

6.003: Signals and Systems

Frequency Response

October 6, 2011

Review

Last time, we saw how a linear, time-invariant (LTI) system can be characterized by its unit-sample/impulse response.

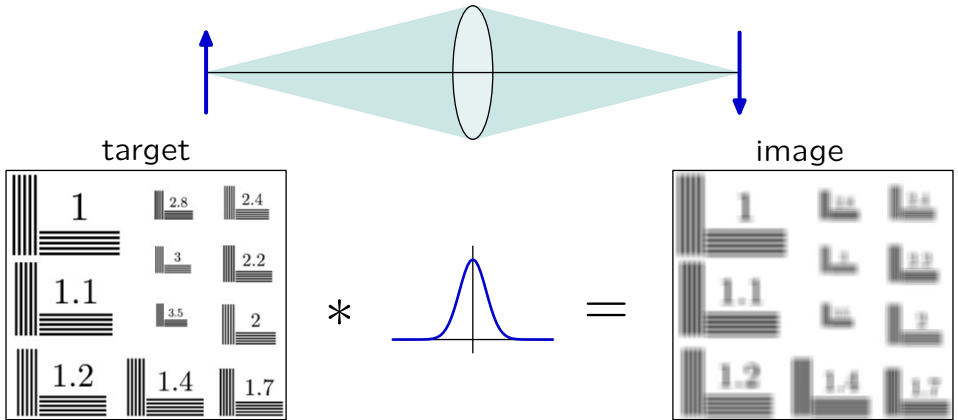
$$\text{DT: } y[n] = (x * h)[n] = \sum_{k=-\infty}^{\infty} x[k]h[n - k]$$

$$\text{CT: } y(t) = (x * h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

Characterizing a system by its unit-sample/impulse response is especially insightful for some systems.

Microscope

Blurring can be represented by convolving the image with the optical “point-spread-function” (3D impulse response).

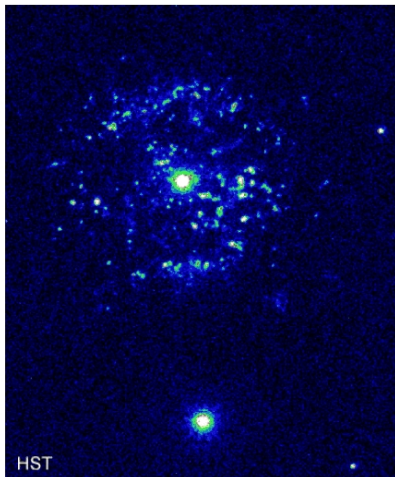
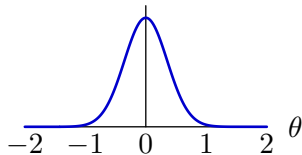


Blurring is inversely related to the diameter of the lens.

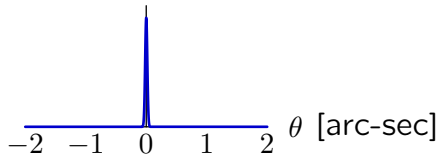
Hubble Space Telescope



optical + atmospheric
blurring



optical blurring



Frequency Response

Today we will investigate a different way to characterize a system: the **frequency response**.

Many systems are naturally described by their responses to sinusoids.

Example: audio systems

Check Yourself

How were frequencies modified in following music clips?

HF: high frequencies

↑: increased

LF: low frequencies

↓: decreased

- | | clip 1 | clip 2 |
|----|-------------------|--------|
| 1. | HF↑ | HF↓ |
| 2. | LF↑ | LF↓ |
| 3. | HF↑ | LF↓ |
| 4. | LF↑ | HF↓ |
| 5. | none of the above | |

Check Yourself

original

clip 1: HF↑ HF↓ LF↑ LF↓ none

original

clip 1: HF↑ HF↓ LF↑ LF↓ none

original

clip 2: HF↑ HF↓ LF↑ LF↓ none

original

clip 2: HF↑ HF↓ LF↑ LF↓ none

- | | clip 1 | clip 2 |
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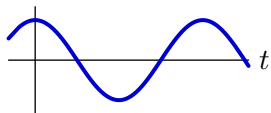
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| 2. | LF↑ | LF↓ |
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| 4. | LF↑ | HF↓ |
| 5. | none of the above | |

Frequency Response Preview

If the input to a linear, time-invariant system is an eternal sinusoid, then the output is also an eternal sinusoid:

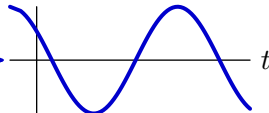
- same frequency
- possibly different amplitude, and
- possibly different phase angle.

$$x(t) = \cos(\omega t)$$



LTI
system

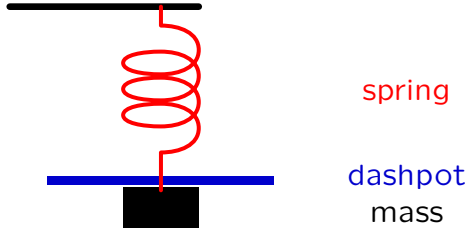
$$y(t) = M \cos(\omega t + \phi)$$



The **frequency response** is a plot of the magnitude M and angle ϕ as a function of frequency ω .

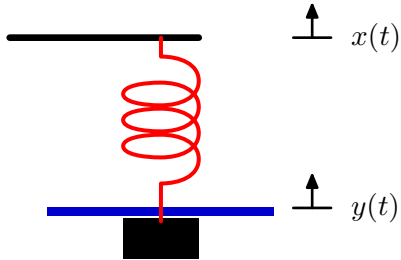
Example

Mass, spring, and dashpot system.



Demonstration

Measure the frequency response of a mass, spring, dashpot system.



Frequency Response

Calculate the frequency response.

