6.003 (Fall 2007)

Quiz 2 *7 November 2007*

Name:

Please circle your section number:

Section	*Instructor*	*Time*
1	Jeffrey Lang	10
2	Jeffrey Lang	11
3	Karen Livescu	11
4	Sanjoy Mahajan	12
5	Antonio Torralba	1
6	Qing Hu	2

Partial credit *will* be given, according to the conceptual features that a proposed answer shares with the correct answer.

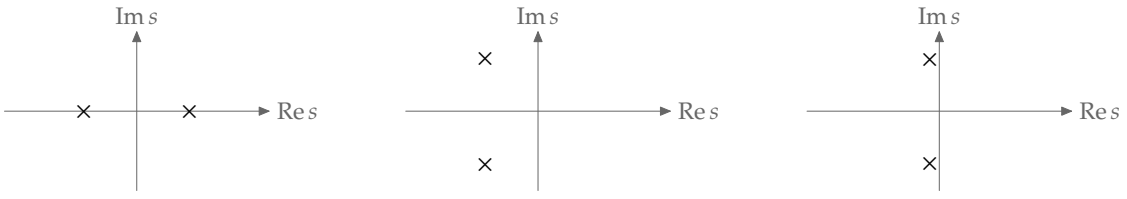
Explanations are not required and do not affect your grade.

- **You have two hours. Have fun!**
- Please put your initials on all subsequent sheets.
- Enter your answers in the boxes.
- This quiz is closed book, but you may use two 8.5 × 11 sheets of paper (four sides).
- No calculators, computers, cell phones, music players, or other aids.

1.	/25 ( )	*4.*	/10 ( )		/ *35* ( )
2.	/25 ( )	*5.*	/20 ( )		/ *45* ( )
3.	/20 ( )	*6.*	/00 ( )		/ *20* ( )
	/*70* ( )		/*30* ( )		/**100** ( )

1. Multiple representations *[25 points]*

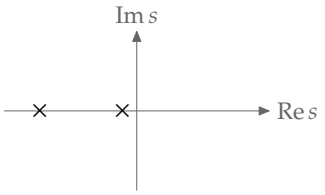
Here are several pole–zero diagrams to use for the parts of this question. On each diagram, the real and imaginary axes have the same scale; however, each diagram has its own scale.



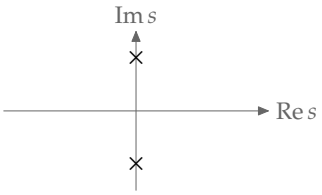
A.

B.

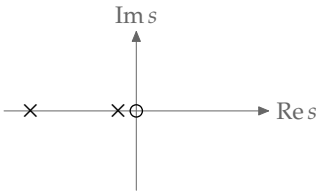
C.



D.



E.



F.

For each part, select the diagram that could be consistent with the information given. Each part has one correct answer. *If you are unsure of the correct answer, you can also select a second answer, in which case you will receive the average of the scores for the two answers.*

a. Differential equation

$$\ddot{y} = x + y,$$

where $x(t)$ is the input signal and $y(t)$ is the output signal.

