

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits
Fall 2002

Quiz 1 Solutions

Name: _____ Recitation Section: _____

Recitation Instructor: _____ Teaching Assistant: _____

Enter all your work and your answers directly in the spaces provided on the printed pages. Make sure that your name is on all sheets. Use the backs of the printed pages as scratch paper, but we will only grade the work that you neatly transfer to the spaces on the printed pages. Answers must be derived or explained, not just simply written down. The quiz is closed book, but **calculators are allowed**.

This quiz contains 9 pages including the cover sheet. Make sure that your quiz contains all 9 pages and that you hand in all 9 pages.

| Problem | Points | Grade | Grader |
|---------|--------|-------|--------|
| 1 | 30 | | |
| 2 | 40 | | |
| 3 | 30 | | |
| Total | 100 | | |

Problem 1: (30 points) For parts (A)-(C), use the associated branch variables as defined in Figure 1.

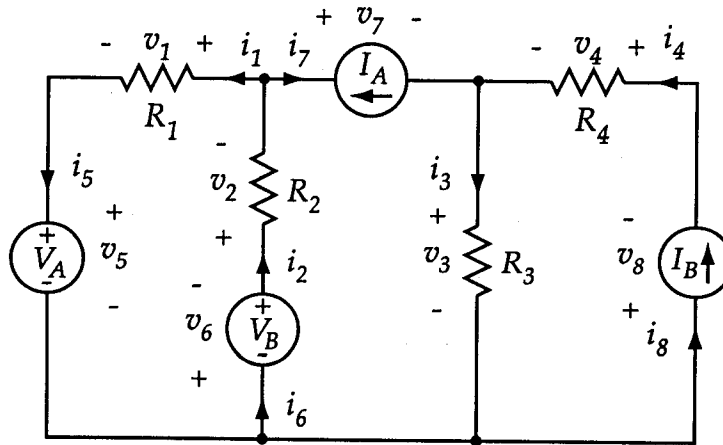


Figure 1: Circuit for Problem 1(A), 1(B) and 1(C)

(A) Write element laws for the resistor R_1 , the current source I_A , and the voltage source V_B .

$$R_1: v_1 = i_1 R_1$$

$$I_A: i_7 = -I_A$$

$$V_B: v_6 = -V_B$$

(B) Write a complete set of independent KVL equations expressed only in terms of the branch voltages v_1, v_2, \dots , and v_8 .

$$v_5 + v_1 + v_2 + v_6 = 0$$

$$v_3 + v_7 + v_2 + v_6 = 0$$

$$v_3 + v_4 + v_8 = 0$$

$$v_5 + v_1 - v_7 + v_4 + v_8 = 0$$

Others possible too...

} any 3

(C) Write a complete set of independent KCL equations expressed only in terms of the branch currents i_1, i_2, \dots , and i_8 .

$$i_1 = i_5$$

$$i_2 = i_1 + i_7$$

$$i_7 + i_4 = i_3$$

$$i_8 = i_4$$

$$i_6 = i_2$$

6 nodes,
5 independent eq's

- (D) The same circuit is shown in Figure 2, labeled for node analysis. Write out the node equations necessary to solve for the three unknown node voltages e_1 , e_2 and e_3 . **DO NOT SOLVE** these equations.

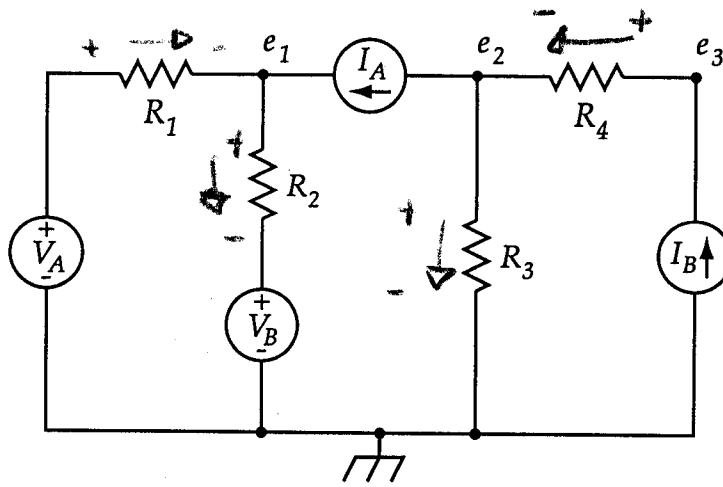


Figure 2: Circuit for Problem 1(D)