Methods

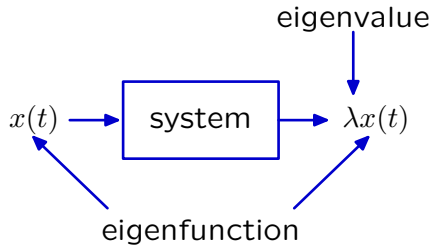
- solve differential equation
 - find particular solution for $x(t) = \cos \omega_0 t$
- find impulse response of system
 - convolve with $x(t) = \cos \omega_0 t$

New method

- use eigenfunctions and eigenvalues

Eigenfunctions

If the output signal is a scalar multiple of the input signal, we refer to the signal as an eigenfunction and the multiplier as the eigenvalue.



Check Yourself: Eigenfunctions

Consider the system described by

$$\dot{y}(t) + 2y(t) = x(t).$$

Determine if each of the following functions is an eigenfunction of this system. If it is, find its eigenvalue.

1. e^{-t} for all time
2. e^t for all time
3. e^{jt} for all time
4. $\cos(t)$ for all time
5. $u(t)$ for all time

Check Yourself: Eigenfunctions

$$\dot{y}(t) + 2y(t) = x(t)$$

1. e^{-t} : $-\lambda e^{-t} + 2\lambda e^{-t} = e^{-t} \rightarrow \lambda = 1$

2. e^t : $\lambda e^t + 2\lambda e^t = e^t \rightarrow \lambda = \frac{1}{3}$

3. e^{jt} : $j\lambda e^{jt} + 2\lambda e^{jt} = e^{jt} \rightarrow \lambda = \frac{1}{j+2}$

4. $\cos t$: $-\lambda \sin t + 2\lambda \cos t = \cos t \rightarrow$ not possible!

5. $u(t)$: $\lambda \delta(t) + 2\lambda u(t) = u(t) \rightarrow$ not possible!

Check Yourself: Eigenfunctions

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$$\dot{y}(t) + 2y(t) = x(t).$$

Determine if each of the following functions is an eigenfunction of this system. If it is, find its eigenvalue.

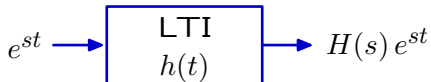
1. e^{-t} for all time \checkmark $\lambda = 1$
2. e^t for all time \checkmark $\lambda = \frac{1}{3}$
3. e^{jt} for all time \checkmark $\lambda = \frac{1}{j+2}$
4. $\cos(t)$ for all time \times
5. $u(t)$ for all time \times

Complex Exponentials

Complex exponentials are eigenfunctions of LTI systems.

If $x(t) = e^{st}$ and $h(t)$ is the impulse response then

$$y(t) = (h * x)(t) = \int_{-\infty}^{\infty} h(\tau) e^{s(t-\tau)} d\tau = e^{st} \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau = H(s) e^{st}$$

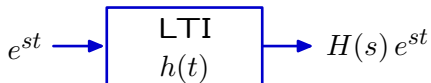


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Eternal sinusoids are sums of complex exponentials.

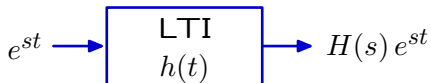
$$\cos \omega_0 t = \frac{1}{2} \left(e^{j\omega_0 t} + e^{-j\omega_0 t} \right)$$

Complex Exponentials

Complex exponentials are eigenfunctions of LTI systems.

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Eternal sinusoids are sums of complex exponentials.

$$\cos \omega_0 t = \frac{1}{2} \left(e^{j\omega_0 t} + e^{-j\omega_0 t} \right)$$

Furthermore, the eigenvalue associated with e^{st} is $H(s)$!

Rational System Functions

Eigenvalues are particularly easy to evaluate for systems represented by linear differential equations with constant coefficients.

Then the system function is a ratio of polynomials in s .

Example:

$$\ddot{y}(t) + 3\dot{y}(t) + 4y(t) = 2\ddot{x}(t) + 7\dot{x}(t) + 8x(t)$$

Then

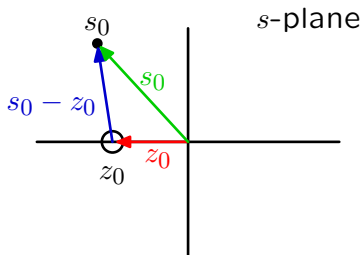
$$H(s) = \frac{2s^2 + 7s + 8}{s^2 + 3s + 4} \equiv \frac{N(s)}{D(s)}$$

Vector Diagrams

The value of $H(s)$ at a point $s = s_0$ can be determined graphically using vectorial analysis.

Factor the numerator and denominator of the system function to make poles and zeros explicit.

$$H(s_0) = K \frac{(s_0 - z_0)(s_0 - z_1)(s_0 - z_2) \cdots}{(s_0 - p_0)(s_0 - p_1)(s_0 - p_2) \cdots}$$



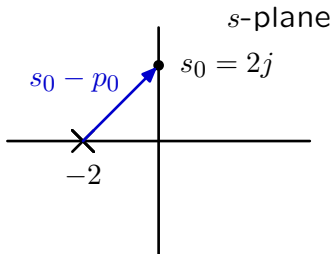
Each factor in the numerator/denominator corresponds to a vector from a zero/pole (here z_0) to s_0 , the point of interest in the s -plane.

Vector Diagrams

Example: Find the response of the system described by

$$H(s) = \frac{1}{s+2}$$

to the input $x(t) = e^{2jt}$ (for all time).



