Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

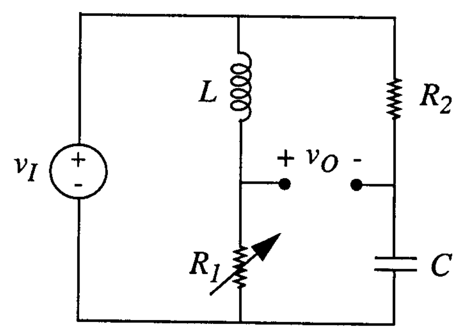
6.002 – Electronic Circuits
Spring 2000

Quiz #4
April 28, 2000
One Hour Closed Book

Professor Jeffrey Freidberg
Department of Nuclear Engineering

Problem #1

1. The circuit below can be used to determine the value of an unknown inductor L.



Assume the values of R_2 and C are known. Also, assume that R_1 is a known variable resistor. The circuit is driven by a sinusoidal voltage $v_I = \mathrm{Re}\{V_i\, e^{j\omega t}\}$.

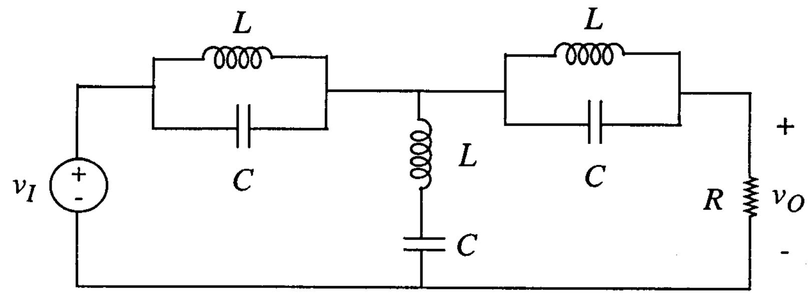
a. Let v_O take the form $v_O = \mathrm{Re}\{V_o\, e^{j\omega t}\}$. Determine V_o in terms of V_i.

b. If you did part (a) correctly, you should be able to show that R_1 can be adjusted to make $V_o = 0$. Using this value of R_1, and the known values of C and R_2, derive an expression for the unknown inductance L.

c. If the frequency ω is doubled, what effect, if any, does this have on the value of R_1 that zero's out V_o?

Problem #2

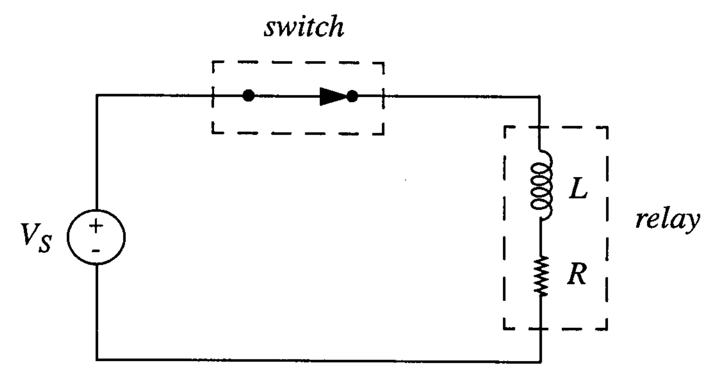
A filter circuit is driven by a sinusoidal voltage as shown below



Is this a (a) low pass, (b) high pass, (c) band pass, or (d) band stop (notch) filter. Explain briefly. No calculations are required.

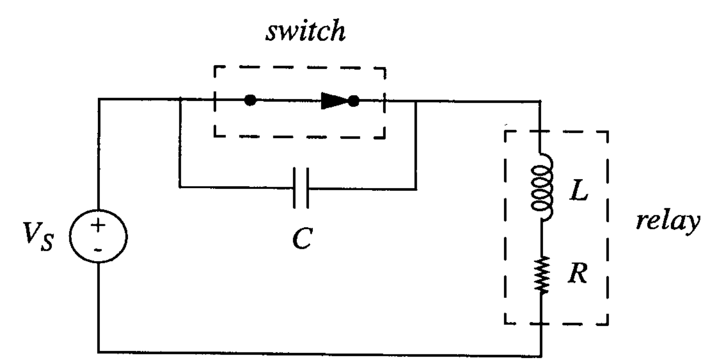
Problem #3

A DC source V_S drives a relay coil as shown below



a. Assume the switch has been closed for a long time and the relay is fully energized. It is desired to de-energize the relay by opening the switch. Why is this a bad way to break the circuit? In particular what happens to the voltage across the switch terminals? (No calculations should be needed for this part of the problem.)

b. To ease the problem in part (a) the circuit is modified as follows



Derive the differential equation and initial conditions describing the behavior of the circuit just after the switch is opened.

c. Solve the differential equation and calculate the voltage across the capacitor. Assume C has been chosen so that the circuit is in the damping regime (i.e. non-ringing). To simplify the analysis assume that $R = 4\Omega$, $L = 1H$, $C = 1/3F$ (unrealistic, but numerically simple values). Sketch the solution.

