MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits
Fall 2002

Quiz 3

Name: ______________________ **Recitation Section:** ____________

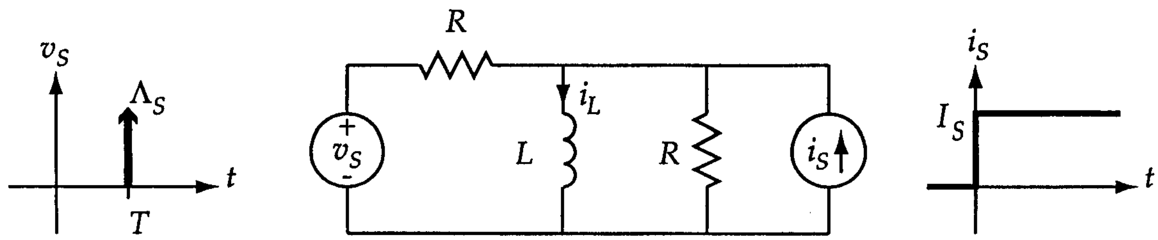
Recitation Instructor: ____________ **Teaching Assistant:** ____________

Make sure that your name is on all sheets. Enter your answers directly in the spaces provided on the printed pages. You may use the back of the previous page as a worksheet. Answers must be derived or explained, not just simply written down. The quiz is closed book, but **calculators are allowed.**

This quiz contains 8 pages including the cover sheet. Make sure that your quiz contains all 8 pages and that you hand in all 8 pages.

Problem	Points	Grade	Grader
1	30		
2	30		
3	40		
Total	100		

Problem 1: (*30 points*) This problem examines the transient response of the circuit shown below. In the circuit, $i_L = 0$ at $t = 0^-$.

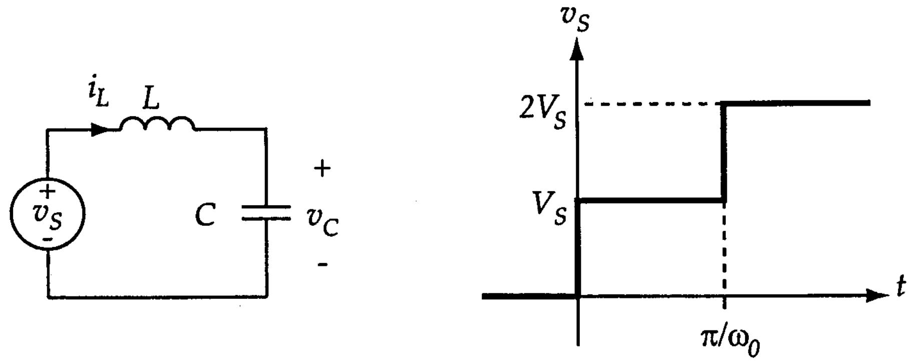


(A) (*10 points*) Determine an expression for i_L due to $i_S = I_S u(t)$; that is, for $\Lambda_S = 0$.

(B) (*10 points*) Determine an expression for i_L due to $v_S = \Lambda_S \delta(t - T)$; that is, for $I_S = 0$.

(C) (*10 points*) Determine an expression for i_L due to i_S and v_S together.

Problem 2: (*30 points*) In the circuit below, $\omega_0 \equiv 1/\sqrt{LC}$, $v_C(0^-) = 0$, and $i_L(0^-) = 0$.

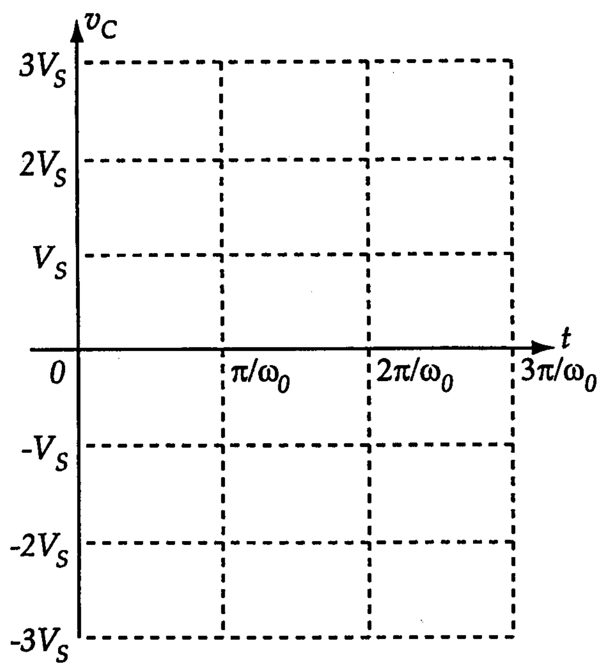


(A) (*15 points*) Determine an expression for $v_C(t)$ for $t > 0$. If you determine $v_C(t)$ by inspection, state your reasoning CLEARLY.