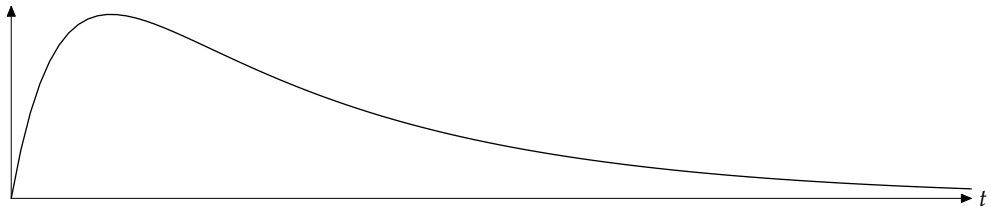
A	B	C	D	E	F

b. System function

$$H(s) = \frac{s}{1 + 8s + 8s^2} .$$

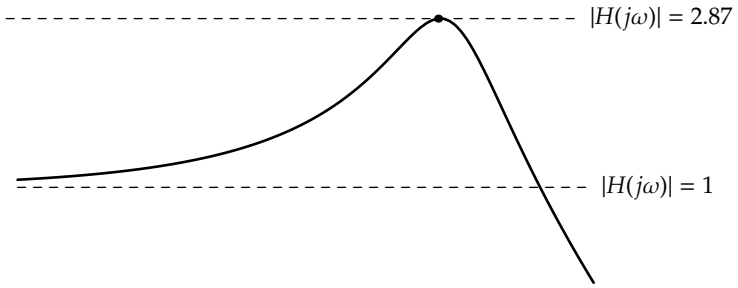
A	B	C	D	E	F

c. Step (*not* impulse) response



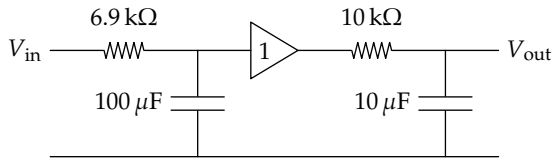
A	B	C	D	E	F

d. $|H(j\omega)|$ (log scale) vs ω (log scale)



A	B	C	D	E	F

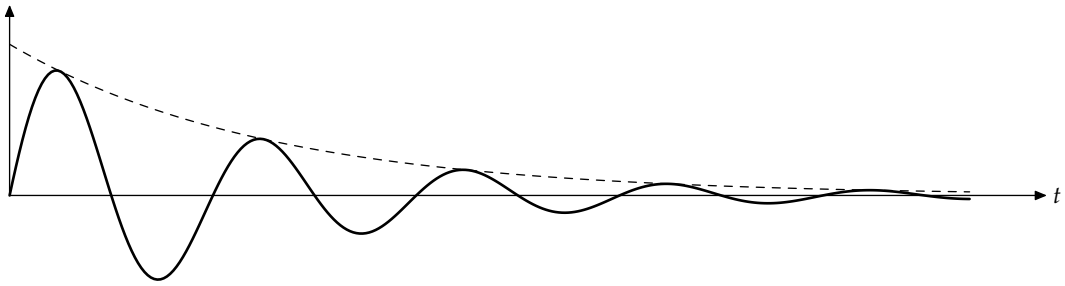
e. Circuit



A	B	C	D	E	F

2. Second-order systems *[25 points]*

a. Here is the impulse response of a second-order system:

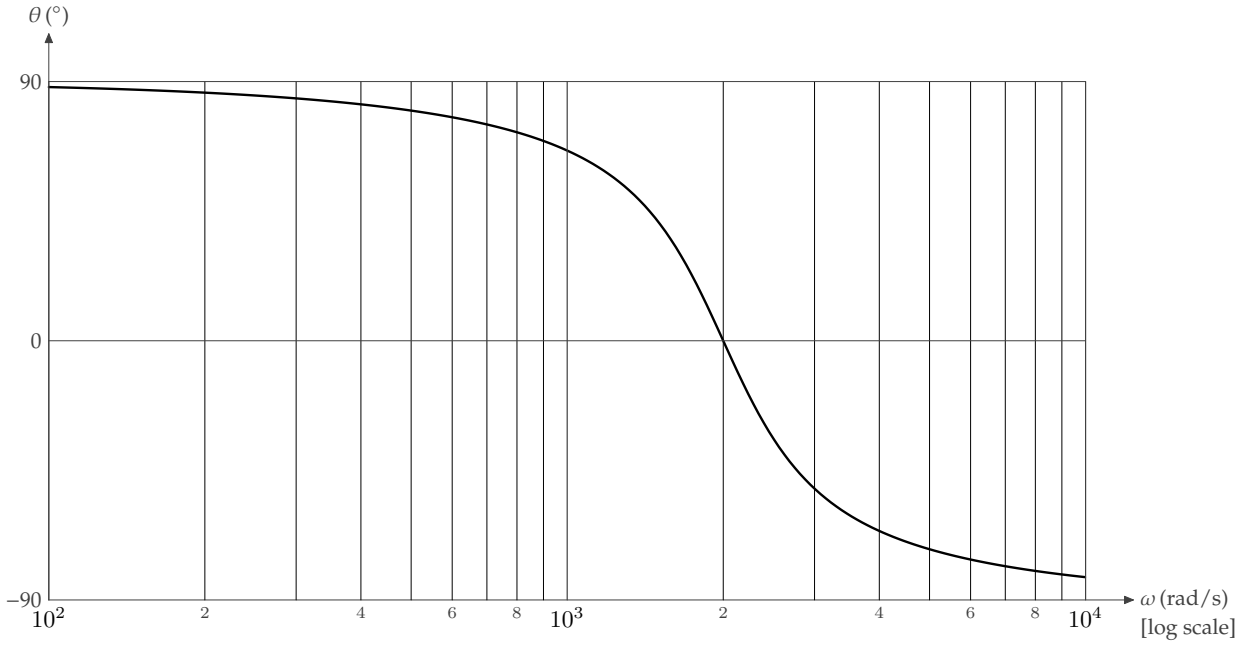


1. Estimate Q, marking your answer as an ✕ on this (logarithmic) scale:

2. Using the usual log–log Bode axes, which magnitude asymptote is flat?

low frequency	high frequency	neither	both

b. Here is the phase plot for another second-order system using the usual Bode linear–log axes:



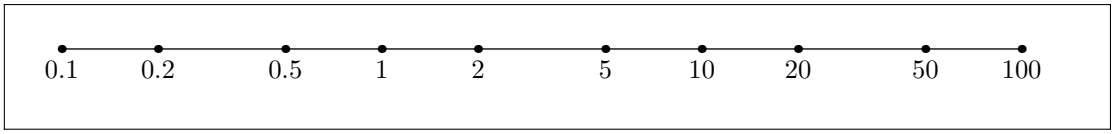
1. Suppose that we draw the *magnitude* plot for this system using the usual log–log Bode axes. At what frequency ω_0 do the high- and low-frequency asymptotes intersect? [include units]

ω_0 =

2. Referring again to the Bode magnitude plot, which asymptote is flat?

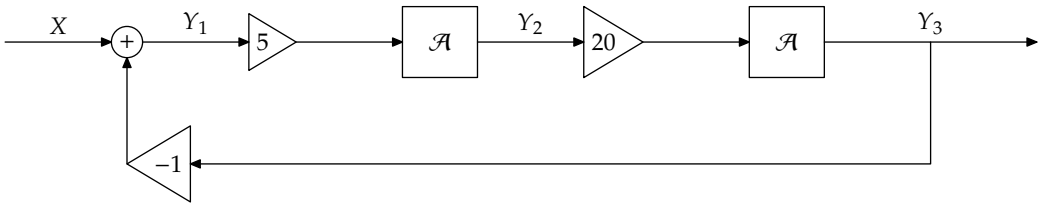
low frequency	high frequency	neither	both

3. Estimate Q, marking your answer as an $\times$ on this (logarithmic) scale:



3. Block diagram *[20 points]*

Here is a system initially at rest, represented by its block diagram:



a. What differential equation relates $y_2(t)$ and $y_3(t)$?

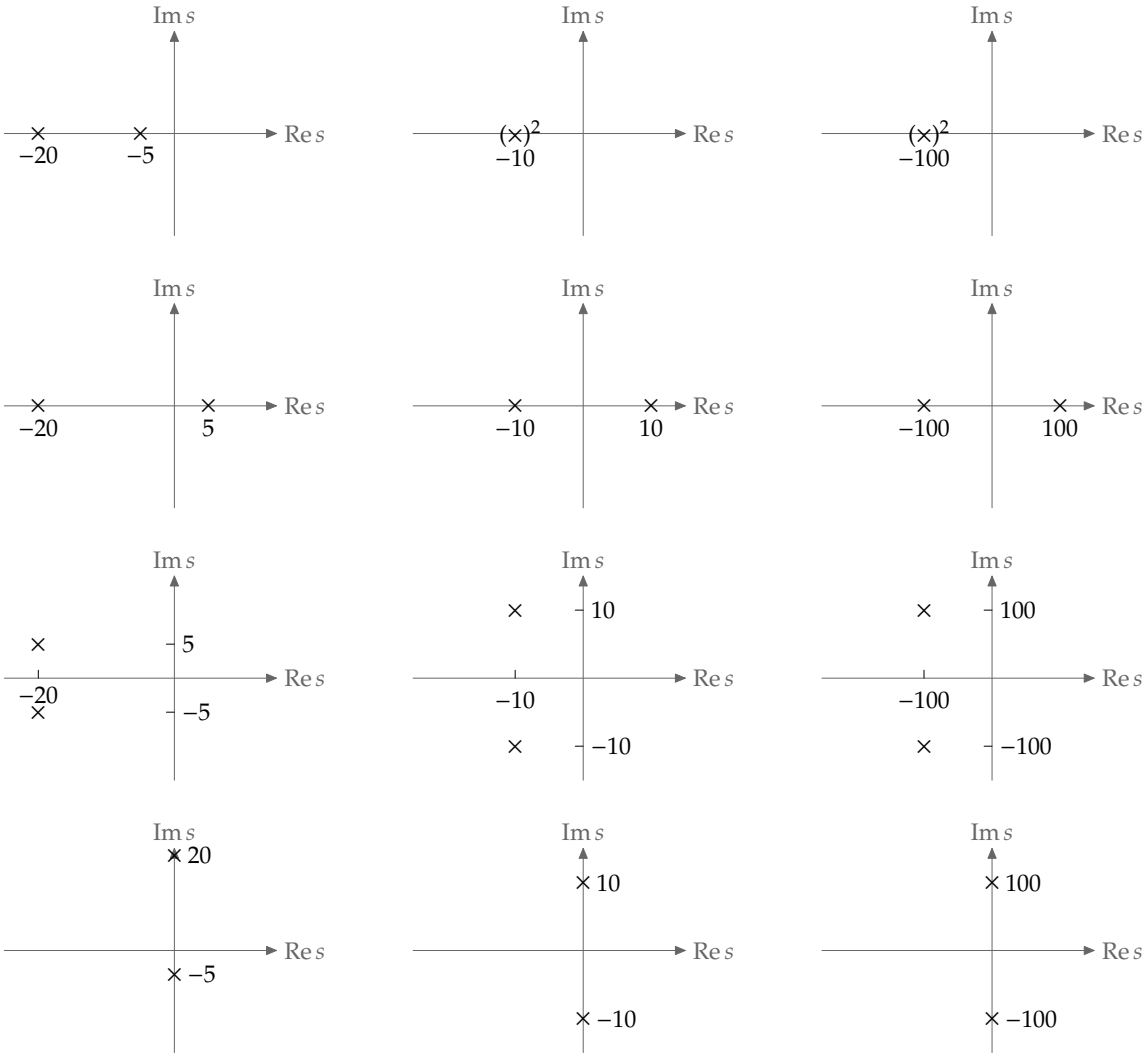
b. For the input signal $\delta(t)$ what is $y_2(0^+)$?

