6.003 (Fall 2009)

Quiz #2 *October 28, 2009*

Name:

Kerberos Username:

Please circle your section number:

Section	*Instructor*	*Time*
1	Marc Baldo	10 am
2	Marc Baldo	11 am
3	Elfar Adalsteinsson	1 pm
4	Elfar Adalsteinsson	2 pm

Partial credit will be given for answers that demonstrate some but not all of the important conceptual issues.

Explanations are not required and will not affect your grade.

You have **two hours.**

Please put your initials on all subsequent sheets.

Enter your answers in the boxes.

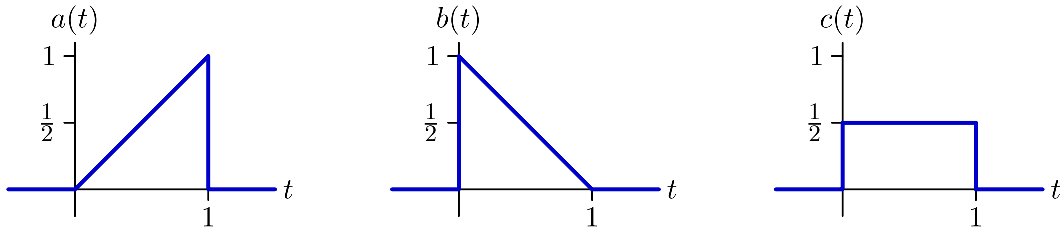
This quiz is closed book, but you may use two 8.5×11 sheets of paper (four sides total).

No calculators, computers, cell phones, music players, or other aids.

1	/18
2	/16
3	/12
4	/20
5	/16
6	/18
Total	/100

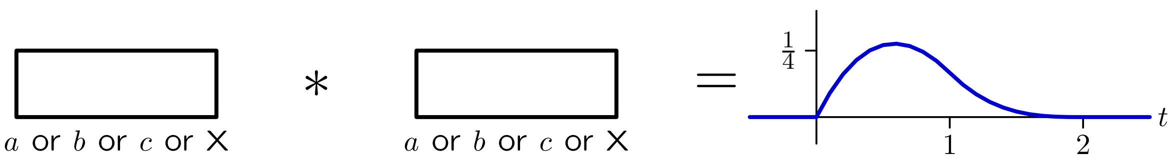
1. Convolutions *[18 points]*

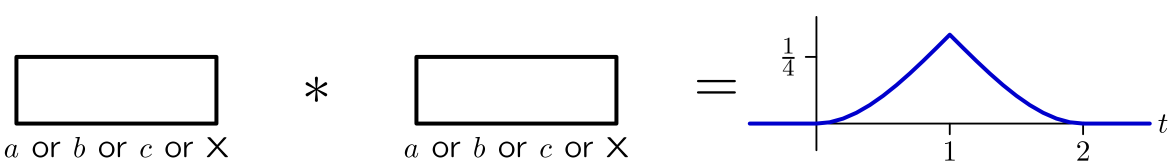
Consider the convolution of two of the following signals.



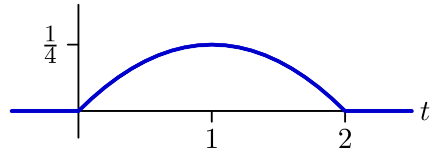
Determine if each of the following signals can be constructed by convolving (a or b or c) with (a or b or c). If it can, indicate which signals should be convolved. If it cannot, put an X in both boxes.

Notice that there are ten possible answers: $(a * a)$, $(a * b)$, $(a * c)$, $(b * a)$, $(b * b)$, $(b * c)$, $(c * a)$, $(c * b)$, $(c * c)$, or (X,X). Notice also that the answer may not be unique.

