Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 Electronic Circuits
Fall 2000

Quiz 2 Solutions

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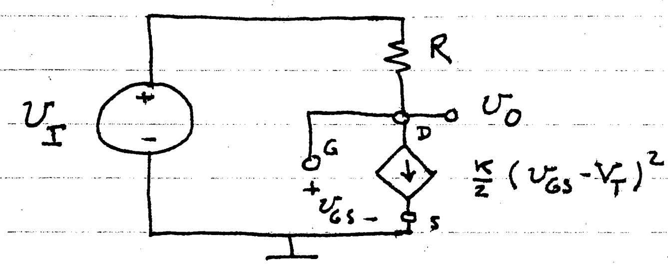
1. Replace MOSFET by its large-signal model valid in the saturation regime:



A) From the circuit, $v_O = v_I - \frac{KR}{2}(v_{GS}-V_T)^2$

but $v_{GS} = v_O$, so $v_O = v_I - \frac{KR}{2}(v_O - V_T)^2$

Plugging the numbers in: $2.5 = 5 - \frac{2 \cdot R(k\Omega)}{2}(2.5-1)^2$

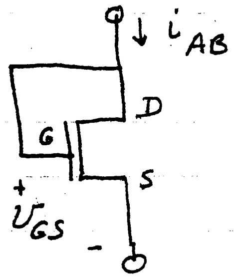
$\Rightarrow R = \frac{2.5}{(1.5)^2} = 1.11\ k\Omega$

B) Requirements for saturation are $v_{GS} > V_T$ and $v_{DS} > v_{GS} - V_T$.

$v_{GS} = v_O = 2.5\ V > V_T = 1\ V$ o.k.

$v_{DS} = v_O = 2.5\ V > v_{GS} - V_T = 1.5\ V$ o.k.

2.



In the saturation regime, the large signal model is

$$i_{AB} = \frac{K}{2}(v_{GS} - V_T)^2$$

Let $v_{GS} = V_{GS} + v_{gs}$ and expand:

$$i_{AB} = \frac{K}{2}(V_{GS} + v_{gs} - V_T)^2$$

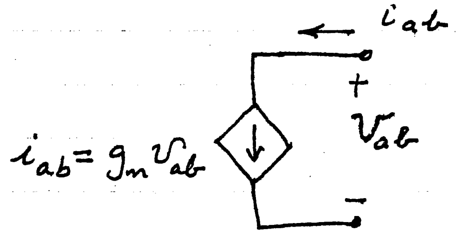
neglect

$$= \frac{K}{2}(V_{GS} - V_T)^2 + K(V_{GS} - V_T)v_{gs} + \cancel{\frac{K}{2}v_{gs}^2}$$

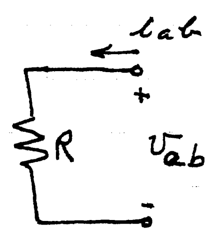
Writing $i_{AB} = I_{AB} + i_{ab}$ and equating bias and small-signal terms:

$$I_{AB} = \frac{K}{2}(V_{GS} - V_T)^2, \quad i_{ab} = K(V_{GS} - V_T)v_{gs}$$

from which we can draw the following models:



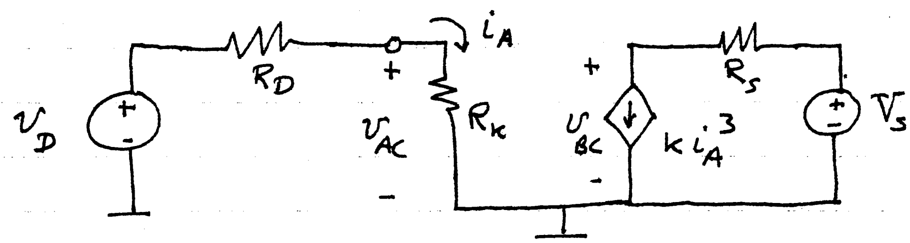
or



$g_m = K(V_{AB} - V_T)$

$R = \frac{1}{g_m} = \frac{1}{K(V_{AB} - V_T)}$

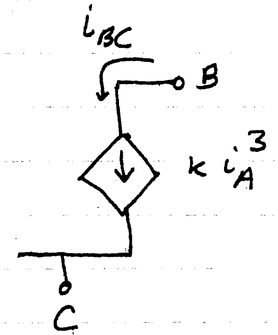
3. A) Substituting the model into the FRAMPLIFIER:



B) $v_0 = v_{BC} = V_s - kR_s\, i_A^3$

$$v_0 = V_s - kR_s\left(\frac{v_D}{R_D+R_K}\right)^3$$

C)



