# Massachusetts Institute of Technology
## Department of Electrical Engineering and Computer Science
## 6.002 --Electronic Circuits
## Quiz #4
## Closed Book

Friday (Cooke-Section), April 28, 2000

Name:__________________

Section Hr: _______

Problem 1 (5pts): The following RL network in Figure 1 is in the sinusoidal steady state. It is driven by an input voltage source, $v_I$ , which results in an output voltage, $v_O$.

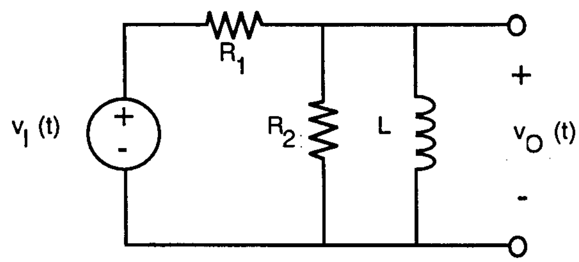
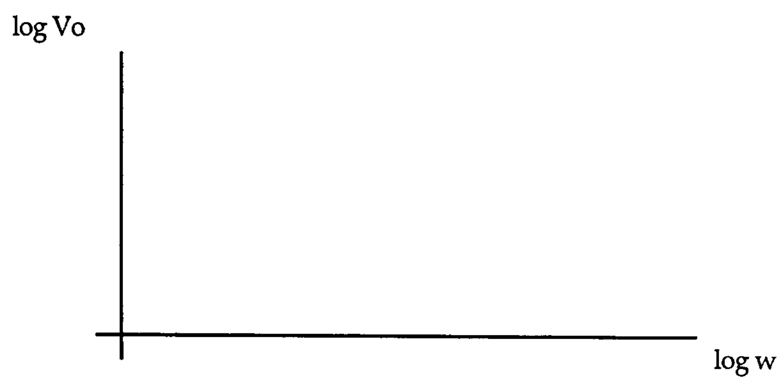



Figure 1

Part 1A Write an expression for the complex magnitude of the output $V_O(jw)$ when $v_I(t) = A\cos(wt)$. Assume the circuit is in sinusoidal steady state.

$V_O(jw) =$

Part 1B Graph the magnitude of the output, $|V_O(jw)|$ vs w, on log-log coordinates (the Bode diagram format) showing the asymptotes and any frequency values where the asymptotes intersect. Does the circuit exhibit high-pass or low-pass filter properties, if so which ?



Problem 2 (10pts): Consider the series connected RLC circuit, Figure 2.

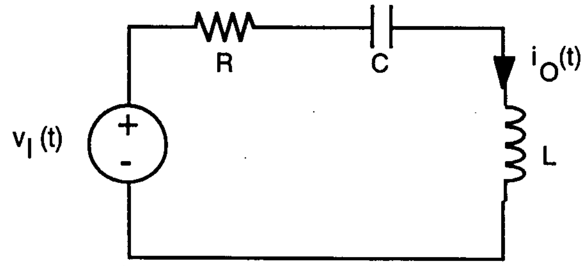



Figure 2

Part 1A Write the time-domain differential equation that relates for the output loop current, $i_O(t)$, in terms of the source voltage, $v_I(t)$, and the circuit elements R, L and C.

Part 1B The circuit is now in the sinusoidal-steady-state, with the voltage source $v_I(t) = V_i \cos(wt)$. Write an expression for the complex magnitude of the loop current, $I_o$, in terms of $V_i$, R, L and C.

Part 1C Write a complete expression with all numerical values for the time-domain output current, $i_O(t)$, **in the sinusoidal steady state,** when: $v_I(t) = 5\cos(10t)$; and R = 2 ohms, C = 0.1F, and L = 0.3 H.

$i_O(t) =$

Problem 3 (5pts): Figure 3 shows the circuit for a resonant system. The voltage source is a step with magnitude K for $t > 0$, and 0 for $t < 0$. The circuit is at rest for $t < 0$, i.e. $v_C(0^-) = 0$, and $i_L(0^-) = 0$