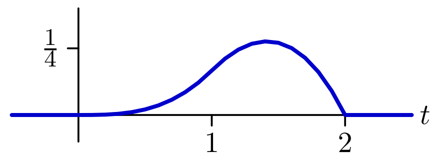




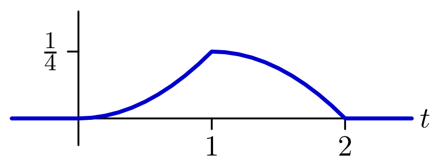
a or b or c or X $\quad * \quad$ a or b or c or X $\quad =$



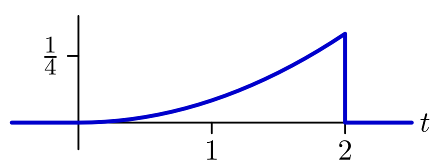
a or b or c or X $\quad * \quad$ a or b or c or X $\quad =$



a or b or c or X $\quad * \quad$ a or b or c or X $\quad =$



a or b or c or X $\quad * \quad$ a or b or c or X $\quad =$



2. Laplace transforms *[16 points]*

Determine if the Laplace transform of each of the following signals exists. If it does, write **yes** in the box. If it does not, write **no** in the box. If you don't know, write **?** in the box.

Grading: +2 points for each correct answer; −2 points for each incorrect answer; 0 points for each ? or blank response.

$x_1(t) = e^{-t}u(t) + e^{-2t}u(t) + e^{-3t}u(t)$ $X_1(s)$ exists? (**yes** or **no** or **?**): ☐

$x_2(t) = e^{-t}u(-t) + e^{-2t}u(t) + e^{-3t}u(t)$ $X_2(s)$ exists? (**yes** or **no** or **?**): ☐

$x_3(t) = e^{-t}u(t) + e^{-2t}u(-t) + e^{-3t}u(t)$ $X_3(s)$ exists? (**yes** or **no** or **?**): ☐

$x_4(t) = e^{-t}u(-t) + e^{-2t}u(-t) + e^{-3t}u(t)$ $X_4(s)$ exists? (**yes** or **no** or **?**): ☐

$x_5(t) = e^{-t}u(t) + e^{-2t}u(t) + e^{-3t}u(-t)$ $X_5(s)$ exists? (**yes** or **no** or **?**): ☐

$x_6(t) = e^{-t}u(-t) + e^{-2t}u(t) + e^{-3t}u(-t)$ $X_6(s)$ exists? (**yes** or **no** or **?**): ☐

$x_7(t) = e^{-t}u(t) + e^{-2t}u(-t) + e^{-3t}u(-t)$ $X_7(s)$ exists? (**yes** or **no** or **?**): ☐

$x_8(t) = e^{-t}u(-t) + e^{-2t}u(-t) + e^{-3t}u(-t)$ $X_8(s)$ exists? (**yes** or **no** or **?**): ☐

3. Impulse response *[12 points]*

Sketch a block diagram for a CT system with impulse response

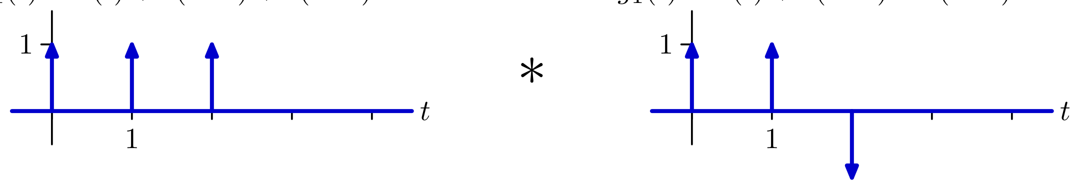
$$h(t) = \left(1 - te^{-t}\right) e^{-2t} u(t)\,.$$

The block diagram should contain only adders, gains, and integrators.

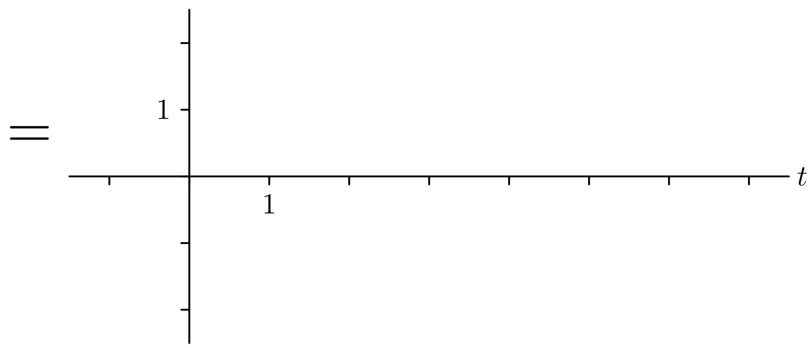
4. Convolutions *[20 points]*

Sketch the signal that results for each of the following parts.

