WITH SOLUTIONS

(S03-054)

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits and Electronics
Spring 2003

Handout S03-048 - Quiz # 2

Thursday April 9, 2003

Name: ______________________________

Recitation Instructor (circle one):

Baldo Hutchinson Kołodziejski Schindall Wilson

Recitation Hour (circle one):

9 10 11 12 1 2

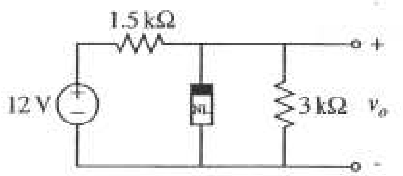
ALL PROBLEMS CARRY THE SAME WEIGHT

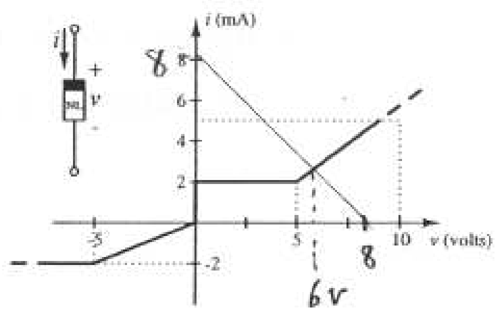
Problem	Points	Score	Grader
1	25		
2	25		
3	25		
4	25		
Total	100		

Name:______________________________

PROBLEM 1

The circuit below contains a nonlinear element whose iv characteristics are shown.





(A) Determine the voltage v_O graphically - show your construction.

REDUCE THE LINEAR PORTION OF THE CIRCUIT TO A THEVENIN EQUIVALENT:

$V_{OC} = 12\frac{3K}{4.5K} = 8V$

$R_{TH} = 1.5K \| 3K = 1K$

USE A LOAD-LINE CONSTRUCTION:

INTERCEPTS: $i = 0 \quad v_o = 8V$

$v_o = 0 \quad i = \frac{8V}{1K} = 8mA$

READ INTERSECTION: $v_o \cong 6V$

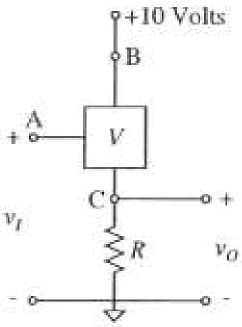
(B) Can this circuit be described by a Thevenin equivalent circuit at the terminals? Explain!

NO THE CONCEPT OF THEVENIN EQUIVALENTS IS BUILT ON THE SUPERPOSITION PRINCIPLE WHICH DEMANDS LINEAR ELEMENTS

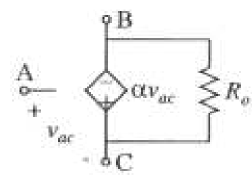
Name:______________________________

PROBLEM 2

This circuit uses a control valve V which has the small-signal model shown.

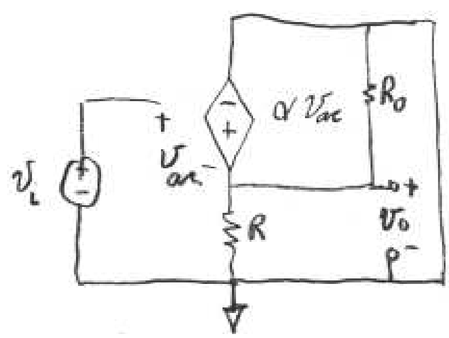


Small-Signal Model of V



Assume that the static component of v_I establishes a suitable operating point at which the small-signal model applies.

(A) Sketch and label a small-signal model of the circuit which can be used to calculate the small-signal voltage gain $A_v = \frac{v_o}{v_i}$ where v_o and v_i are the small-signal components of v_O and v_I.



Name:____________________________

(B) Express v_{ac} in terms of v_i and v_o.

$$v_{ac} = v_i - v_o$$

For Extra Credit: (one-third the value of a problem)

(C) Derive an expression for $A_v = \frac{v_o}{v_i}$, the incremental voltage gain.

$$v_o = \alpha v_{ac} = \alpha (v_i - v_o)$$

$$v_o (1+\alpha) = \alpha v_i$$

$$\frac{v_o}{v_i} = A_v = \frac{\alpha}{1+\alpha}$$

NOTE THAT R AND R_O ARE IN PARALLEL AND THE VOLTAGE ACROSS THEM IS v_o

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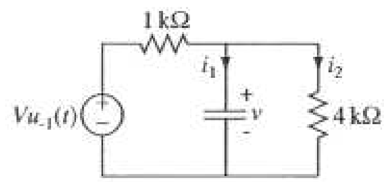
PROBLEM 3

For each of the circuits below, determine the initial and final (asymptotic) values of the indicated variables.