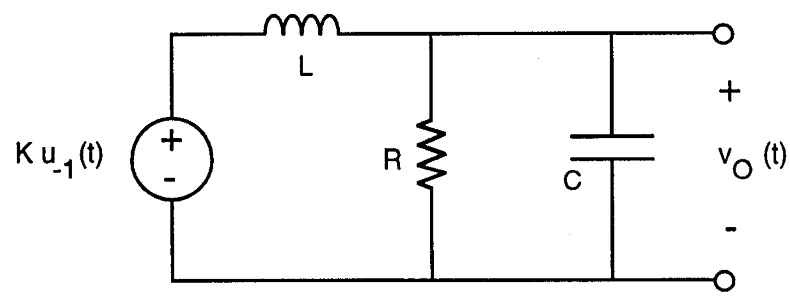


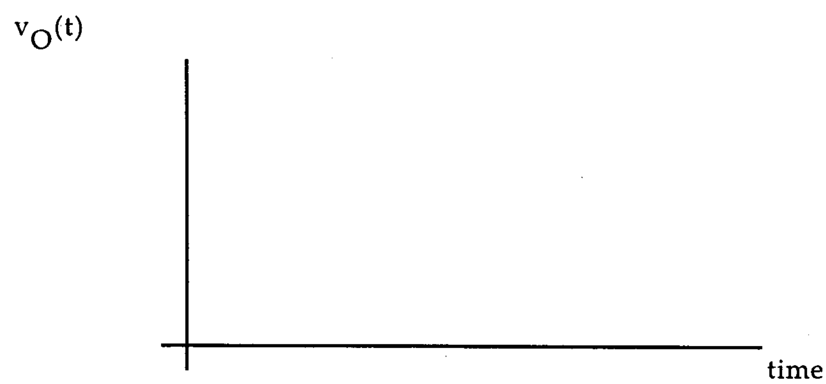

Figure 3

Part 3A (i) What is the value of $v_O(0^+)$ ?

(ii) What is the value of the derivative of the output, $dv_O/dt$ , at $t = 0^+$ ?

(iii) What is the final value of the output $v_O(\infty)$ ?

Part 3B Sketch and label (with approximate values) the form of the time response given that R, L and C values yield an oscillatory characteristic, (i.e. damping is small).




OPTIONAL (no points, for 'fun' only)

Part 3C Do you think this circuit is classified as a series or parallel resonant network, Provide a proof for your choice.

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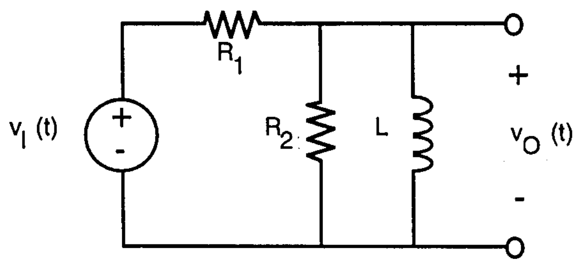


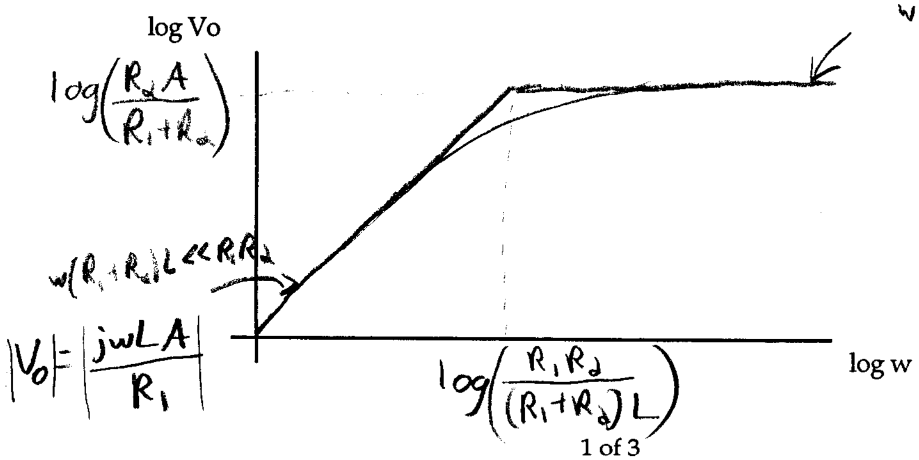

Figure 1

Part 1A Write an expression for the complex magnitude of the output $V_O(jw)$ when $v_I(t) = A\cos(wt)$. Assume the circuit is in sinusoidal steady state.

$$V_O(jw) = A\frac{R_2 \| jwL}{R_1 + R_2 \| jwL} = \boxed{\frac{jw R_2 L A}{R_1 R_2 + jw(R_1+R_2)L}}$$

Part 1B Graph the magnitude of the output, $|V_O(jw)|$ vs w, on log-log coordinates (the Bode diagram format) showing the asymptotes and any frequency values where the asymptotes intersect. Does the circuit exhibit high-pass or low-pass filter properties, if so which ?

$w(R_1+R_2)L >> R_1R_2$
$V_O = \frac{R_2 A}{R_1+R_2}$



Problem 2 (10pts): Consider the series connected RLC circuit, Figure 2.

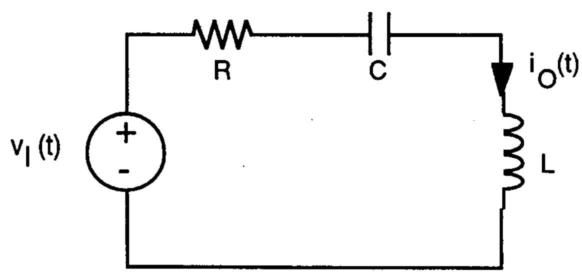



Figure 2

Part 1A Write the time-domain differential equation that relates for the output loop current, $i_O(t)$, in terms of the source voltage, $v_I(t)$, and the circuit elements R, L and C.

$$v_I = i_O R + L\frac{di_O}{dt} + \frac{1}{C}\int i_O\,dt$$

$$\frac{1}{L}\frac{dv_I}{dt} = \frac{d^2 i_O}{dt^2} + \frac{R}{L}\frac{di_O}{dt} + \frac{1}{LC}i_O$$