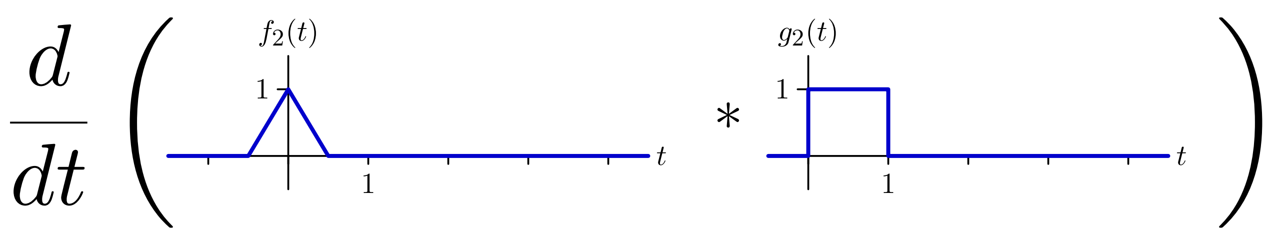
$f_1(t) = \delta(t) + \delta(t-1) + \delta(t-2)$ $\quad * \quad$ $g_1(t) = \delta(t) + \delta(t-1) - \delta(t-2)$



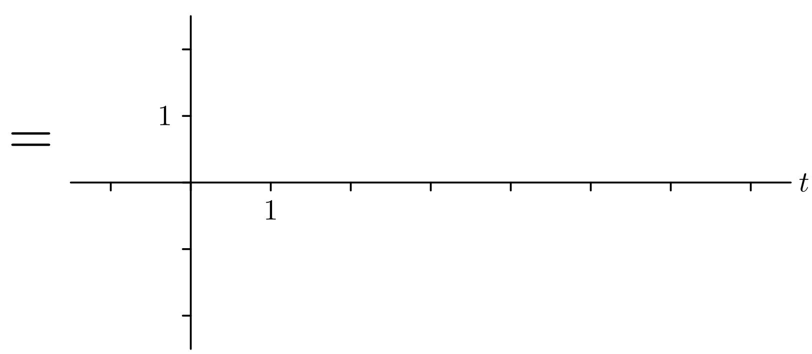
$(f_1 * g_1)(t)$



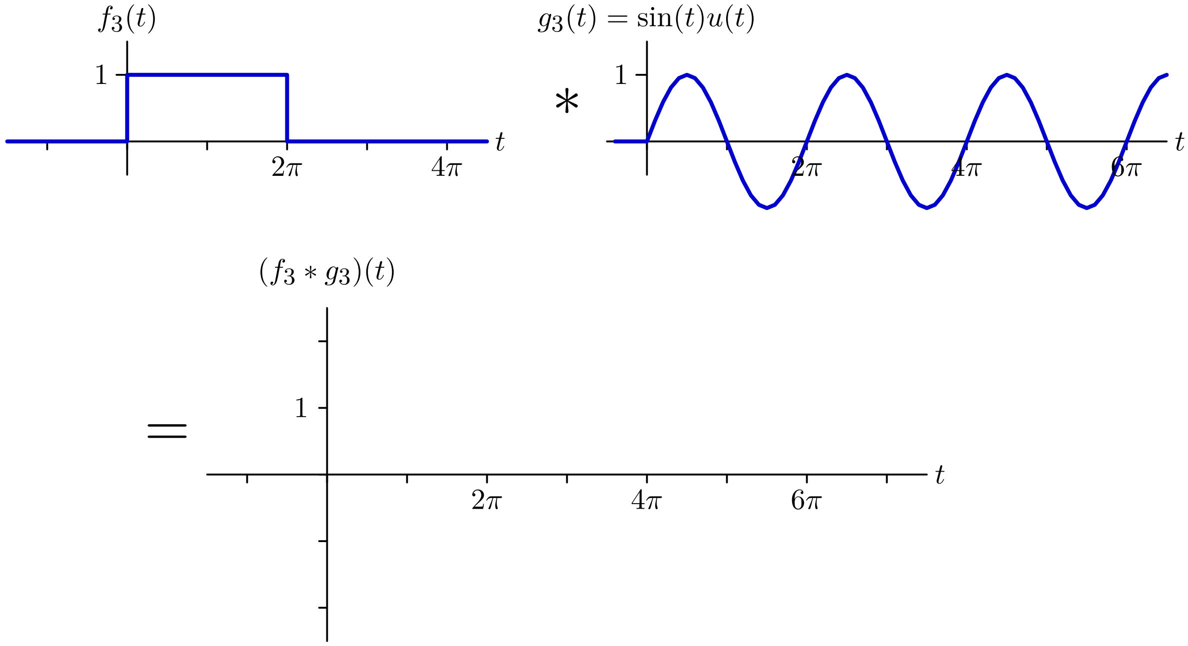
Label the important features of your results!