$y_2(0^+) =$

c. For the input signal $\delta(t)$ what is $y_3(0^+)$?

$y_3(0^+) =$

d. Circle the correct pole–zero diagram for Y_3/X. *If you are unsure of the correct answer, you can also select a second answer, in which case you will receive the average of the scores for the two answers.*



4. Cascade *[10 points]*

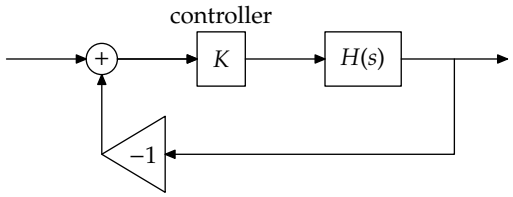
a. For the system

$$H(s) = \frac{1}{(1+\tau s)^4}$$

find the phase of the frequency response at $\omega = \sqrt{3}/\tau$.

phase =

b. Now wrap the system in proportional feedback:



At what controller gain K does the whole feedback system first become unstable as K sweeps from 0 to ∞?

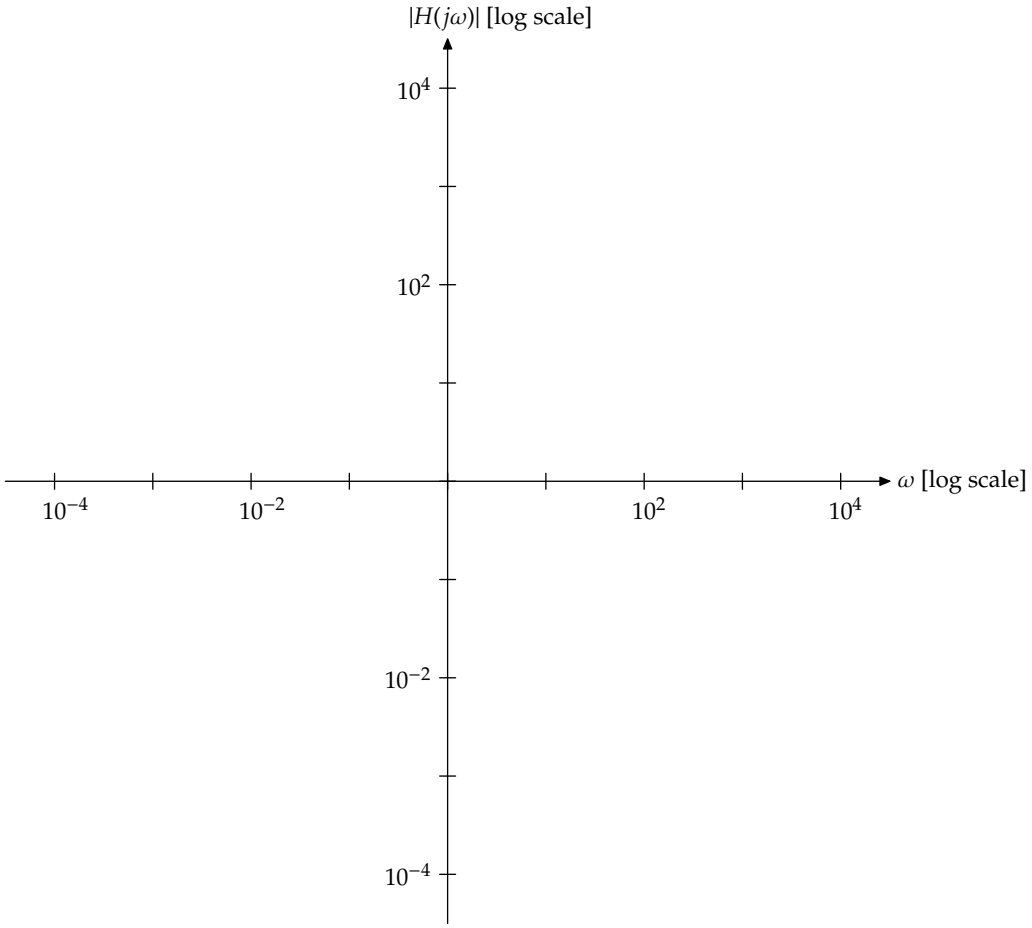
$K =$

5. System analysis *[20 points]*

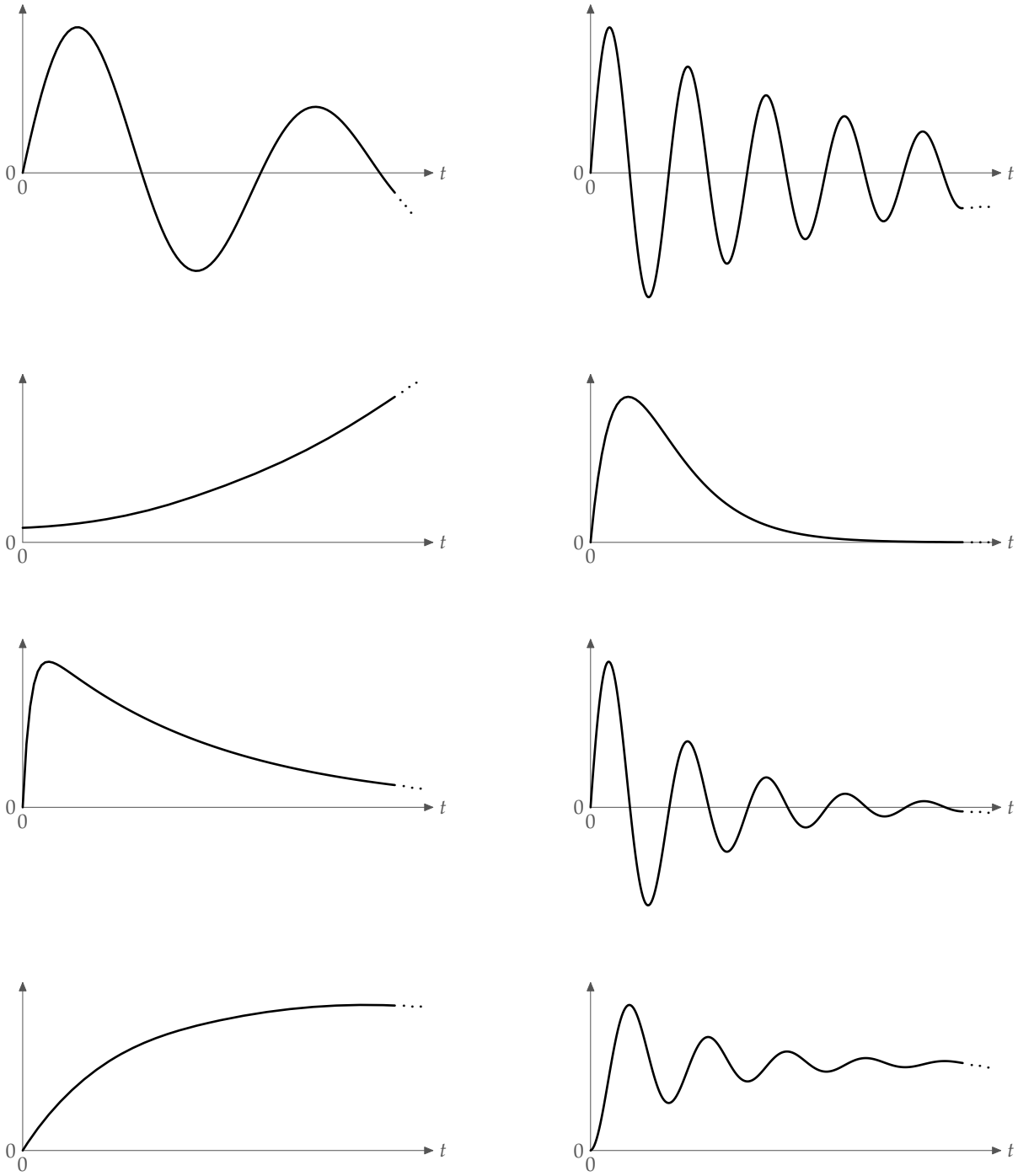
Consider the system represented by the system function