Let $i_A = I_A + i_a$

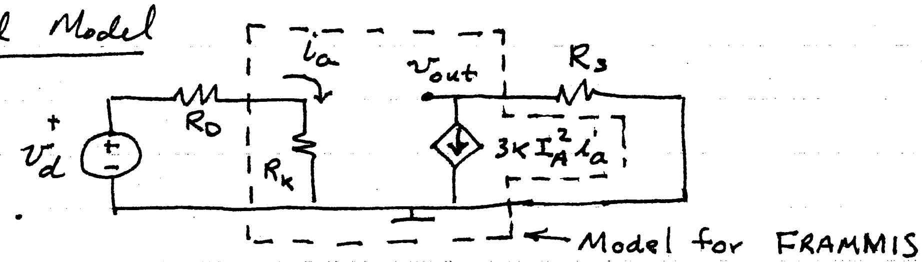
Then $i_{BC} = k(I_A + i_a)^3$

$= kI_A^3 + 3kI_A^2 i_a + \underbrace{\cdots\cdots}_{\text{neglect!}}$

or, with $i_{BC} = I_{BC} + i_{bc}$

$i_{bc} = 3kI_A^2 i_a$

Incremental Model



Model (2) is clearly the correct choice.

D) From the above model

$$v_o = -3kR_s I_A^2 \frac{v_d}{R_D+R_K} \Rightarrow A = -\frac{3kR_s I_A^2}{R_D+R_K}$$

Note: $I_A = \dfrac{V_D}{R_D+R_K}$.

4. A) Saturation discipline requires $v_{DS} > v_{GS} - V_T$

In this problem, $v_C > v_I - V_T$

B) From the i-v relationship of the capacitor

$$C \frac{dv_C}{dt} = -i_{DS} = -\frac{K}{2}(v_I - V_T)^2$$

which is valid for $v_I > V_T$. For $v_I < V_T$, $i_{DS} = 0$

and the differential eq is $C \frac{dv_C}{dt} = 0$.

C) For $0 < t < 1\ \mu s$, $v_I = 0$, so $\frac{dv_C}{dt} = 0$ and

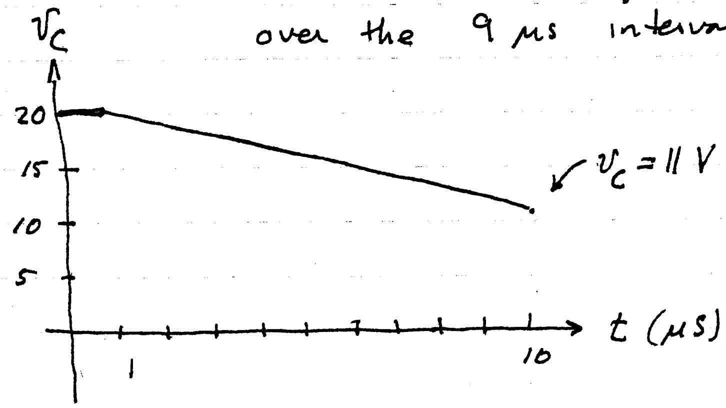
$v_C = \text{const.} = v_C(0) = 20\ V.$

For $1 < t < 10\ \mu s$

$$10^{-7} \frac{dv_C}{dt} = -\frac{2 \times 10^{-3}}{2}(10)^2 \qquad \text{(units !)}$$

$$\frac{dv_C}{dt} = -10^6\ \frac{V}{s} \Rightarrow v_C = 20 - 10^6(t - 10^{-6})$$

v_C decays linearly from 20 V to 11 V over the 9 μs interval.



5. A) When switch closes, v_1 decays from its initial value to 0 with time constant $RC = 10^5 \times 10^{-4} = 10$ sec

Taking $t=0$ as time switch closes, $v_1 = 1000\, e^{-t/10}$

which reaches 50 when $50 = 1000\, e^{-t_1/10}$

or $t_1 = 10 \ln 20 \approx 30$ sec.

B) Writing a KVL around the loop

$$Ri + v_2 - v_1 = 0$$

Again taking $t=0$ as the time the switch closes,

$$v_2 = \frac{1}{C_2} \int_0^t i\, dt'$$

$$v_1 = 1000 - \frac{1}{C_1} \int_0^t i\, dt'$$

so

$$Ri + \frac{1}{C_2} \int_0^t i\, dt' + \frac{1}{C_1} \int_0^t i\, dt' = 1000$$

and, differentiating,

$$\Rightarrow R \frac{di}{dt} + \frac{1}{C_{eq}} i = 0, \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

C) $$R C_{eq} = R \frac{C_1 C_2}{C_1 + C_2} = R \cdot \frac{C_1^2/2}{3C_1/2} = \frac{RC_1}{3} = 3.33 \text{ sec.}$$

D) At $t=0$, $v_2 = 0$, $v_1 = 1000$ so $i = \frac{1000}{10^5} = 10^{-2}$ A.

E) From B, $v_1 = 1000 - 10^4 \int_0^t i(t')\, dt'$

while from C) and D) $i = 10^{-2} e^{-t/3.33}$ A.

so $v_1 = 1000 - 10^2 \times 3.33 \left(1 - e^{-t/3.33}\right) \Rightarrow 667$ V at $t = \infty$ No Good!