$\frac{d}{dt}\Big(f_2 * g_2\Big)(t)$



Label the important features of your results!



Label the important features of your results!

Given

$$f_4[n] = 2^n u[-n] \qquad \text{and} \qquad g_4[n] = \left(\frac{1}{3}\right)^n u[n]$$

enter the following numbers:

$(f_4 * g_4)[-2] =$ [ ]

$(f_4 * g_4)[-1] =$ [ ]

$(f_4 * g_4)[0] =$ [ ]

$(f_4 * g_4)[1] =$ [ ]

$(f_4 * g_4)[2] =$ [ ]

5. Z transform *[16 points]*

Let $X(z)$ represent the Z transform of $x[n]$, and let $r_0 < |z| < r_1$ represent its region of convergence (ROC).

Let $x[n]$ be represented as the sum of even and odd parts

$$x[n] = x_e[n] + x_o[n]$$

where $x_e[n] = x_e[-n]$ and $x_o[n] = -x_o[-n]$.

a. Under what conditions does the Z transform of $x_e[n]$ exist?

conditions: | |
|---|
| |

b. Assuming the conditions given in part a, find an expression for the Z transform of $x_e[n]$, including its region of convergence.

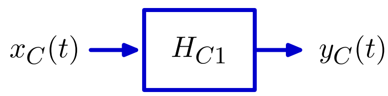
Z transform:	
ROC:	

6. DT approximation of a CT system *[18 points]*

Let H_{C1} represent a **causal** CT system that is described by

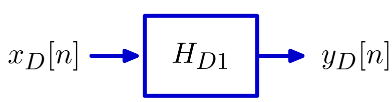
$$\dot{y}_C(t) + 3y_C(t) = x_C(t)$$

where $x_C(t)$ represents the input signal and $y_C(t)$ represents the output signal.



a. Determine the pole(s) of H_{C1}, and enter them in the box below.

Your task is to design a **causal** DT system H_{D1} to approximate the behavior of H_{C1}.



Let $x_D[n] = x_C(nT)$ and $y_D[n] = y_C(nT)$ where T is a constant that represents the time between samples. Then approximate the derivative as

$$\frac{dy_C(t)}{dt} = \frac{y_C(t+T) - y_C(t)}{T} .$$

b. Determine an expression for the pole(s) of H_{D1}, and enter the expression in the box below.

c. Determine the range of values of T for which H_{D1} is stable and enter the range in the box below.

Now consider a second-order **causal** CT system H_{C2}, which is described by

$$\ddot{y}_C(t) + 100 y_C(t) = x_C(t)\,.$$

d. Determine the pole(s) of H_{C2}, and enter them in the box below.

Design a **causal** DT system H_{D2} to approximate the behavior of H_{C2}. Approximate derivatives as before:

$$\dot{y_C}(t) = \frac{dy_C(t)}{dt} = \frac{y_C(t+T) - y_C(t)}{T} \quad \text{and}$$

$$\frac{d^2 y_C(t)}{dt^2} = \frac{\dot{y_C}(t+T) - \dot{y_C}(t)}{T}\,.$$

e. Determine an expression for the pole(s) of H_{D2}, and enter the expression in the box below.

f. Determine the range of values of T for which H_{D2} stable and enter the range in the box below.

Worksheet (intentionally blank)

Worksheet (intentionally blank)