The denominator of $H(s)|_{s=2j}$ is $2j+2$, a vector with length $2\sqrt{2}$ and angle $\pi/4$. Therefore, the response of the system is

$$y(t) = H(2j)e^{2jt} = \frac{1}{2\sqrt{2}}e^{-\frac{j\pi}{4}}e^{2jt}.$$

Vector Diagrams

The value of $H(s)$ at a point $s = s_0$ can be determined by combining the contributions of the vectors associated with each of the poles and zeros.

$$H(s_0) = K \frac{(s_0 - z_0)(s_0 - z_1)(s_0 - z_2) \cdots}{(s_0 - p_0)(s_0 - p_1)(s_0 - p_2) \cdots}$$

The magnitude is determined by the product of the magnitudes.

$$|H(s_0)| = |K| \frac{|(s_0 - z_0)||s_0 - z_1)||s_0 - z_2| \cdots}{|(s_0 - p_0)||s_0 - p_1)||s_0 - p_2| \cdots}$$

The angle is determined by the sum of the angles.

$$\angle H(s_0) = \angle K + \angle(s_0 - z_0) + \angle(s_0 - z_1) + \cdots - \angle(s_0 - p_0) - \angle(s_0 - p_1) - \cdots$$

Frequency Response

Response to eternal sinusoids.

Let $x(t) = \cos \omega_0 t$ (for all time). Then

$$x(t) = \frac{1}{2} \left(e^{j\omega_0 t} + e^{-j\omega_0 t} \right)$$

and the response to a sum is the sum of the responses.

$$y(t) = \frac{1}{2} \left(H(j\omega_0) e^{j\omega_0 t} + H(-j\omega_0) e^{-j\omega_0 t} \right)$$

Conjugate Symmetry

The complex conjugate of $H(j\omega)$ is $H(-j\omega)$.

The system function is the Laplace transform of the impulse response:

$$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st} dt$$

where $h(t)$ is a real-valued function of t for physical systems.

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t} dt$$

$$H(-j\omega) = \int_{-\infty}^{\infty} h(t)e^{j\omega t} dt \equiv (H(j\omega))^*$$

Frequency Response

Response to eternal sinusoids.

Let $x(t) = \cos \omega_0 t$ (for all time), which can be written as

$$x(t) = \frac{1}{2} \left(e^{j\omega_0 t} + e^{-j\omega_0 t} \right)$$

The response to a sum is the sum of the responses,

$$y(t) = \frac{1}{2} \left(H(j\omega_0) e^{j\omega_0 t} + H(-j\omega_0) e^{-j\omega_0 t} \right)$$

$$= \operatorname{Re} \left\{ H(j\omega_0) e^{j\omega_0 t} \right\}$$

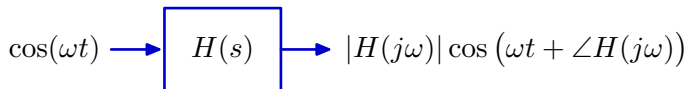
$$= \operatorname{Re} \left\{ |H(j\omega_0)| e^{j\angle H(j\omega_0)} e^{j\omega_0 t} \right\}$$

$$= |H(j\omega_0)| \operatorname{Re} \left\{ e^{j\omega_0 t + j\angle H(j\omega_0)} \right\}$$

$$y(t) = |H(j\omega_0)| \cos(\omega_0 t + \angle H(j\omega_0)).$$

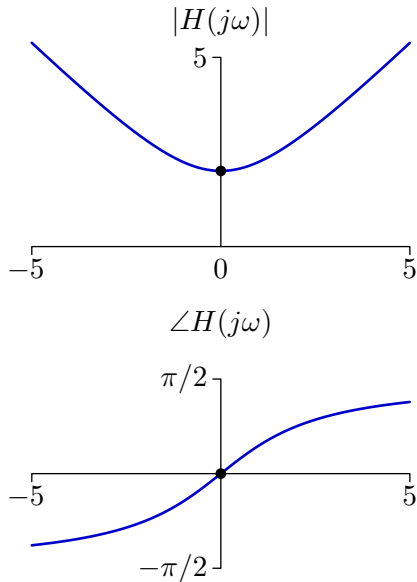
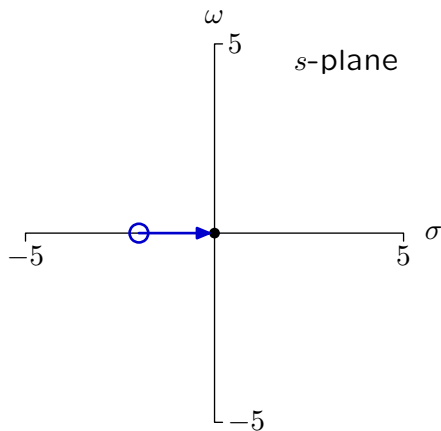
Frequency Response

The magnitude and phase of the response of a system to an eternal cosine signal is the magnitude and phase of the system function evaluated at $s = j\omega$.



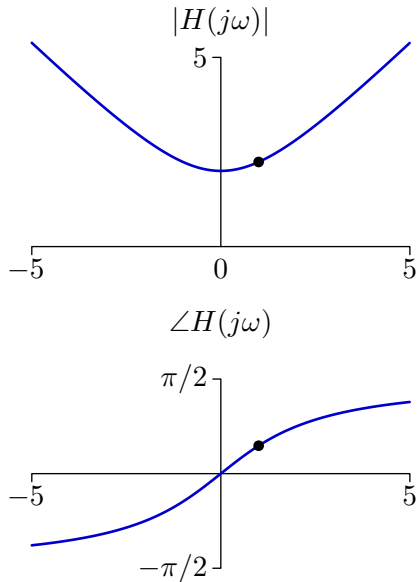
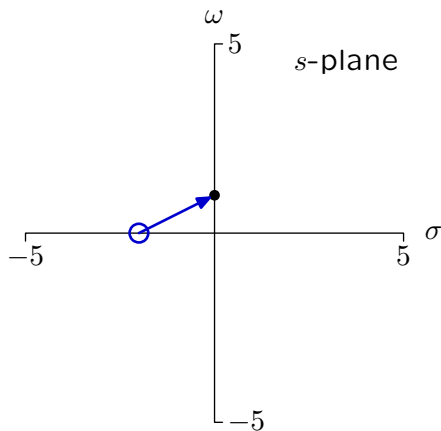
Vector Diagrams