Part 1B The circuit is now in the sinusoidal-steady-state, with the voltage source $v_I(t) = V_i\cos(wt)$. Write an expression for the complex magnitude of the loop current, $I_o$, in terms of $V_i$, R, L and C.

$$I_o = \frac{V_i}{R + \frac{1}{j\omega C} + j\omega L} = \frac{V_i\, j\omega C}{j\omega RC + j^2\omega^2 LC + 1}$$

$$I_o = \frac{j\omega V_i C}{1 - \omega^2 LC + j\omega RC}$$

Part 1C Write a complete expression with all numerical values for the time-domain output current, $i_O(t)$, **in the sinusoidal steady state,** when: $v_I(t) = 5\cos(10t)$; and R = 2 ohms, C = 0.1F, and L = 0.3 H.

$\omega = 10$

$V_i = 5$

$$|I_o| = \frac{(10)(5)(0.1F)}{\sqrt{[1-(10^2)(0.3H)(0.1F)]^2 + [(10)(2\Omega)(0.1F)]^2}} = 1.768\,A$$

$i_O(t) =$

$$\angle I_o = 90^\circ - \tan^{-1}\left(\frac{(10)(2\Omega)(0.1F)}{1-(10^2)(0.3H)(0.1F)}\right) = 135^\circ \ \left(\frac{3\pi}{4}\right)$$

$$i_O(t) = (1.768\,A)\cos\left(10t + \frac{3\pi}{4}\right)$$

**Problem 3** (5pts): Figure 3 shows the circuit for a resonant system. The voltage source is a step with magnitude K for t > 0, and 0 for t < 0. The circuit is at rest for t < 0, i.e. $v_C(0^-) = 0$, and $i_L(0^-) = 0$

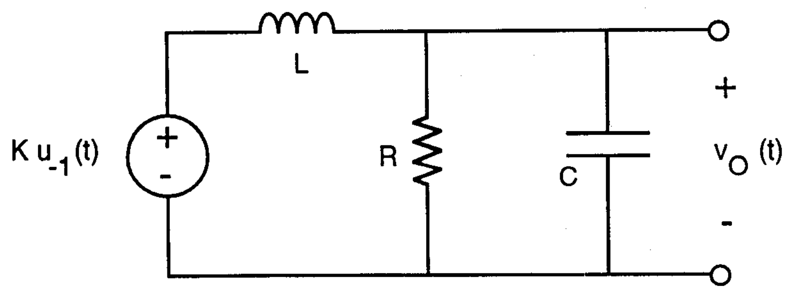



Figure 3

**Part 3A**

(i) What is the value of $v_O(0^+)$ ? $= V_C(0^-) = \boxed{0.}$ continuity requirement.

(ii) What is the value of the derivative of the output, $dv_O/dt$ , at $t = 0^+$ ?

$i_L(0^+) = 0$, and $v_R(0^+) = 0$, so $i_C(0^+) = 0$. $\frac{dv_O}{dt} = \frac{1}{C} i_C = \boxed{0}$

(iii) What is the final value of the output $v_O(\infty)$ ?

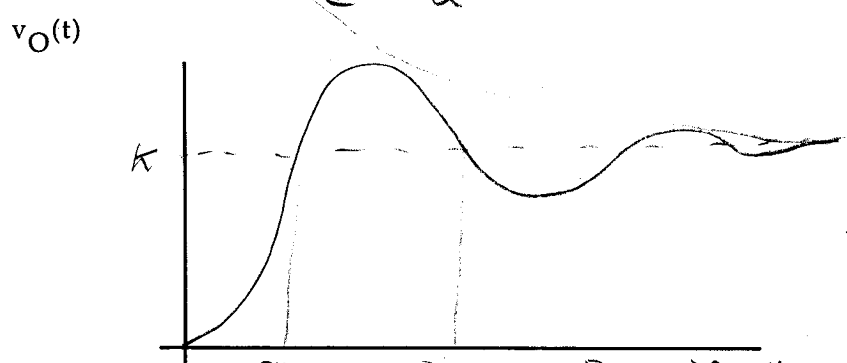
L => short, C => open (assuming DC steady state)

$\boxed{v_O(\infty) = K}$

**Part 3B** Sketch and label (with approximate values) the form of the time response given that R, L and C values yield an oscillatory characteristic, (i.e. damping is small).

$\tau = \frac{1}{\alpha}$ $\alpha = \frac{1}{2RC}$ $w_d = \sqrt{w_0^2 - \alpha^2}$

$w_0 = \sqrt{\frac{1}{LC}}$




$\frac{\pi}{2w_d}$ $\frac{3\pi}{2w_d}$ $\frac{5\pi}{2w_d}$ $\frac{7\pi}{2w_d}$

OPTIONAL (no points, for 'fun' only)

**Part 3C** Do you think this circuit is classified as a series or parallel resonant network, Provide a proof for your choice.

Parallel => set source = 0.
(proof ?)