$$\frac{V_A - e_1}{R_1} - \frac{e_1 - V_B}{R_2} + I_A = 0$$

$$\frac{e_3 - e_2}{R_4} - \frac{e_2}{R_3} - I_A = 0$$

$$I_B - \frac{e_3 - e_2}{R_4} = 0$$

OR:
$$e_1 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{V_A}{R_1} + \frac{V_B}{R_2} + I_A$$

$$e_2 \left(\frac{1}{R_3} + \frac{1}{R_4} \right) - \frac{e_3}{R_4} = -I_A$$

$$\frac{e_3}{R_4} - \frac{e_2}{R_4} = I_B$$

(E) The current i_1 in Figure 3 can be written in the form:

$$i_1 = aV_A + bV_B + cI_A + dI_B$$

Determine the coefficients a , b , c and d in terms of the resistor variables in the circuit.

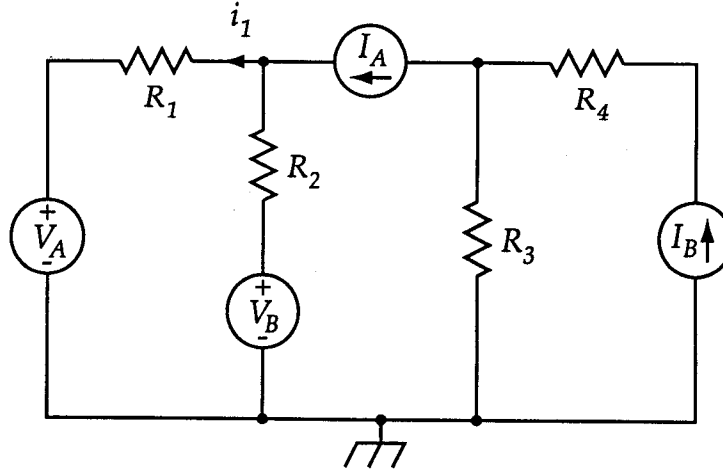
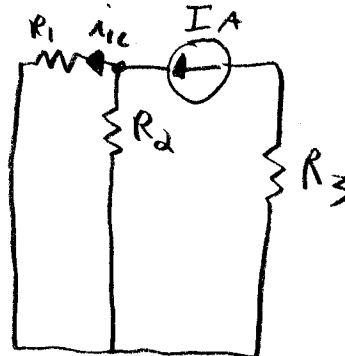
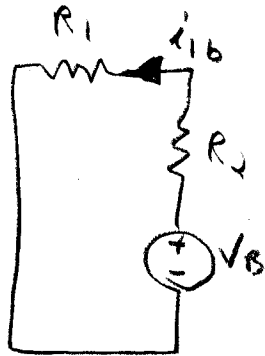
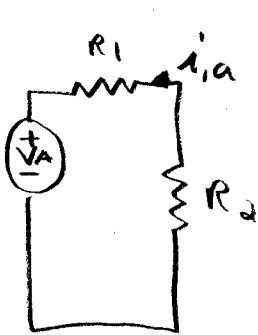


Figure 3: Circuit for Problem 1(E)

Superposition!



$$a = \frac{1}{R_1 + R_2}$$

$$b = \frac{1}{R_1 + R_2}$$

$$c = \frac{R_2}{R_1 + R_2}$$

$$d = 0$$

Problem 2: (40 points) Network 1, shown in Figure 4, is described by its $v-i$ relationship measured at the terminals. Network 2, shown in Figure 5, is described by a schematic diagram of its components.

(A) Find the Thévenin and Norton equivalent circuits that have the same $v-i$ relationship as Network 1.