$v_C(t) =$ ______________________________ $0 \leq t \leq \pi/\omega_0$

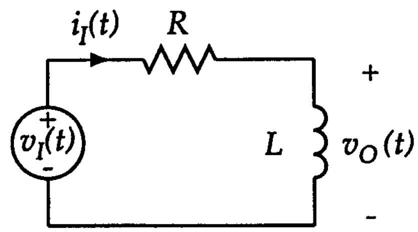
$v_C(t) =$ ______________________________ $t > \pi/\omega_0$

(B) (*15 points*) Sketch and dimension $v_C(t)$ for $t > 0$.



Problem 3: (*40 points*) All circuits below operate in the sinusoidal steady state. For each circuit, determine the input impedance $Z(j\omega)$, the specified transfer function $H(j\omega)$, and the parameters V_o and ϕ which define $v_O(t) \equiv V_o \cos(\omega t + \phi)$, where V_o is a real, positive number.

(A) (*10 points*)



$v_I(t) = V_i \cos(\omega t)$

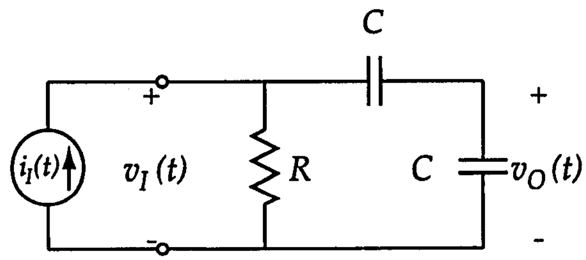
$Z(j\omega) = \frac{V_i}{\hat{I}_i} =$ ____________________

$H(j\omega) = \frac{\hat{V}_o}{V_i} =$ ____________________

$V_o =$ ____________________

$\phi =$ ____________________

(B) (*15 points*)



$i_I(t) = I_i \cos(\omega t)$

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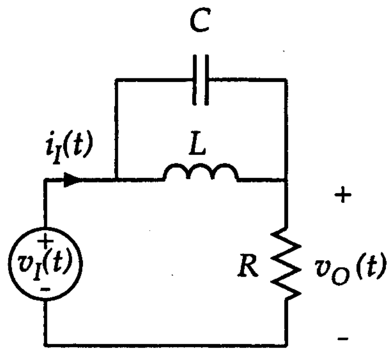
$H(j\omega) = \frac{\hat{V}_o}{I_i} =$ ______________________

$V_o =$ ______________________

$\phi =$ ______________________

(C) (*15 points*)



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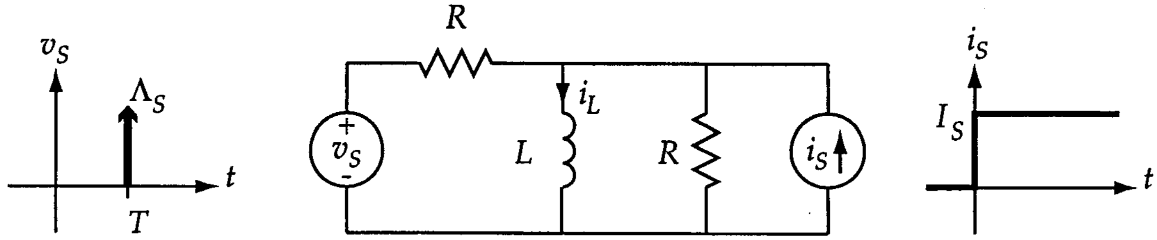
Problem 1: (*30 points*) This problem examines the transient response of the circuit shown below. In the circuit, $i_L = 0$ at $t = 0^-$.



(A) (*10 points*) Determine an expression for i_L due to $i_S = I_S u(t)$; that is, for $\Lambda_S = 0$.