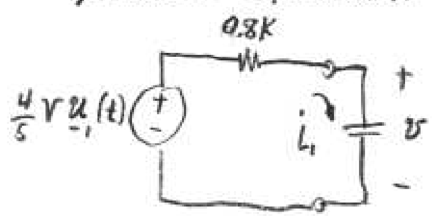
(A)



For $t<0$, $v=0$
v is in Volts

VARIABLE	$t=0^+$	$t\to\infty$	UNITS
v	0	0.8	V
i_1	1	0	mA
i_2	0	0.2	mA

THEVENIN EQUIVALENT SEEN BY C:

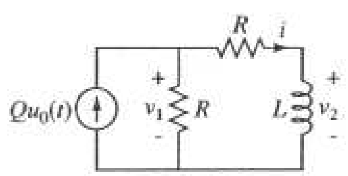


THE CAPACITOR VOLTAGE STARTS AT ZERO, RISE TO 0.8V

i_1 STARTS AT $\frac{0.8V}{0.8K} = 1\,mA$ AND FALLS TO ZERO

$i_2 = \frac{v}{4K}$, STARTS AT ZERO, RISES TO $\frac{0.8V}{4K} = 0.2\,mA$

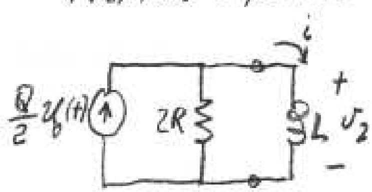
(B)



For $t<0$, $i=-I$
Q is in Coulombs
R is in Ohms
I is in Amps

VARIABLE	$t=0^+$	$t\to\infty$
i	$\frac{QR}{L} - I$	0
v_1	$-R\left(\frac{QR}{L}-I\right)$	0
v_2	$-2R\left(\frac{QR}{L}-I\right)$	0

NORTON EQUIVALENT SEEN BY L:



THE CURRENT IMPULSE PRODUCES A VOLTAGE IMPULSE $\frac{Q}{2} \times 2R = QR$ ACROSS L AND 2R. THIS ESTABLISHES AN INDUCTOR CURRENT $i(0+) = \left(\frac{QR}{L} - I\right)$. THIS CURRENT DECAYS TO ZERO THROUGH 2R ESTABLISHING A VOLTAGE

$v_2(0+) = -2R\left(\frac{QR}{L} - I\right)$

OBVIOUSLY IN ORIGINAL CIRCUIT $v_1 = v_2/2 = -R\left(\frac{QR}{L} - I\right)$

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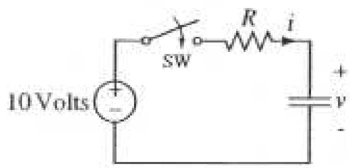
PROBLEM 4

The capacitor in the circuit below is initially charged to the voltage $v = -5$ Volts. At $t = 0$ the switch closes.



$v = -5$ V for $t < 0$
$R = 10\,k\Omega = 10^4\,\Omega$
$C = 1\,\mu F = 10^{-6}\,F$

INITIAL VOLTAGE = −5 V

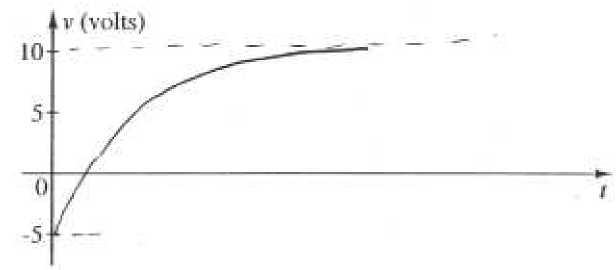
INITIAL CURRENT:

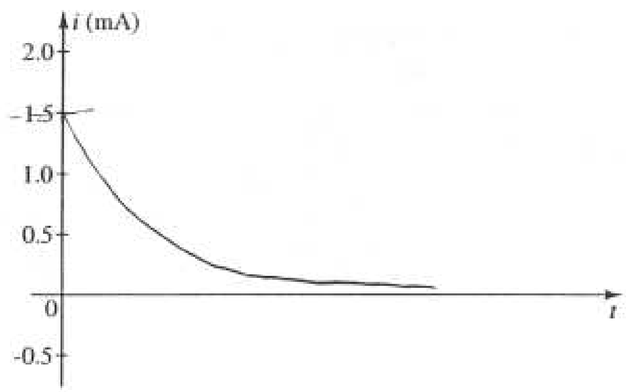
$$i = \frac{10-(-5)}{10K} = 1.5\,mA$$

DECAYS TO ZERO

FINAL CURRENT = 0

FINAL VOLTAGE = +10





(A) Without detailed analysis of the circuit, sketch $v(t)$ and $i(t)$ for $t > 0$. Label initial values and asymptotes.

(B) Determine the time constant with which the circuit responds.

$$\tau = RC = 10^4 \times 10^{-6} = 10^{-2}\ SEC \qquad \underline{\tau = 10\ mSEC}$$

(C) Express <u>either</u> $v(t)$ <u>or</u> $i(t)$ as a function of time. If you can do this without first developing an analytical solution, fine.

$$i(t) = 1.5 e^{-t/\tau}\ mA$$

$$v(t) = -5 + 15(1 - e^{-t/\tau})\ V \quad \text{OR} \quad 10 - 15 e^{-t/\tau}\ V$$