6.003 Homework #6

*Due at the beginning of recitation on **October 19, 2011**.*

Problems

1. Maximum gain

For each of the following systems, find the frequency ω_m for which the magnitude of the gain is greatest.

a. $\dfrac{1}{1+s+s^2}$

$\omega_m = \boxed{}$

b. $\dfrac{s}{1+s+s^2}$

$\omega_m = \boxed{}$

c. $\dfrac{s^2}{1+s+s^2}$

$\omega_m = \boxed{}$

Compare the ω_m for these systems and make sure that you can explain qualitatively any similarities or differences.

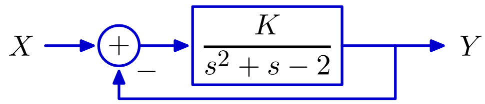
2. Phase

For a second-order system with poles at -1 and -4 (and no zeros), find the frequency at which the phase is $-90°$, using any method except for the vector method. Then illustrate and confirm that result using the vector method.

$\omega = \boxed{}$

3. CT stability

Consider the following feedback system in which the box represents a causal LTI CT system that is represented by its system function.



a. Determine the range of K for which this feedback system is stable.

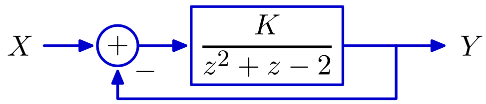
range of K:

b. Determine the range of K for which this feedback system has real-valued poles.

range of K:

4. DT stability

Consider the following feedback system in which the box represents a causal LTI DT system that is represented by its system function.



a. Determine the range of K for which this feedback system is stable.

range of K:

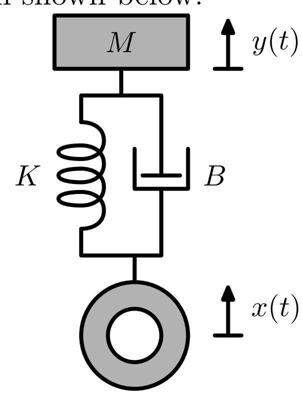
b. Determine the range of K for which this feedback system has real-valued poles.

range of K:

Engineering Design Problems

5. Automotive suspension

Wheels are attached to an automobile through a suspension system that is designed to minimize the vibrations of the passenger compartment that result when traveling over bumpy terrain. The suspension system consists of a spring and shock absorber that are both compressed when the wheel passes over a bump, so that the sudden motion of the wheel is not directly transmitted to the passenger compartment. The spring generates a force to hold the passenger compartment at a desired distance above the surface of the road, and the shock absorber adds frictional damping. In this problem, you will determine how much damping is desireable by analyzing a simple model of an automobile's suspension system shown below.



The model consists of a mass M that represents the mass of the car, which is connected through a spring and dashpot to the wheel. The vertical displacement of the wheel from it's equilibrium position is taken as the input $x(t)$. The vertical displacement of the mass from it's equilibrium position is taken as the output $y(t)$. The spring is assumed to obey Hooke's law, so that the force it generates is a constant K times the amount that the spring is compressed relative to it's equilibrium compression. The shock absorber is assumed to generate a force that is a constant B times the velocity with which the shock absorber is compressed. Notice that by referring $x(t)$ and $y(t)$ to their equilibrium positions, the force due to gravity can be ignored. Assume that $M = 1$ and $K = 1$.

a. Determine the differential equation that relates the input $x(t)$ and output $y(t)$.

b. Determine and plot the impulse response of the system when $B = 0$. Based on this result, give a physical explanation of the problem that would result if there were no shock absorber in the system.

c. Determine an expression for the smallest positive damping constant B for which the poles of the system have real values. Sketch the impulse response of the system for this value of B. Based on this result, give a physical explanation of how the shock absorber improves performance of the suspension system.

d. Consider what would happen if B were very large. Sketch the impulse response for the system if $B = 100$. Describe how this response might be less desireable than that in part c. Provide a physical explanation for how a stiff shock absorber can degrade system performance.

6. Dial tones

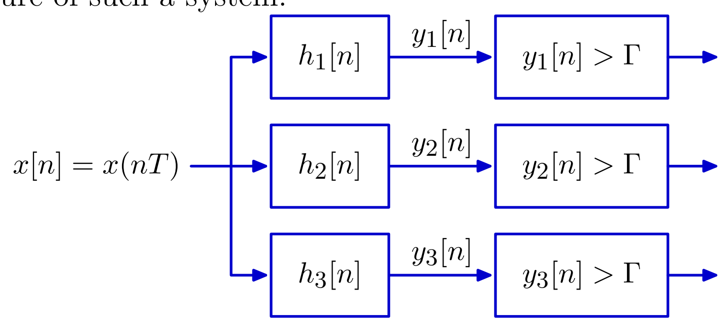
Pressing the buttons on a touch-tone phone generates tones that are used for dialing. Each button produces a pair of tones of the form

$$x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$$

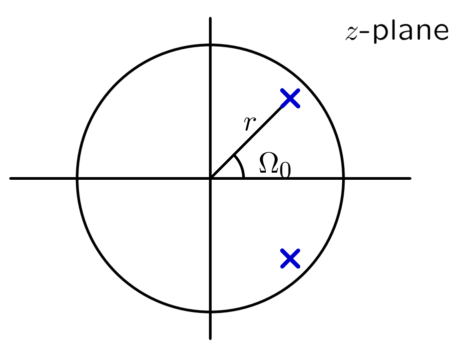
where f_1 and f_2 code the row and column of the button as shown in the following table.

	f_2[Hz]		
f_1[Hz]	1209	1336	1477
697	1	2	3
770	4	5	6
852	7	8	9
941	$*$	0	#

This problem concerns the design of a system to detect the row and column numbers that were pressed by analyzing the signal $x(t)$. The following block diagram illustrates the basic structure of such a system.



The input $x(t)$ is first sampled with $T = 10^{-4}$ seconds. The samples are then passed through LTI systems that generate intermediate signals so that $y_1[n]$ is large when a button in column 1 is pressed, $y_2[n]$ is large when a button in column 2 is pressed, and $y_3[n]$ is large when a button in column 3 is pressed. These intermediate signals are then passed through detectors that determine when the signals are bigger than a threshold value Γ. Your task is to design the LTI systems. Each should consist of a system with 2 poles of the form shown in the following pole-zero diagram.



Such systems can be simulated by finding the difference equation that corresponds to the system and then iteratively solving that difference equation.

a. Determine values of r and Ω_0 so that the $h_1[n]$ system generates a large response when the "1" key is pressed and a small response when the "2" or "'3" keys are pressed. Your solution should work not only when the input consists of a single key press but also when it consists of sequences of key presses (as when dialing a phone number). Submit hardcopies of your code to generate $y_1[n]$ along with a plot of $y_1[n]$.

b. Describe how the choice of Ω_0 affects the output signal $y_1[n]$.

c. Describe how the choice of r affects the output signal $y_1[n]$. In particular, what limits the maximum acceptable value of r? Also, what limits the minimum acceptable value of r?