$t = 0^+$, $i_L = 0$ (continuity)

$t = \infty$, $i_L = I_S$ (inductor short)

$$\tau = \frac{2L}{R}$$

$$\boxed{i_L = I_S\left(1 - e^{-Rt/2L}\right)u(t)}$$

(B) (*10 points*) Determine an expression for i_L due to $v_S = \Lambda_S \delta(t-T)$; that is, for $I_S = 0$.

to an impulse, inductor looks like an open circuit.

$$v_L = \frac{R}{R+R} \Lambda_S \delta(t-T) = \frac{\Lambda_S}{2} \delta(t-T)$$

for $T^- < t < T^+$

$$v_L = L \frac{di_L}{dt}$$

$$\frac{1}{L}\int_{T^-}^{T^+} v_L \, dt = i_L(T^+) - i_L(T^-)$$

$$\frac{\Lambda_S}{2L} = i_L(T^+) - 0$$

$$i_L(T^+) = \frac{\Lambda_S}{2L}$$

initial condition that afterwards decays homogenously

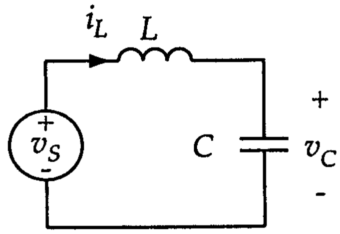
$$i_L = \frac{\Lambda_S}{2L} e^{-R(t-T)/2L} u(t-T)$$

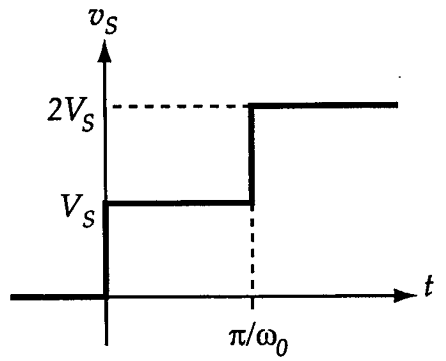
(C) (*10 points*) Determine an expression for i_L due to i_S and v_S together.

superposition!

$$i_L = I_S(1 - e^{-Rt/2L}) u(t) + \frac{\Lambda_S}{2L} e^{-R(t-T)/2L} u(t-T)$$

Problem 2: (*30 points*) In the circuit below, $\omega_0 \equiv 1/\sqrt{LC}$, $v_C(0^-) = 0$, and $i_L(0^-) = 0$.





(A) (*15 points*) Determine an expression for $v_C(t)$ for $t > 0$. If you determine $v_C(t)$ by inspection, state your reasoning CLEARLY.

for $t<0$, no oscillation because no energy is in either the capacitor ($v_C(0^-)=0$) or the inductor ($i_L(0^-)=0$).

The first voltage step will initially fall across L, as v_C must remain zero at $t=0^+$

$i_L(0^+)=0$, so $\frac{dv_C}{dt}(0^+)=0$.

KCL: $\frac{1}{L}\int(V_S - v_C)\,dt = C\frac{dv_C}{dt} \Rightarrow \frac{V_S}{LC} = \frac{d^2v_C}{dt^2} + \frac{1}{LC}v_C$

particular solution, $v_C = V_S$.

total soln: $v_C = A\cos(\omega_0 t + \phi) + V_S$

initial cond: $0 = A\cos(\phi) + V_S$ $\quad 0 = -A\omega_0\sin(\phi)$

$\phi = 0$, $A = -V_S$

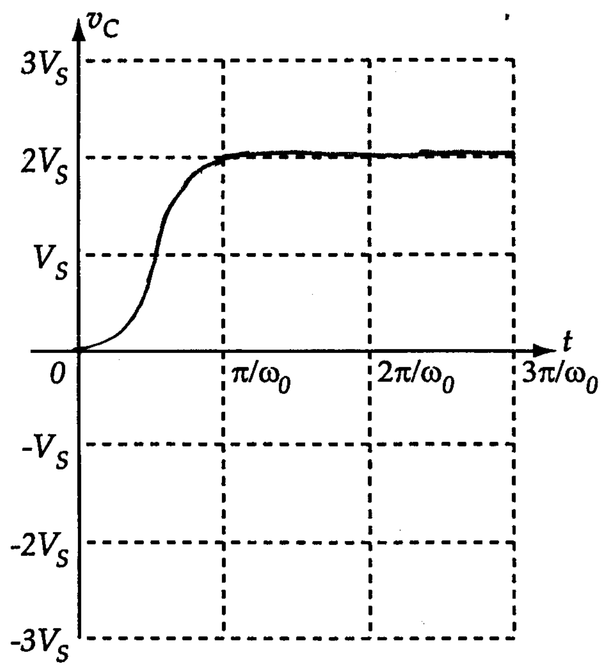
(continued →)

$v_C(t) =$ $V_S(1-\cos(\omega_0 t))$ $\qquad 0 \le t \le \pi/\omega_0$

$v_C(t) =$ $2V_S$ $\qquad t > \pi/\omega_0$

(B) (*15 points*) Sketch and dimension $v_C(t)$ for $t > 0$.

?