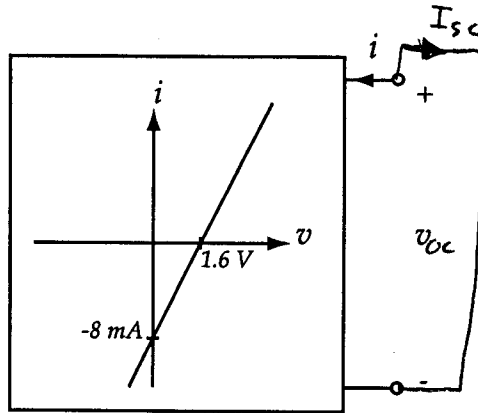


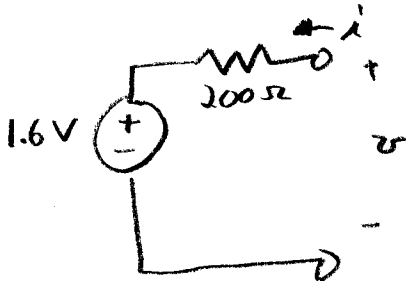
Figure 4: Network for Problem 2(A)

$$V_{OC} = 1.6 \text{ V}$$

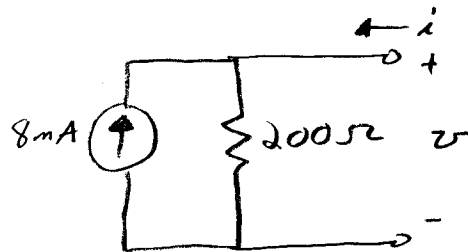
$$I_{SC} = 8 \text{ mA}$$

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = 200 \Omega$$

Thévenin:



Norton:



(B) Find the Thévenin and Norton equivalent circuits that have the same $v-i$ relationship as Network 2.

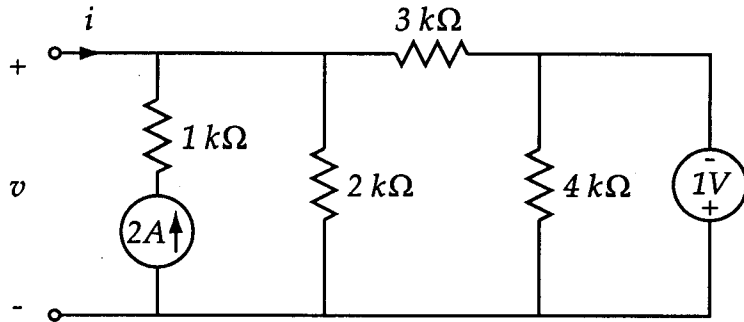
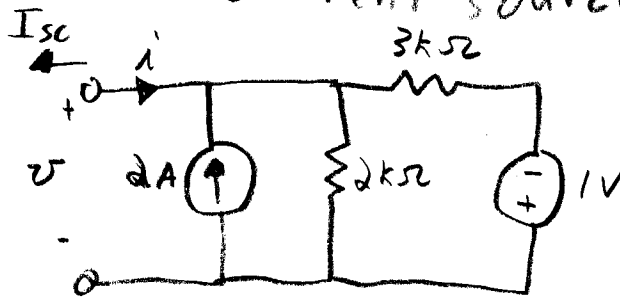


Figure 5: Network for Problem 2(B)

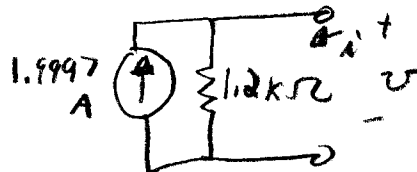
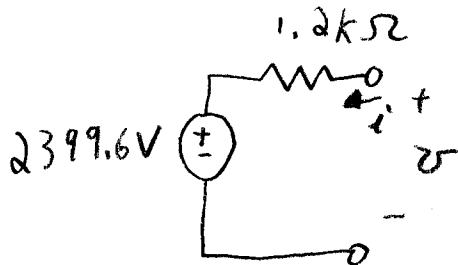
Simplify: Resistor in // with Voltage Source does nothing, neither does resistor in series with current source.



$$R_{TH} = 2k\Omega \parallel 3k\Omega = 1.2k\Omega$$

$$I_{SC} = 2A - \frac{1V}{3k\Omega} = 1.999667A$$

$$V_{OC} = 2399.6V$$



(C) Suppose that the two networks are connected together through a resistor as shown in Figure 6. Find the current i_1 and the voltage v_1 .