$$H(s) = s - z_1$$



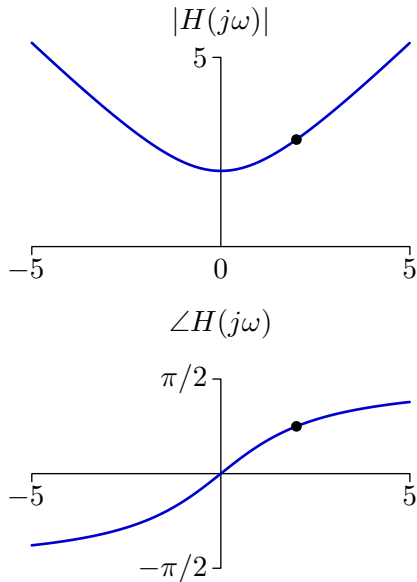
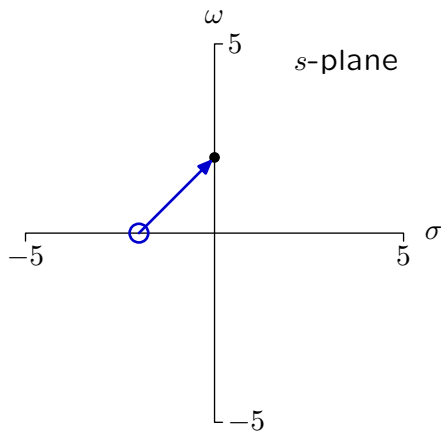
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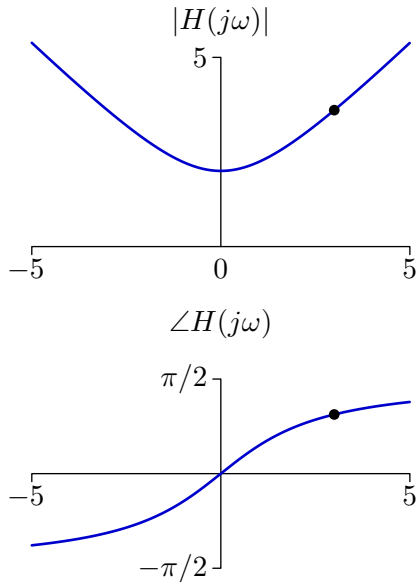
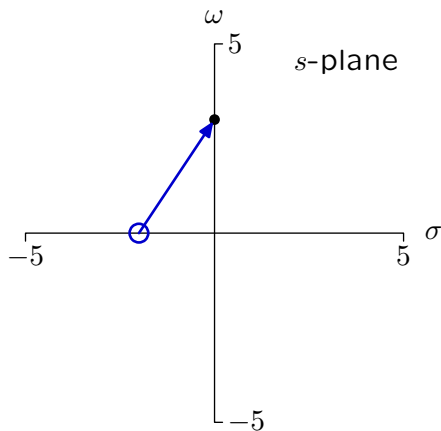
Vector Diagrams

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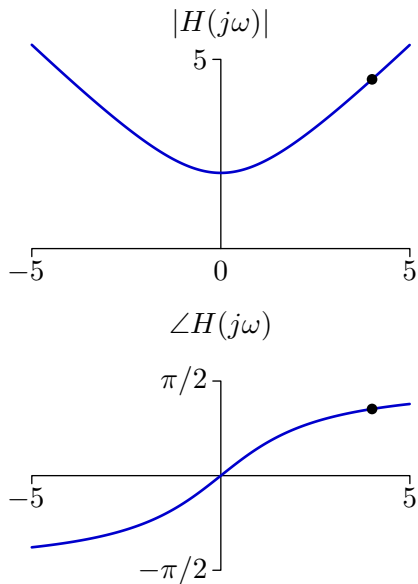
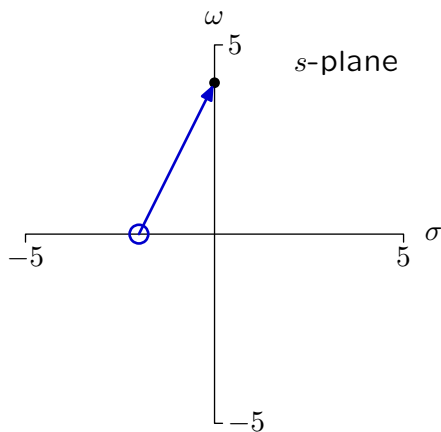
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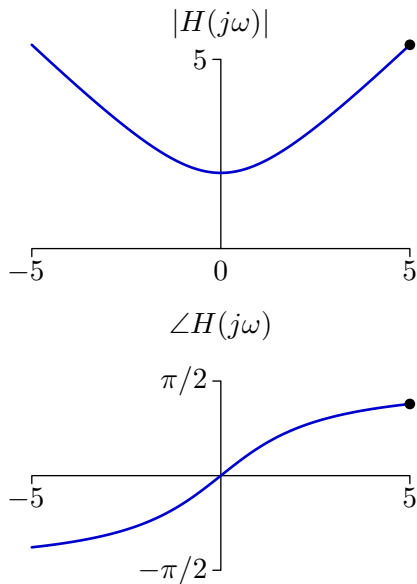
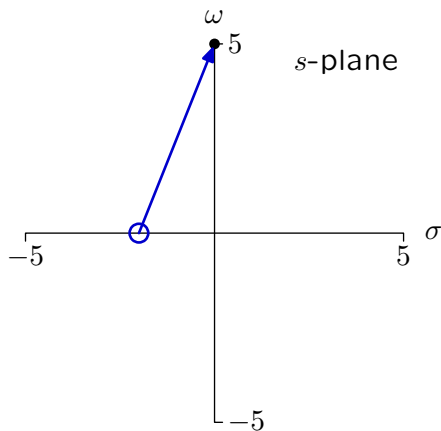
Vector Diagrams

