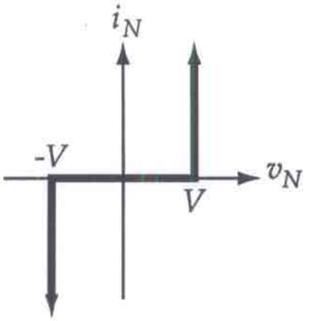
(iii) If the resistance R is increased, the time at which the stored energy falls to half its value at time T

increases. **decreases.** **remains the same.**

Problem 3: (*16 points*)

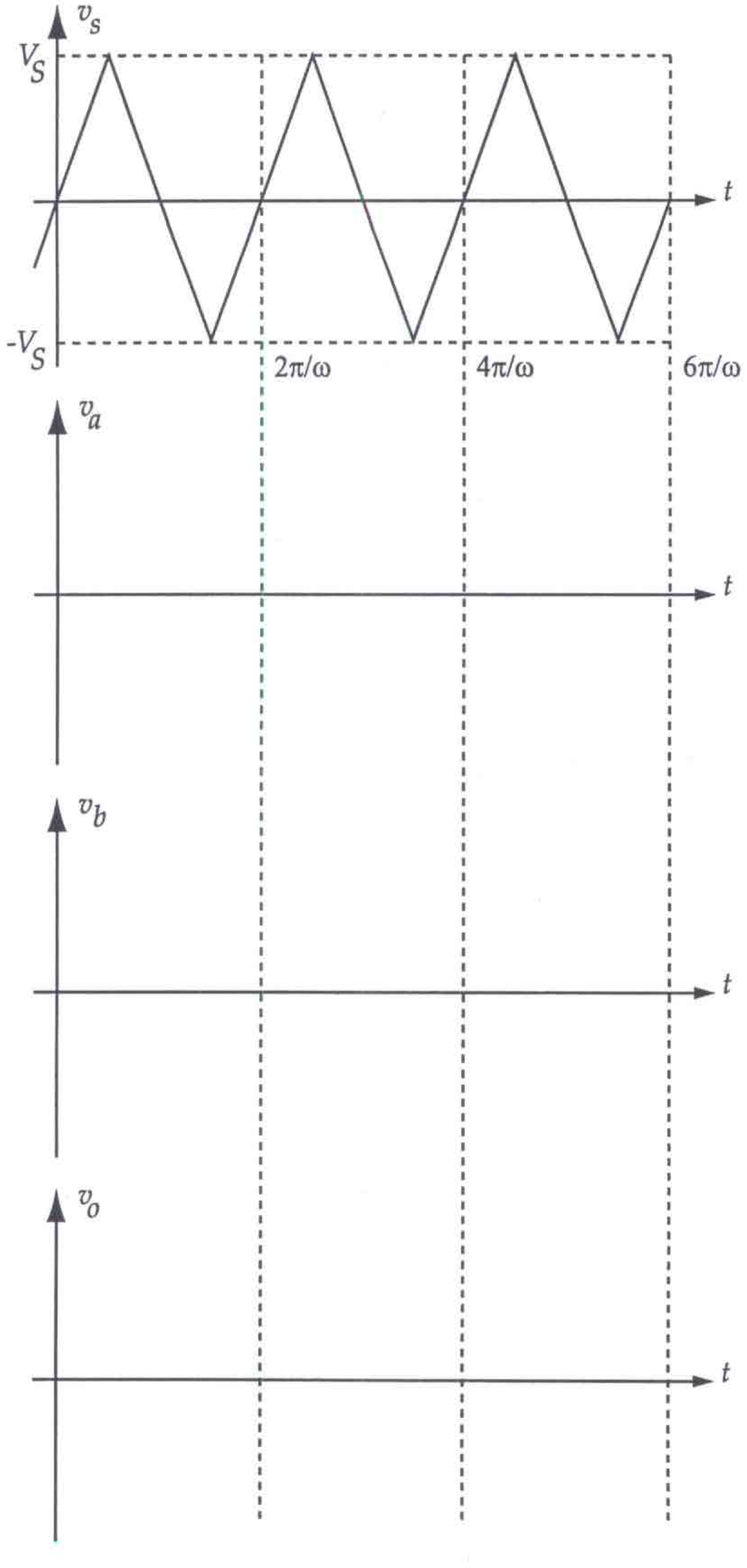


The source $v_s(t)$ has the waveform shown in the graph on the next page. The nonlinear element N has the i-v relation sketched below. Assume that the op-amps are ideal (i.e., infinite gain, no input current, zero output resistance, no saturation).

