Problem 1 (5pts): The following RL network in Figure 1 is in the sinusoidal steady state. It is driven by an input voltage source, \( v_1 \), which results in an output voltage, \( v_O \).

![Figure 1](image_url)

**Part 1A** Write an expression for the complex magnitude of the output \( V_o(jw) \) when \( v_1(t) = A \cos(wt) \). Assume the circuit is in sinusoidal steady state.

\[ V_o(jw) = \]

**Part 1B** Graph the magnitude of the output, \( |V_o(jw)| \) vs \( w \), on log-log coordinates (the Bode diagram format) showing the asymptotes and any frequency values where the asymptotes intersect. Does the circuit exhibit high-pass or low-pass filter properties, if so which ?

\[ \log V_o \]

\[ \log w \]
Problem 2 (10pts): Consider the series connected RLC circuit, Figure 2.

Figure 2

Part 1A Write the time-domain differential equation that relates for the output loop current, \( i_O(t) \), in terms of the source voltage, \( v_I(t) \), and the circuit elements \( R, L \) and \( C \).

Part 1B The circuit is now in the sinusoidal-steady-state, with the voltage source \( v_I(t) = V_I \cos(\omega t) \). Write an expression for the complex magnitude of the loop current, \( I_O \), in terms of \( V_I, R, L \), and \( C \).

Part 1C Write a complete expression with all numerical values for the time-domain output current, \( i_O(t) \), in the sinusoidal steady state, when: \( v_I(t) = 5 \cos(10t) \); and \( R = 2 \) ohms, \( C = 0.1 \) F, and \( L = 0.3 \) H.

\[
i_O(t) =
\]
**Problem 3** (5pts): Figure 3 shows the circuit for a resonant system. The voltage source is a step with magnitude $K$ for $t > 0$, and 0 for $t < 0$. The circuit is at rest for $t < 0$, i.e. $v_C(0^-) = 0$, and $i_L(0^-) = 0$. 

![Circuit Diagram]

Figure 3

**Part 3A**

(i) What is the value of $v_O(0^+)$?

(ii) What is the value of the derivative of the output, $\frac{dv_O}{dt}$, at $t = 0^+$?

(iii) What is the final value of the output $v_O(\infty)$?

**Part 3B**

Sketch and label (with approximate values) the form of the time response given that $R$, $L$ and $C$ values yield an oscillatory characteristic, (i.e. damping is small).

**Optional** (no points, for 'fun' only)

**Part 3C**

Do you think this circuit is classified as a series or parallel resonant network, Provide a proof for your choice.
Problem 1 (5pts): The following RL network in Figure 1 is in the sinusoidal steady state. It is driven by an input voltage source, \( v_1(t) \), which results in an output voltage, \( v_0(t) \).

![RL Network Diagram](image)

**Figure 1**

**Part 1A** Write an expression for the complex magnitude of the output \( V_0(j\omega) \) when \( v_1(t) = A \cos(\omega t) \). Assume the circuit is in sinusoidal steady state.

\[
V_0(j\omega) = A \frac{R_d \omega L}{R_1 + R_d + j\omega L} = \frac{\frac{\omega L R_d A}{R_1 R_d + j\omega(R_1 + R_d)L}}
\]

**Part 1B** Graph the magnitude of the output, \(|V_0(j\omega)|\) vs \(\omega\), on log-log coordinates (the Bode diagram format) showing the asymptotes and any frequency values where the asymptotes intersect. Does the circuit exhibit high-pass or low-pass filter properties, if so which?

\[
\frac{w(R_1 + R_d)L}{R_1 + R_d} \gg R_1 R_d
\]

\[
V_0 = \frac{R_d A}{R_1 + R_d}
\]

\[
\log V_0 \quad \log(\frac{R_d A}{R_1 + R_d})
\]

\[
\omega(R_1 + R_d)L < R_1 R_d
\]

\[
\log(\frac{R_1 R_d}{R_1 + R_d} L)
\]
Problem 2 (10pts): Consider the series connected RLC circuit, Figure 2.

![RLC Circuit Diagram]

**Figure 2**

**Part 1A** Write the time-domain differential equation that relates for the output loop current, \( i_o(t) \), in terms of the source voltage, \( v_1(t) \), and the circuit elements \( R, L \) and \( C \).

\[
\begin{align*}
\mathcal{L}_I &= i_o R + L \frac{d i_o}{d t} + \frac{1}{C} \int i_o \, dt \\
\frac{1}{L} \frac{d v_1}{d t} &= \frac{d^2 i_o}{d t^2} + \frac{R}{L} \frac{d i_o}{d t} + \frac{1}{LC} i_o
\end{align*}
\]

**Part 1B** The circuit is now in the sinusoidal-steady-state, with the voltage source \( v_1(t) = V_i \cos(\omega t) \).

Write an expression for the complex magnitude of the loop current, \( I_o \), in terms of \( V_i, R, L \) and \( C \).

\[
I_o = \frac{V_i}{R + \frac{j \omega L}{C} + j \omega L} = \frac{V_i}{\sqrt{\omega^2 LC + 1}}
\]

\[
I_o = \frac{j \omega V_i C}{\sqrt{1 - \omega^2 LC}}
\]

**Part 1C** Write a complete expression with all numerical values for the time-domain output current, \( i_o(t) \), in the sinusoidal steady, when: \( v_1(t) = 5 \cos(10t) \); and \( R = 2 \) ohms, \( C = 0.1 \) F, and \( L = 0.3 \) H.

\[
\begin{align*}
\omega &= 10 \\
V_i &= 5 \\
I_o &= \left| I_o \right| = \frac{(10)(5)(0.1F)}{\sqrt{[1 - (10^2)(0.3H)(0.1F)] + [(10)(2.52)(0.1F)]}} = 1.768 A \\
i_o(t) &= 4I_o = 90^\circ - \tan^{-1} \left( \frac{(10)(2.52)(0.1F)}{1 - (10^2)(0.3H)(0.1F)} \right) = 135^\circ \left( \frac{3\pi}{4} \right) \\
i_o(t) &= (1.768 A) \cos \left( 10t + \frac{3\pi}{4} \right)
\end{align*}
\]
Problem 3 (5pts): Figure 3 shows the circuit for a resonant system. The voltage source is a step with magnitude $K$ for $t > 0$, and 0 for $t < 0$. The circuit is at rest for $t < 0$, i.e. $v_C(0^-) = 0$, and $i_L(0^-) = 0$

![Circuit Diagram]

Figure 3

Part 3A

(i) What is the value of $v_O(0^+)$? $v_C(0^-) = 0$. Continuity requirement.

(ii) What is the value of the derivative of the output, $\frac{dv_O}{dt}$, at $t = 0^+$?

(iii) What is the final value of the output $v_O(\infty)$?

L $\Rightarrow$ Short, C $\Rightarrow$ Open (assuming DC steady state)

$\boxed{v_O(\infty) = K}$

Part 3B

Sketch and label (with approximate values) the form of the time response given that R, L and C values yield an oscillatory characteristic, (i.e. damping is small).

$\zeta = \frac{1}{\alpha}$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

$\omega_0 = \sqrt{\frac{1}{LC}}$

OPTIONAL (no points, for ‘fun’ only)

Part 3C

Do you think this circuit is classified as a series or parallel resonant network? Provide a proof for your choice.

$\text{Parallel =)}$ set source $= 0$.

($\rho$ root?)