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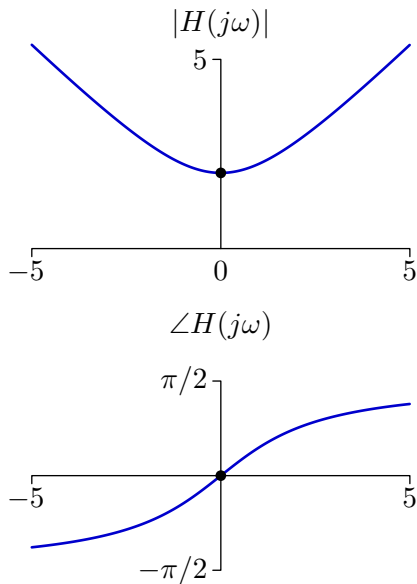
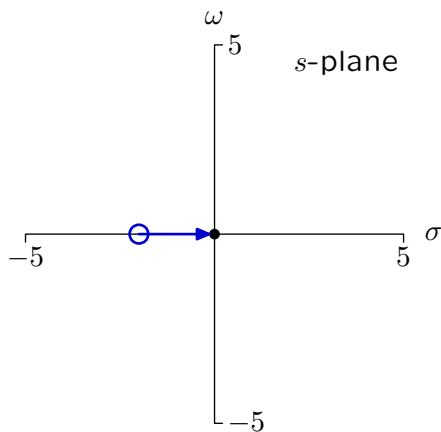
Vector Diagrams

$$H(s) = s - z_1$$



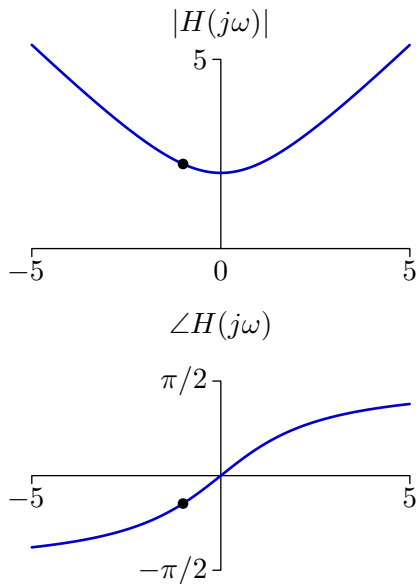
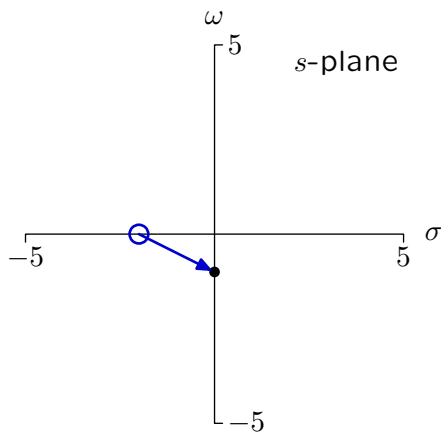
Vector Diagrams

$$H(s) = s - z_1$$



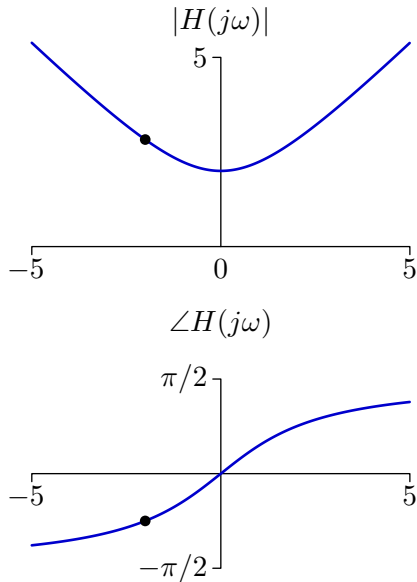
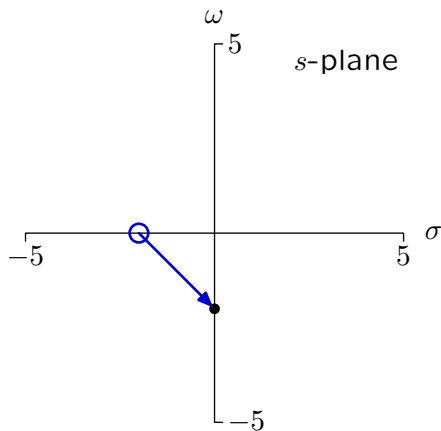
Vector Diagrams

$$H(s) = s - z_1$$



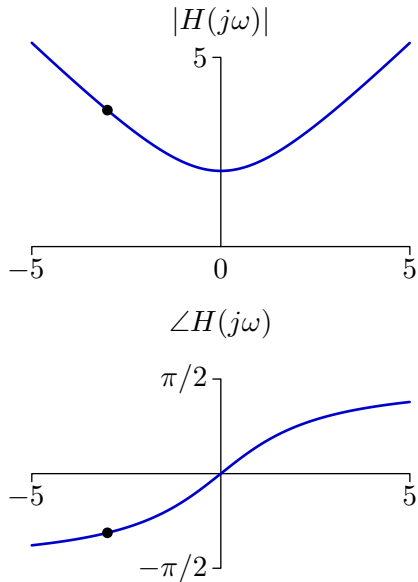
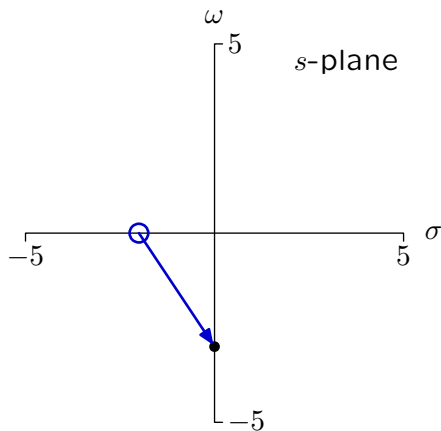
Vector Diagrams