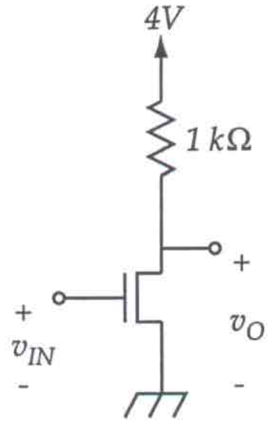


On the axes provided, sketch the waveforms of $v_a(t)$, $v_b(t)$ and $v_o(t)$. Follow the time divisions indicated in the graph and label the amplitude of your waveforms. If any of your graphs contains impulses, be sure to indicate their area.

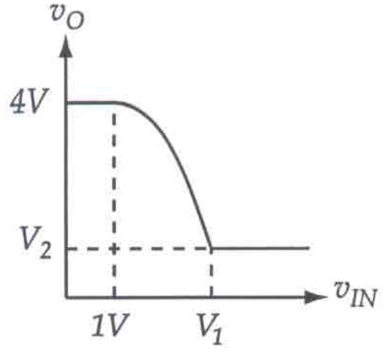
Name: ______________________________



Problem 4: *(20 points)*



(a)



(b)

A simple MOSFET inverter is shown above. The input-output relationship is also shown above **(NOT to scale)**, where the MOSFET has been modeled using the familiar square-law relationship

$$i_{DS} = \frac{K}{2}(v_{GS} - V_T)^2$$

in the saturation region, and the triode region is compressed onto the single curve

$$i_{DS} = \frac{K}{2}v_{DS}^2$$

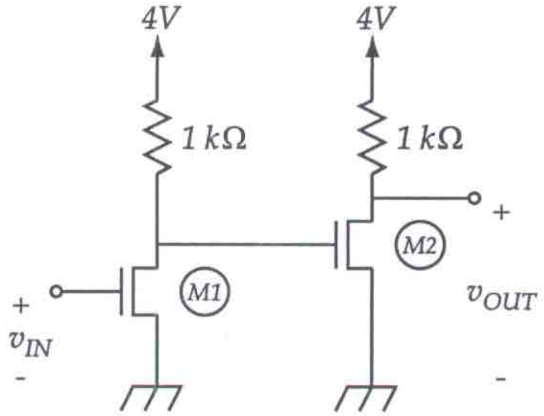
The MOSFET parameters are $V_T = 1V$ and $K = 1mA/V^2$.

(A) Determine the voltages V_1 and V_2 in the input-output graph for this inverter.

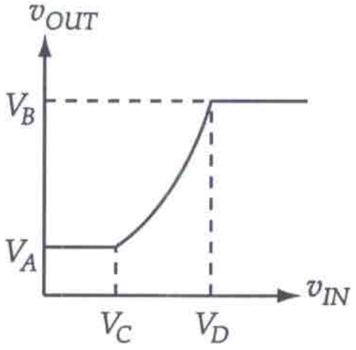
$V_1 =$ ____________ V

$V_2 =$ ____________ V

Two identical inverters from part (A) are cascaded to form the buffer circuit shown below.



The input-output relationship is qualitatively sketched below **(NOT to scale)**.



(B) Determine the voltages V_A, V_B, V_C and V_D.

$V_A =$ ____________ V

$V_B =$ ____________ V

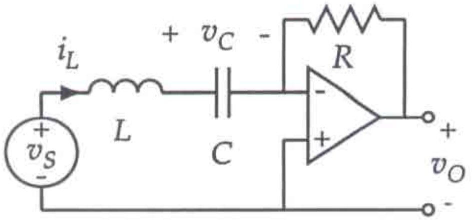
$V_C =$ ____________ V

$V_D =$ ____________ V

(C) For a given input voltage each transistor in the buffer circuit will be in one of three regions of operation: cutoff (CO), saturation (SAT) and triode region (TR). As the input voltage v_{IN} is increased from 0 to $4V$, the buffer passes through four states. In the following table, indicate the four states by circling the region of operation of the transistors for each range of input voltage.