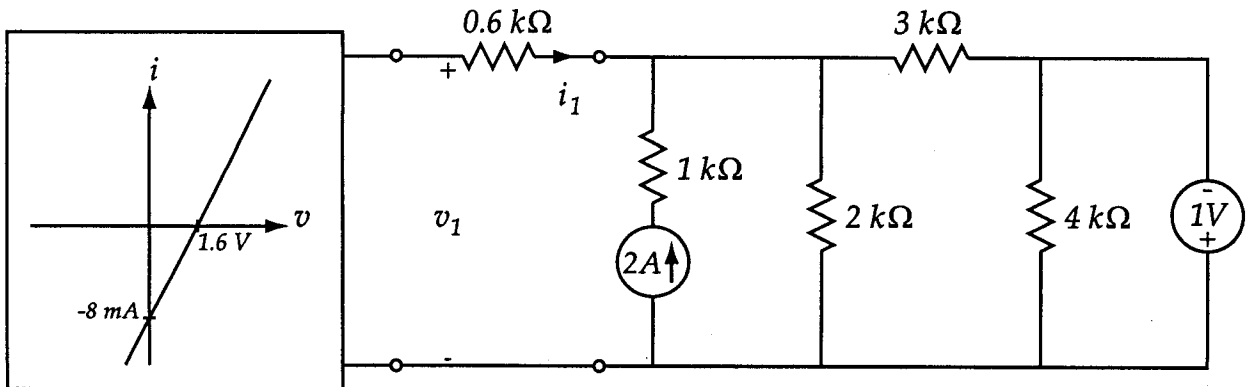
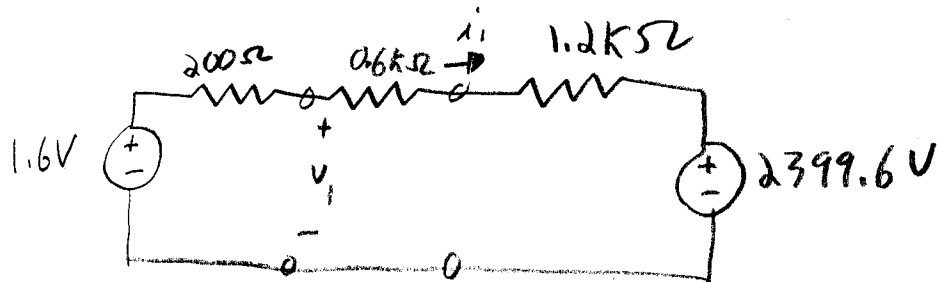


Figure 6: Network for Problem 2(C)



$$i_1 = \frac{1.6V - 2399.6V}{200 + 0.6k + 1.2k} = -1.199A$$

$$v_1 = 1.6V + (1.199A)(200\Omega) = 241.4V$$

$$i_1 = \underline{-1.199A}$$

$$v_1 = \underline{241.4V}$$

(D) Suppose that the two networks are connected together through a resistor as shown in Figure 7. Find the current i_2 and the voltage v_2 .

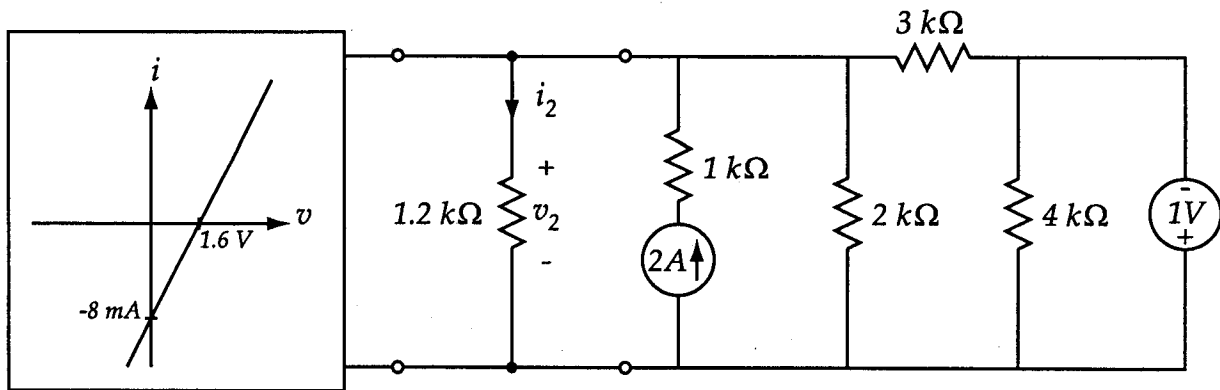
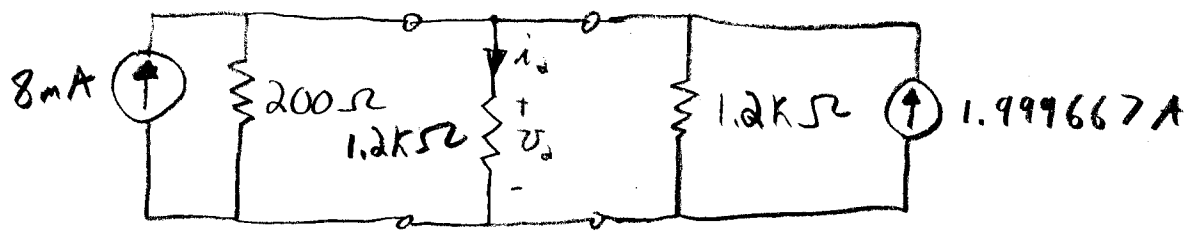