$$H(s) = s - z_1$$



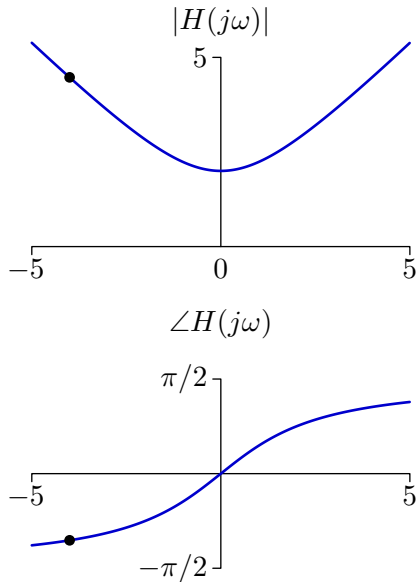
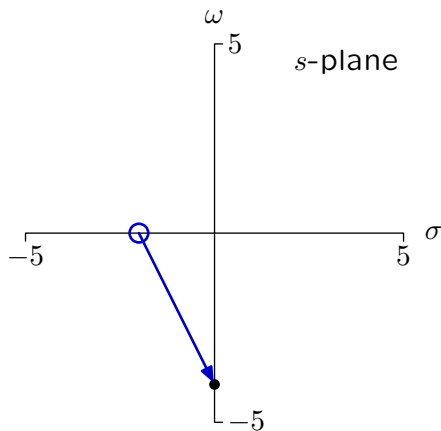
Vector Diagrams

$$H(s) = s - z_1$$



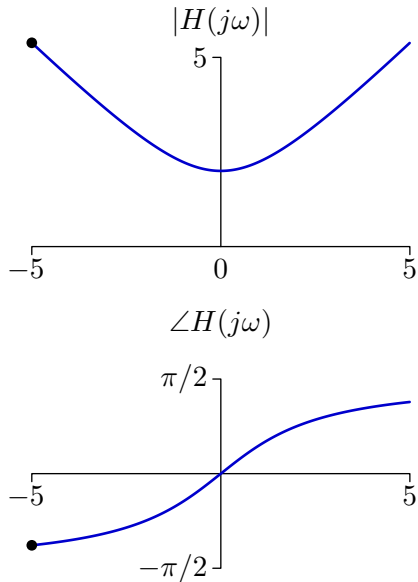
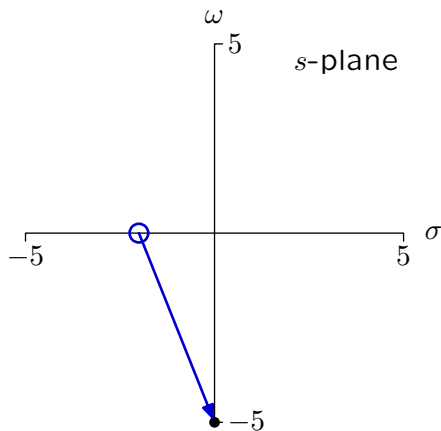
Vector Diagrams

$$H(s) = s - z_1$$



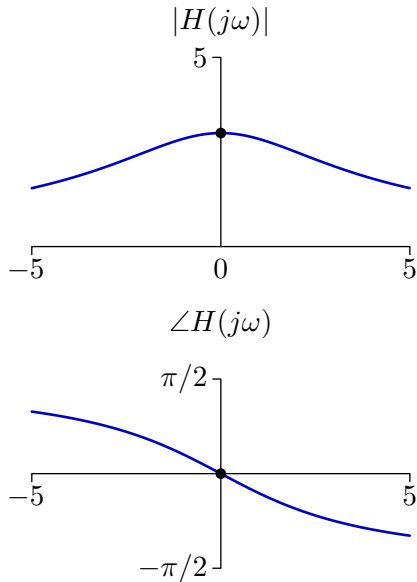
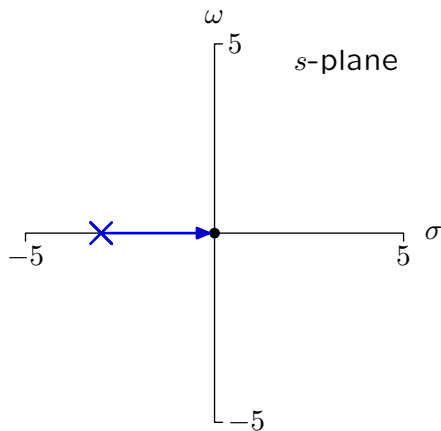
Vector Diagrams