$$H(s) = \frac{4 + s + 4s^2}{s^2}.$$

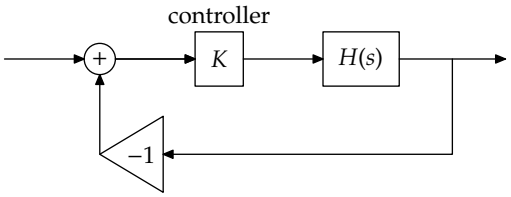
a. On these log–log axes, sketch the Bode magnitude plot giving the straight-line asymptotes and the smooth curve.



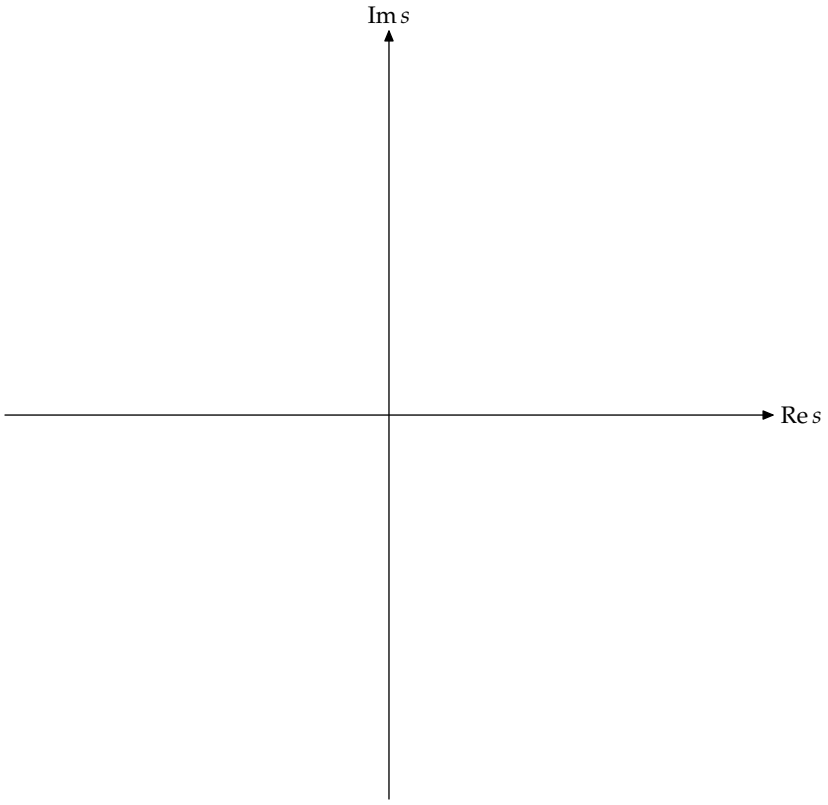
b. Circle the correct step response (*not* impulse response). The axes are all linear but each has its own scale. *If you are unsure of the correct answer, you can also select a second answer, in which case you will receive the average of the scores for the two answers.*



c. Now you wrap the system in a proportional-control feedback loop:



Sketch the location of the poles of the overall system as K sweeps from 0 to ∞:



6. *Bonus question:* Square roots forever *[extra credit of 5 points]*

$$2 \times \sqrt{2} \times \sqrt{\sqrt{2}} \times \sqrt{\sqrt{\sqrt{2}}} \times \cdots =$$