→ continued:

at $t = \frac{\pi}{\omega_0}^-$, $v_C = 2V_s$ $\frac{dv_C}{dt} = 0$ ⇒ same for $t = \frac{\pi}{\omega_0}^+$

v_C and i_L don't change instantly

for $t > \pi/\omega_0$, $\frac{2V_s}{LC} = \frac{d^2 v_C}{dt^2} + \frac{1}{LC} v_C$

particular, $v_C = 2V_s$

total: $v_C = B\cos(\omega_0 t + \phi_d) + 2V_s$

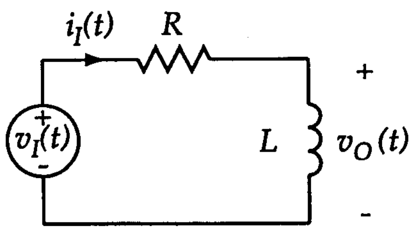
initial cond: $2V_s = B\cos(\phi_d) + 2V_s$, $0 = -B\omega_0 \sin(\phi_d)$

$\phi_d = 0$, $B = 0$!

soln = $2V_s = v_C$ for $t > \pi/\omega_0$

Problem 3: (*40 points*) All circuits below operate in the sinusoidal steady state. For each circuit, determine the input impedance $Z(j\omega)$, the specified transfer function $H(j\omega)$, and the parameters V_o and ϕ which define $v_O(t) \equiv V_o \cos(\omega t + \phi)$, where V_o is a real, positive number.

(A) (*10 points*)



$v_I(t) = V_i \cos(\omega t)$

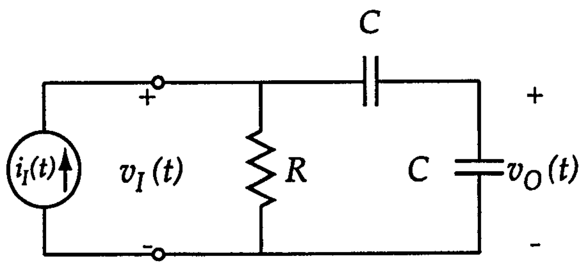
$Z(j\omega) = \frac{V_i}{\hat{I}_i} =$ $R + j\omega L$

$H(j\omega) = \frac{\hat{V}_o}{V_i} =$ $\frac{j\omega L}{R + j\omega L}$

$V_o =$ $\frac{V_i|\omega L|}{\sqrt{R^2 + \omega^2 L^2}}$

$\phi =$ $90^\circ - \tan^{-1}\left(\frac{\omega L}{R}\right)$

(B) (*15 points*)



$i_I(t) = I_i \cos(\omega t)$

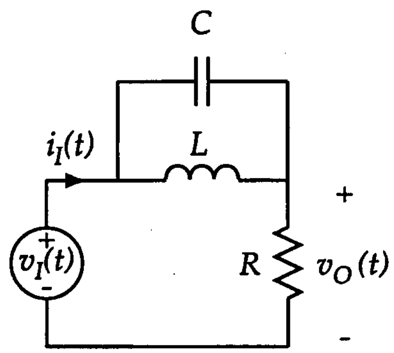
$Z(j\omega) = \frac{\hat{V}_i}{I_i} =$ $R \| \frac{2}{j\omega C} = \frac{R}{\frac{j\omega RC}{2} + 1}$

$H(j\omega) = \frac{\hat{V}_o}{I_i} =$ $\left(\frac{R}{R + \frac{2}{j\omega C}}\right)\left(\frac{1}{j\omega C}\right) = \frac{R}{j\omega RC + 2}$

$V_o = \frac{I_i |R|}{\sqrt{(\omega RC)^2 + 4}}$

$\phi =$ $-\tan^{-1}\left(\frac{\omega RC}{2}\right)$

(C) (*15 points*)



$v_I(t) = V_i \cos(\omega t)$

$Z(j\omega) = \frac{\hat{V}_i}{\hat{I}_i} = R + j\omega L \,||\, \frac{1}{j\omega C} = \boxed{R + \frac{j\omega L}{1-\omega^2 LC}}$

$H(j\omega) = \frac{\hat{V}_o}{\hat{V}_i} = \frac{R}{R + j\omega L \,||\, \frac{1}{j\omega C}} = \boxed{\frac{R-\omega^2 LRC}{R-\omega^2 LRC + j\omega L}}$

$V_o = \frac{V_i\,|R-\omega^2 LRC|}{\sqrt{(R-\omega^2 LRC)^2 + \omega^2 L^2}}$

$\phi = -\tan^{-1}\left(\frac{\omega L}{R-\omega^2 LRC}\right)$