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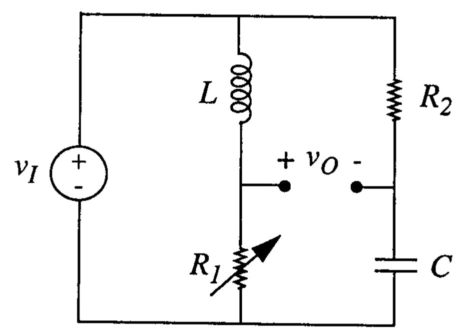
Problem #1

1. The circuit below can be used to determine the value of an unknown inductor L.



Assume the values of R_2 and C are known. Also, assume that R_1 is a known variable resistor. The circuit is driven by a sinusoidal voltage $v_I = \text{Re}\{V_i e^{j\omega t}\}$.

a. Let v_O take the form $v_O = \text{Re}\{V_o e^{j\omega t}\}$. Determine V_o in terms of V_i.

$$V_o = \frac{R_1}{R_1 + j\omega L} V_i - \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R_2} V_i$$

$$V_o = \left(\frac{R_1}{R_1 + j\omega L} - \frac{1}{1 + j\omega R_2 C} \right) V_i$$

$$\boxed{V_o = \frac{j\omega (R_1 R_2 C - L)}{(R_1 + j\omega L)(1 + j\omega R_2 C)} V_i}$$

b. If you did part (a) correctly, you should be able to show that R_1 can be adjusted to make $V_o = 0$. Using this value of R_1, and the known values of C and R_2, derive an expression for the unknown inductance L.

$$R_1 R_2 C - L = 0$$

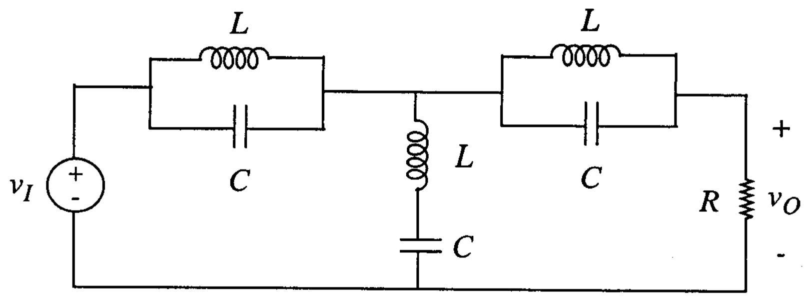
$$\boxed{L = R_1 R_2 C} \quad \left(R_1 = \frac{L}{R_2 C} \right)$$

c. If the frequency ω is doubled, what effect, if any, does this have on the value of R_1 that zero's out V_o?

none

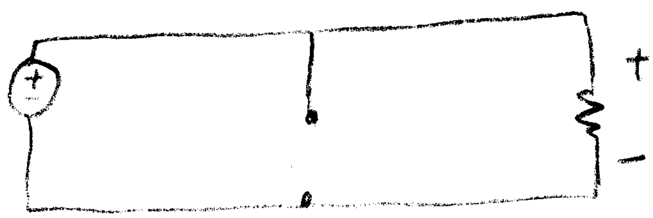
Problem #2

A filter circuit is driven by a sinusoidal voltage as shown below



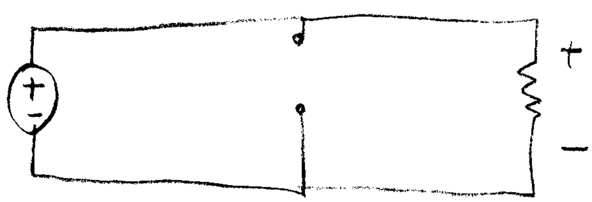
Is this a (a) low pass, (b) high pass, (c) band pass, or (d) band stop (notch) filter. Explain briefly. No calculations are required.

