# 6.003 (Fall 2007)

# Quiz 1 *3 October 2007*

**Name:**

**Please circle your section number:**

| *Section* | *Instructor* | *Time* |
|---|---|---|
| 1 | Jeffrey Lang | 10 |
| 2 | Jeffrey Lang | 11 |
| 3 | Karen Livescu | 11 |
| 4 | Sanjoy Mahajan | 12 |
| 5 | Antonio Torralba | 1 |
| 6 | Qing Hu | 2 |

**Explanations are not required and do not affect your grade.**

- **You have two hours. Have fun!**
- Please put your initials on all subsequent sheets.
- Enter your answers in the boxes.
- This quiz is closed book, but you may use one $8.5 \times 11$ sheet of paper (two sides).
- No calculators, computers, cell phones, music players, or other aids.

| | | | | | |
|---|---|---|---|---|---|
| *1.* | /20 ( ) | *4.* | /20 ( ) | | / *40* ( ) |
| *2.* | /20 ( ) | *5.* | /20 ( ) | | / *40* ( ) |
| *3.* | /20 ( ) | | | | / *20* ( ) |
| | */60* ( ) | | */40* ( ) | | $\boxed{/\mathbf{100}}$ ( ) |

## 1. Third-order system *[20 points]*

Find the third-order (linear, constant-coefficient) difference equation for the system whose impulse response is the signal $Y$ where

$$y[n] = \begin{cases} (n+1)^2 & \text{if } n \geq 0; \\ 0 & \text{otherwise.} \end{cases}$$

$$\boxed{y[n] = \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

## 2. Pole–zero diagrams *[20 points]*

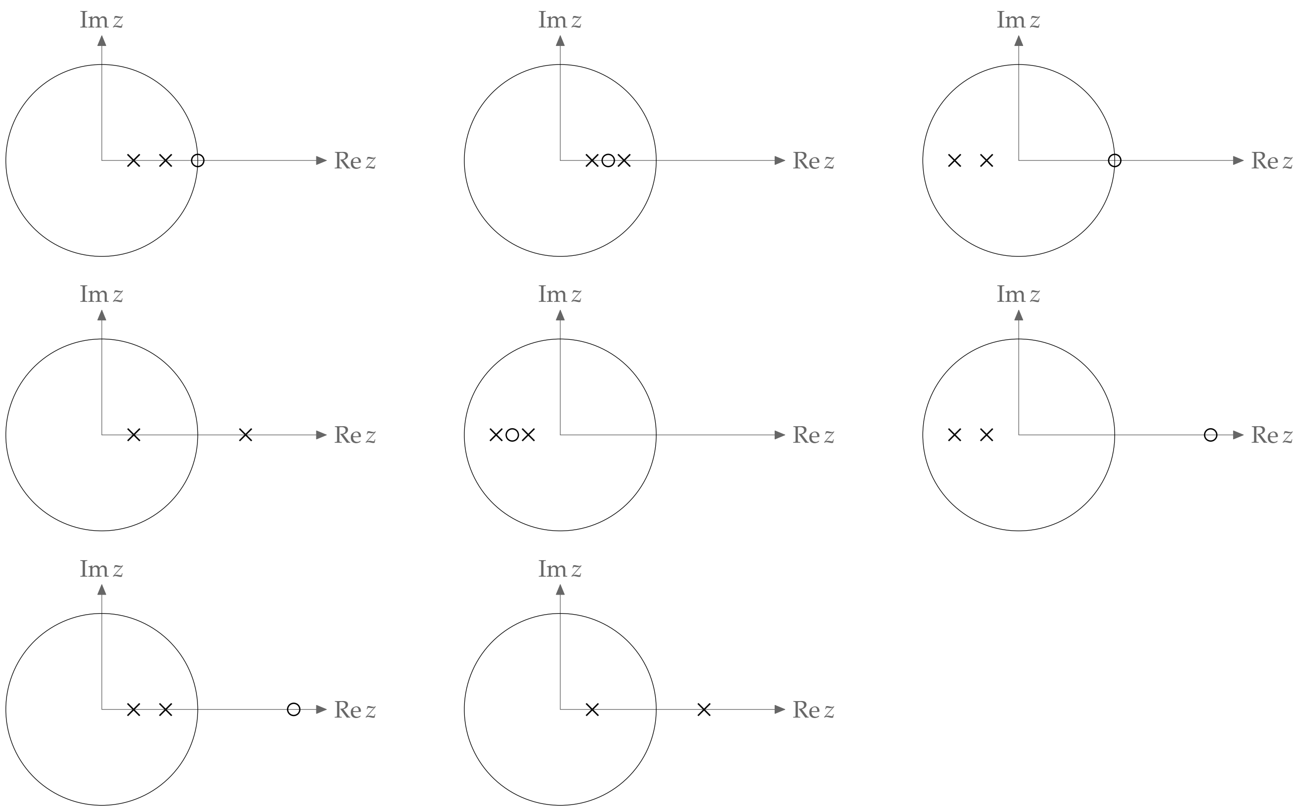
Here is a system of difference equations:

$$a[n] = \frac{1}{3}a[n-1] + x[n],$$

$$\frac{3}{2}c[n] = c[n-1] + x[n],$$

$$y[n] = 2a[n] + 3c[n].$$

**a.** Circle the correct pole–zero diagram for $Y/X$. The unit circle is shown on each diagram.

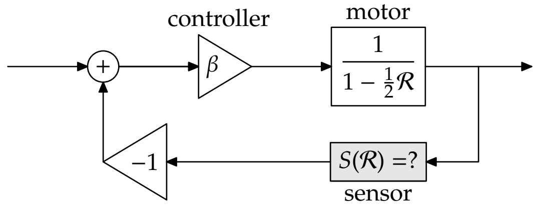


**b.** Estimate, to within 0.01%, the ratio $y[2007]/y[2005]$ when the input signal $X$ is the impulse $1, 0, 0, 0, \ldots$ ($n = 0, 1, 2, 3, \ldots$) with the system at rest prior to $n = 0$.

$$\frac{y[2007]}{y[2005]} \simeq$$

## 3. Feedback control *[20 points]*

Here is a proportional-feedback system to control a motor ($\beta > 0$):




It has the system functional

$$\frac{\beta}{1-\frac{1}{2}\mathcal{R}+\beta\mathcal{R}^2}.$$

**a.** Find the sensor functional $S(\mathcal{R})$.

| |
|---|
| $S(\mathcal{R}) =$ |

**b.** Find $\beta_0$, the smallest positive $\beta$ for which the feedback system has a pole on the unit circle.

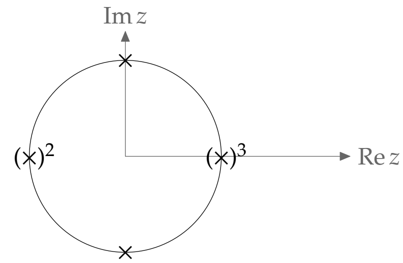
| |
|---|
| $\beta_0 =$ |

**c.** Now change the motor functional to $\mathcal{R}/(1 - \mathcal{R}/2)$. Let $\beta_1$ be the smallest positive $\beta$ for which this feedback system has a pole on the unit circle. Circle the correct option from the following list:

1. $\beta_1 = \beta_0$ because only sensor delays destabilize a feedback system.
2. $\beta_1 = \beta_0$ because the extra delay does not change the poles of the motor.
3. $\beta_1 = \beta_0$ because an extra delay in the forward path just delays the output but does not affect loop stability.
4. $\beta_1 > \beta_0$ because an extra delay in the forward path postpones any instability by one time step.
5. $\beta_1 > \beta_0$ because a delay gives the controller time to stabilize the system.
6. $\beta_1 < \beta_0$ because any delay in the controller, motor, or sensor reduces the stability of a feedback system.
7. $\beta_1 < \beta_0$ because a sensor delay adds a zero to the loop functional.

## 4. Lots of poles *[20 points]*

The figure shows a system's pole–zero diagram. All the poles are on the unit circle. The '2' and '3' means that the adjacent pole, marked with parentheses, is a repeated pole of order 2 or 3 respectively.




Which of the following choices represents the order of growth of this system's impulse response for large $n$? Circle your choice and give the required information.

1. $y[n]$ is periodic. If you choose this option, give the period:

2. $y[n] \sim An^k$ (where $A$ is a constant). If you choose this option, give $k$:

3. $y[n] \sim Az^n$ (where $A$ is a constant). If you choose this option, give $z$:

4. None of the above. If you choose this option, give a closed-form asymptotic expression for $y[n]$:

## 5. Complex sum *[20 points]*

Each diagram below shows the unit circle in the complex plane, with the origin labeled with a dot.

Each diagram illustrates the sum

$$S = \sum_{n=0}^{100} \alpha^n.$$

Circle the diagram for which $\alpha = 0.8 + 0.2j$.