$$H(s) = s - z_1$$



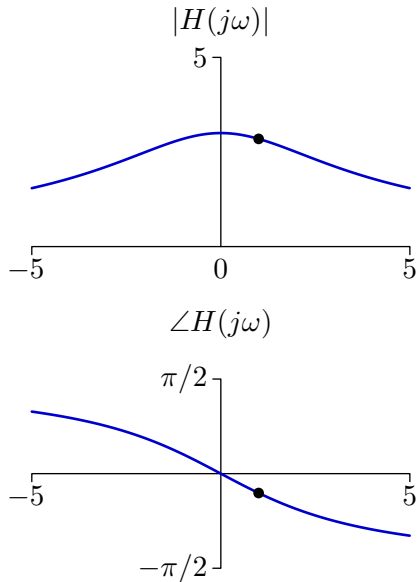
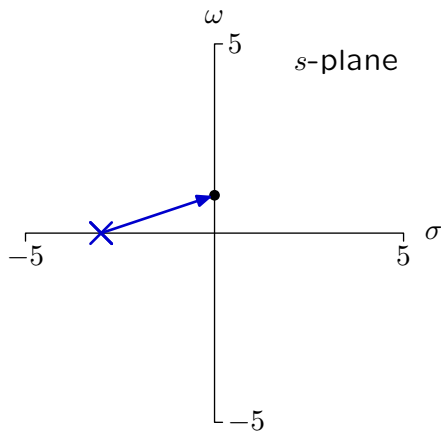
Vector Diagrams

$$H(s) = \frac{9}{s - p_1}$$



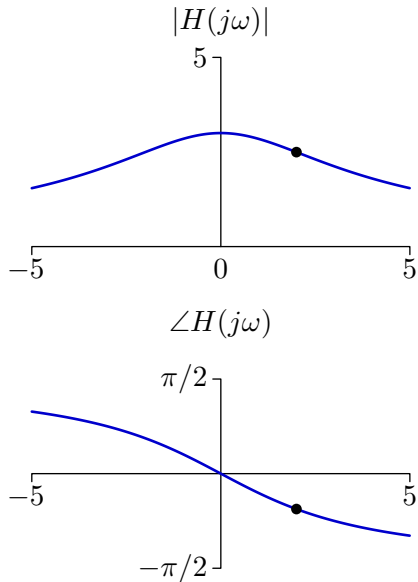
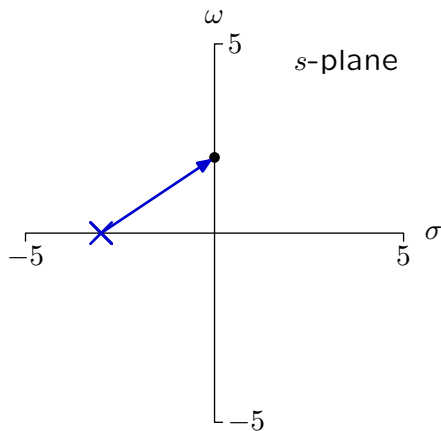
Vector Diagrams

$$H(s) = \frac{9}{s - p_1}$$



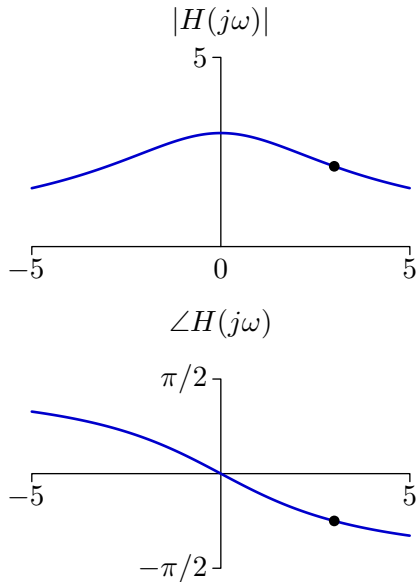
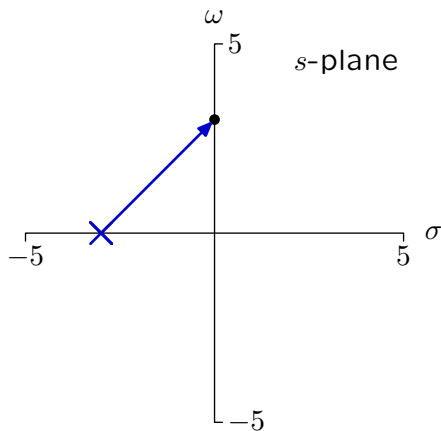
Vector Diagrams

$$H(s) = \frac{9}{s - p_1}$$



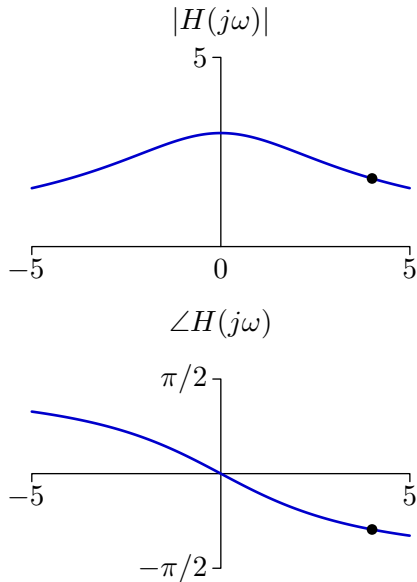
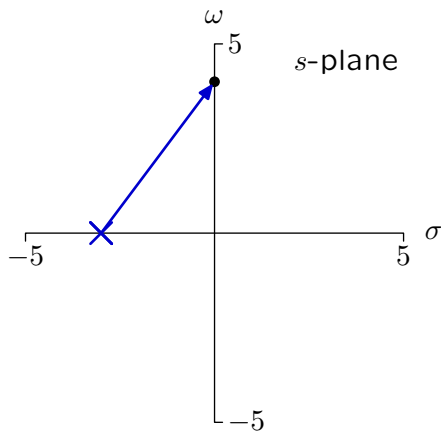
Vector Diagrams