Input Voltage	M1			M2		
$0V < v_{IN} < V_T$	CO	SAT	TR	CO	SAT	TR
$V_T < v_{IN} < V_C$	CO	SAT	TR	CO	SAT	TR
$V_C < v_{IN} < V_D$	CO	SAT	TR	CO	SAT	TR
$V_D < v_{IN} < 4V$	CO	SAT	TR	CO	SAT	TR

Problem 5: (*24 points*) Match the indicated variable to the form of its response for $t > 0^+$ by circling the appropriate letter corresponding to the choices offered in the last page. *Note: The parameters* V_S, I_S, Λ_S, Q_S, V_0 *and* I_0 *are all positive.*

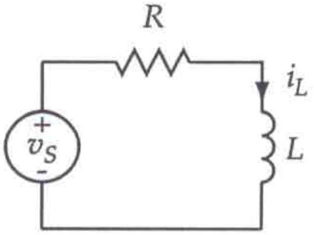


$v_S = V_S u(t)$
$i_L(0^-) = 0 \qquad v_C(0^-) = 0$

Form of $v_O(t)$:

a b c d e f

g h i j k l

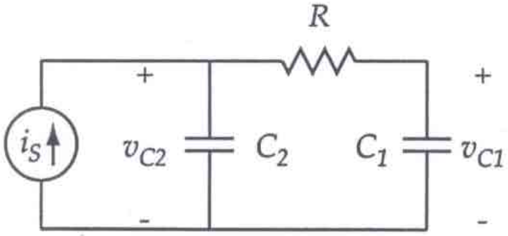


$v_S = \Lambda_S \delta(t)$
$i_L(0^-) = 0$

Form of $i_L(t)$:

a b c d e f

g h i j k l

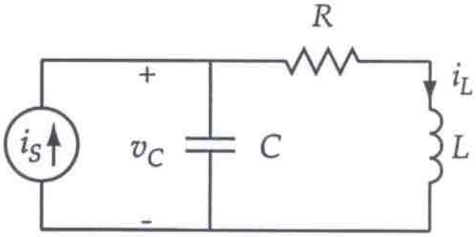


$i_S = Q_S \delta(t)$
$v_{C1}(0^-) = 0 \qquad v_{C2}(0^-) = 0$

Form of $v_{C1}(t)$:

a b c d e f

g h i j k l

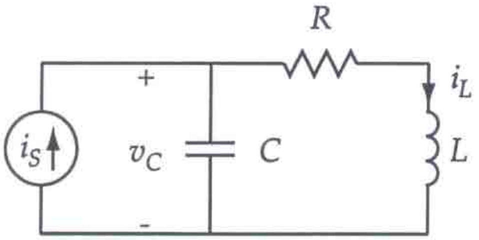


$i_S = I_S u(t) \qquad \frac{1}{\sqrt{LC}} > \frac{R}{2L}$
$i_L(0^-) = 0 \qquad v_C(0^-) = 0$

Form of $i_L(t)$:

a b c d e f

g h i j k l

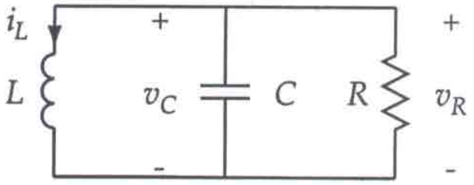


$i_S = Q_S\delta(t)$ $\frac{1}{\sqrt{LC}} > \frac{R}{2L}$
$v_C(0^-) = 0$ $i_L(0^-) = 0$

Form of $i_L(t)$:

a b c d e f

g h i j k l

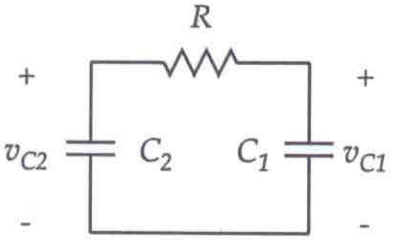


$i_L(0^-) = I_0$ $\frac{1}{\sqrt{LC}} > \frac{1}{2RC}$
$v_C(0^-) = 0$

Form of $v_R(t)$:

a b c d e f

g h i j k l

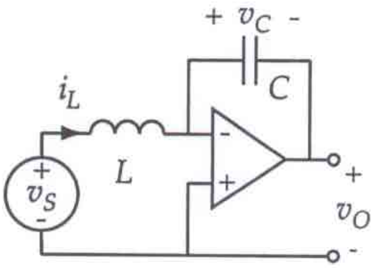


$v_{C1}(0^-) = V_0$
$v_{C2}(0^-) = 0$

Form of $v_{C1}(t)$:

a b c d e f

g h i j k l



$v_S = V_S u(t)$
$i_L(0^-) = 0$ $v_C(0^-) = 0$

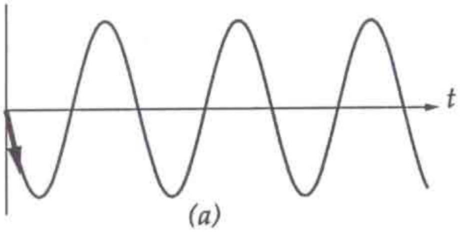
Form of $v_O(t)$:

a b c d e f

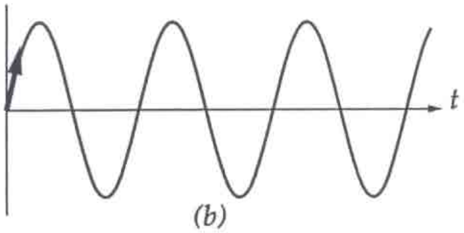
g h i j k l

NOTE: YOU MAY TEAR THIS PAGE OFF. ALL WRITING ON THIS PAGE WILL BE IGNORED.

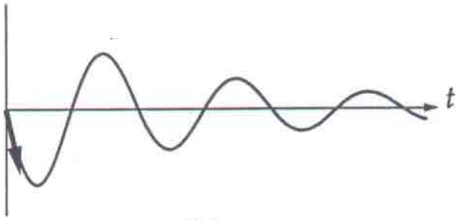
The waveforms may represent current or voltages as a function of time for $t > 0^+$. The arrows represent the slope of the curve at $t = 0^+$.



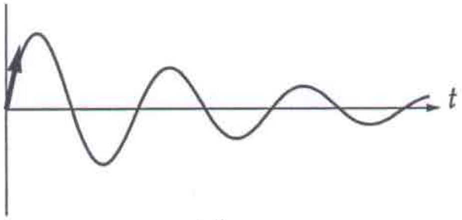
(a)



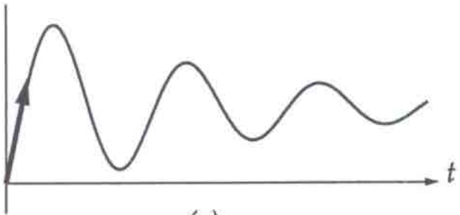
(b)



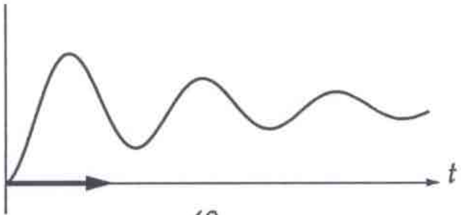
(c)



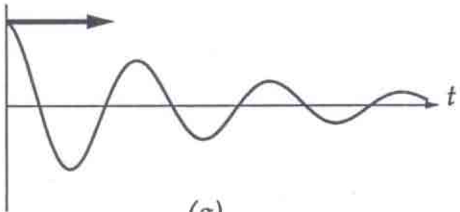
(d)



(e)



(f)



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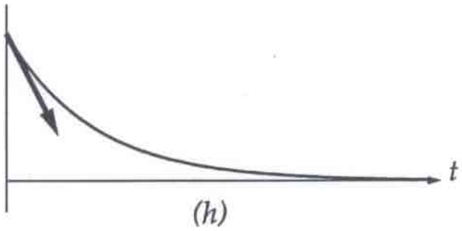

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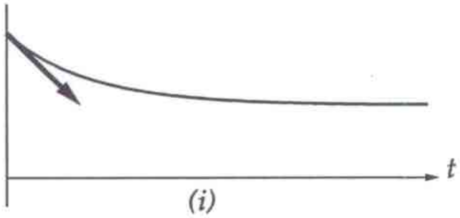
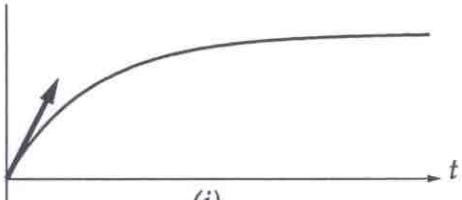
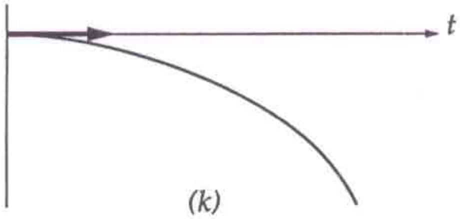


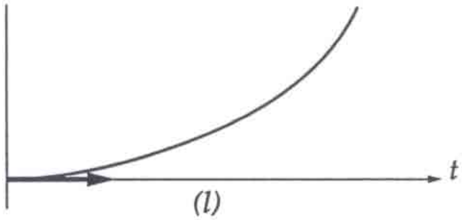
(i)



(j)



(k)



(l)