Low freq: C's open, L's short



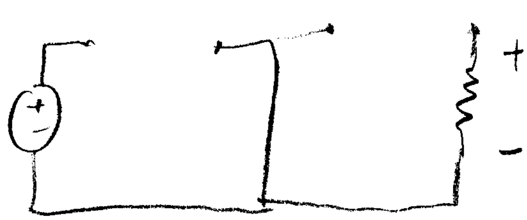
gain = 1

High freq: C's short, L's open



gain = 1

At resonant freq $\sqrt{\frac{1}{LC}}$, series LC is short, parallel LC is open

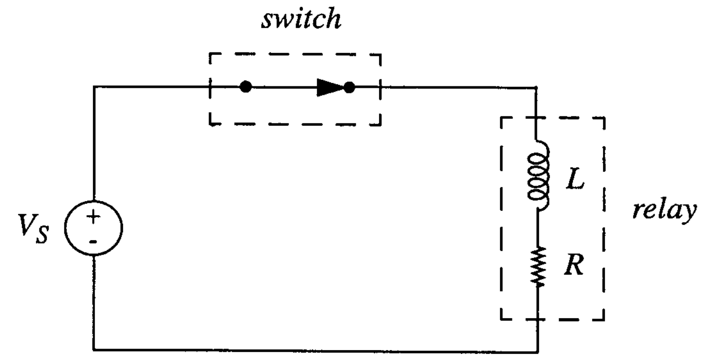


gain = 0

$\boxed{\text{d) notch filter}}$

Problem #3

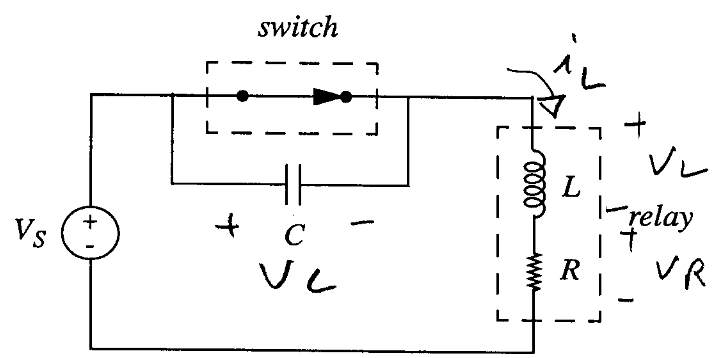
A DC source V_S drives a relay coil as shown below



a. Assume the switch has been closed for a long time and the relay is fully energized. It is desired to de-energize the relay by opening the switch. Why is this a bad way to break the circuit? In particular what happens to the voltage across the switch terminals? (No calculations should be needed for this part of the problem.)

fully energized = constant current.
opening the switch => current step
$V_L = L\frac{di}{dt}$, derivative of a step is an impulse. $V_{switch} = V_S - V_L - V_R$, and $V_R = 0$, so impulse (infinite voltage) falls across switch. (sparks!)

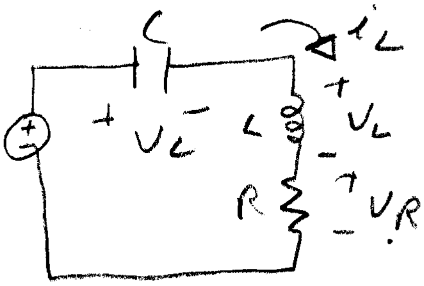
b. To ease the problem in part (a) the circuit is modified as follows



Derive the differential equation and initial conditions describing the behavior of the circuit just after the switch is opened.

Just before switch is open, in DC steady state, $i_L = \frac{V_S}{R}$, $V_R = V_S$, $V_C = 0$, $V_L = 0$

after opened:



continuity $\Rightarrow$ $\boxed{i_L = \frac{V_S}{R}}$, $V_C = 0$

KVL $\Rightarrow$ $V_L = 0$

inductor $\Rightarrow$ $\boxed{di_L/dt = 0}$

$$V_S = R i_L + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$0 = R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C} i \Rightarrow \boxed{\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0}$$

c. Solve the differential equation and calculate the voltage across the capacitor. Assume C has been chosen so that the circuit is in the damping regime (i.e. non-ringing). To simplify the analysis assume that R = 4Ω, L = 1H, C = 1/3F (unrealistic, but numerically simple values). Sketch the solution.

Characteristic equation $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \qquad \alpha = \frac{R}{2L} \quad \omega_0 = \sqrt{\frac{1}{LC}}$$

damping, $\alpha \geq \omega_0$ $\quad \alpha = 2 \quad \omega_0 = \sqrt{3}$

$$s = -2 \pm \sqrt{4-3} = -3, -1$$

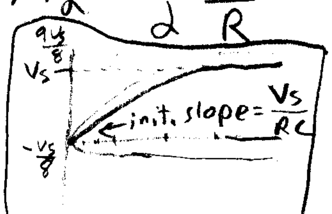
$$i = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

initial conditions:

$\frac{V_S}{R} = A_1 + A_2 \qquad 0 = -3A_1 - A_2$

$A_1 = \frac{-V_S}{2R} \qquad A_2 = \frac{3}{2}\frac{V_S}{R}$

$$i = \frac{-V_S}{2R}e^{-3t} + \frac{3}{2}\frac{V_S}{R}e^{-t}$$



$$v_C = \frac{1}{C}\int_0^t i\,dt = \frac{3}{2}\frac{V_S}{RC}(1-e^{-t}) - \frac{V_S}{6RC}(1-e^{-3t})$$

$$\boxed{v_C = V_S\left[\frac{9}{8}(1-e^{-t}) - \frac{1}{8}(1-e^{-3t})\right] \quad t>0}$$