Figure 7: Network for Problem 2(D)



$$i_2 = \frac{\frac{1}{1.2k\Omega}}{\frac{1}{200\Omega} + \frac{1}{1.2k\Omega} + \frac{1}{1.2k\Omega}} (8mA + 1.999667A) = 0.2509583A \approx 251mA$$

$$v_2 = i_2(1.2k\Omega) = 301.15V$$

$$i_2 = \underline{251mA}$$

$$v_2 = \underline{301.15V}$$

Problem 3: (30 points) Determine all node potentials in the network shown in Figure 8 in terms of the conductances of the resistors ($G_A, G_B, G_C, G_D, G_E,$ and G_F), the current sources (I_C and I_F), and the voltage sources (V_A, V_B, V_D and V_E).

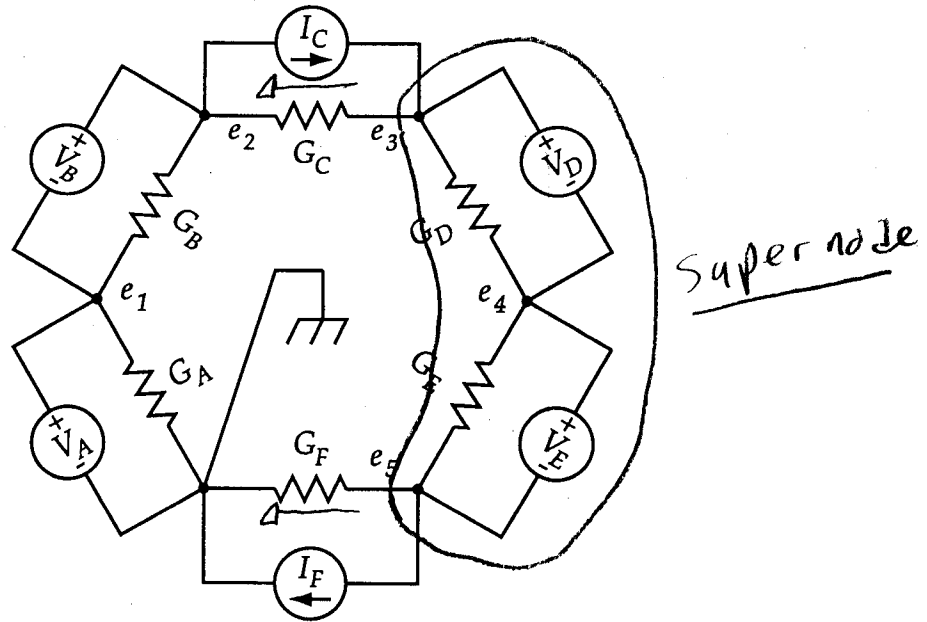


Figure 8: Circuit for Problem 3

$$e_4 = e_5 + V_E$$

$$e_3 = e_5 + V_E + V_D$$

$$I_C - (e_3 - e_2)G_C - I_F - e_5 G_F = 0$$

$$I_C - e_5 G_C - [V_E + V_D]G_C + [V_A + V_B]G_C - I_F - e_5 G_F = 0$$

$$e_1 = \frac{V_A}{G_A}$$

$$e_2 = \frac{V_A + V_B}{G_A + G_B}$$

$$e_3 = \frac{e_5 + V_E + V_D}{G_C}$$

$$e_4 = \frac{e_5 + V_E}{G_D}$$

$$e_5 = \frac{I_C - I_F + [V_A + V_B - V_E - V_D]G_C}{G_C + G_F}$$

$$e_5 [G_C + G_F] = I_C + [V_A + V_B - V_E - V_D]G_C - I_F$$