$$H(s) = \frac{9}{s - p_1}$$



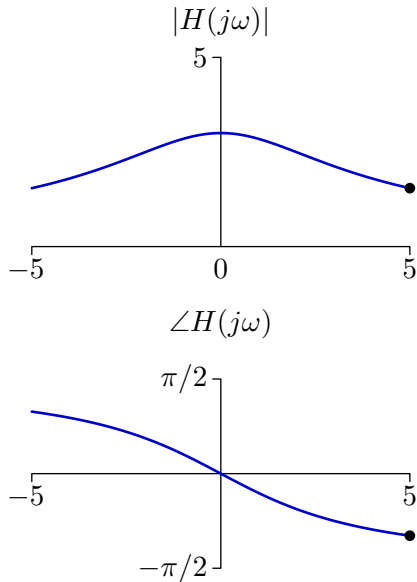
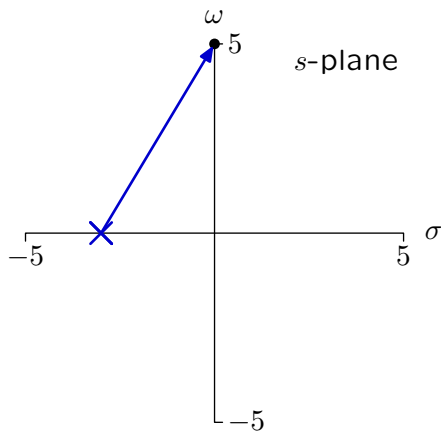
Vector Diagrams

$$H(s) = \frac{9}{s - p_1}$$



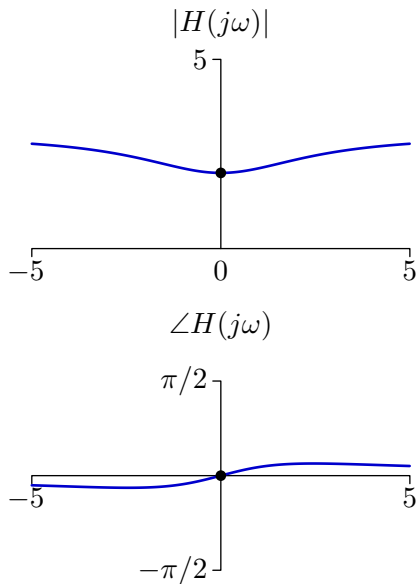
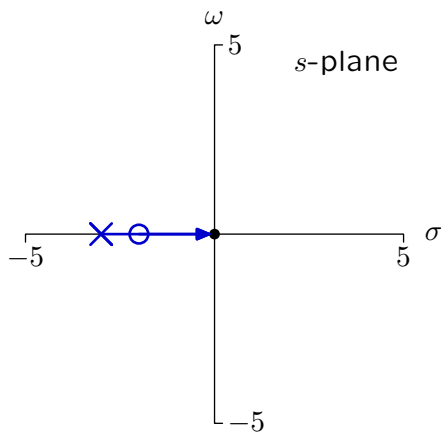
Vector Diagrams

$$H(s) = \frac{9}{s - p_1}$$



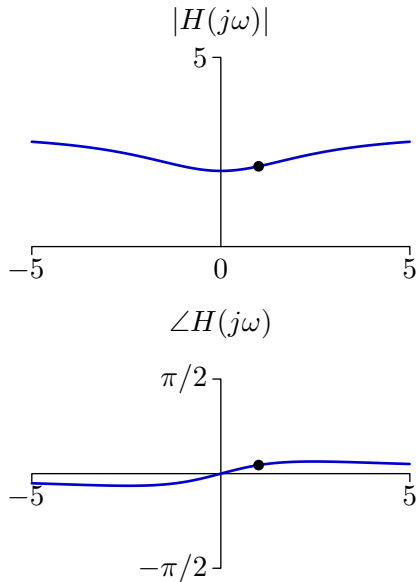
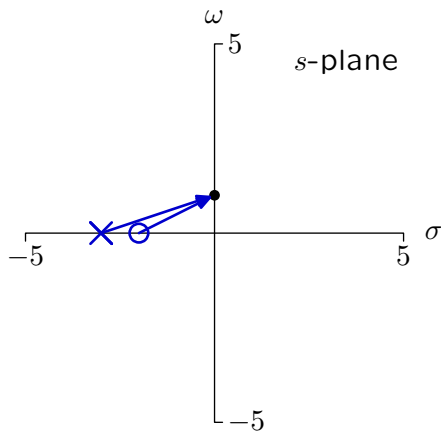
Vector Diagrams

$$H(s) = 3 \frac{s - z_1}{s - p_1}$$



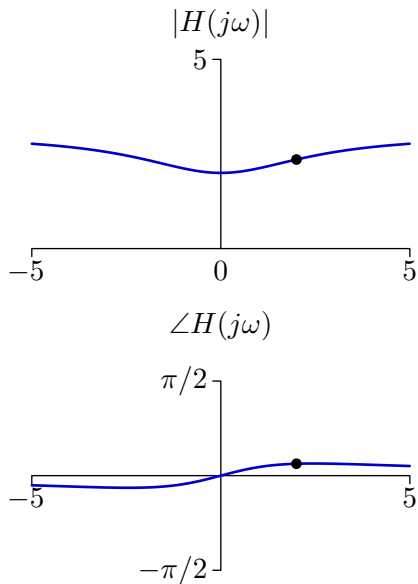
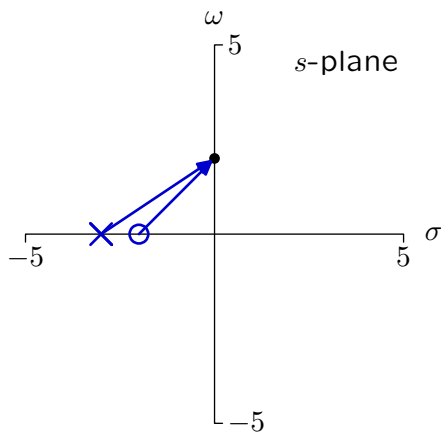
Vector Diagrams

$$H(s) = 3 \frac{s - z_1}{s - p_1}$$



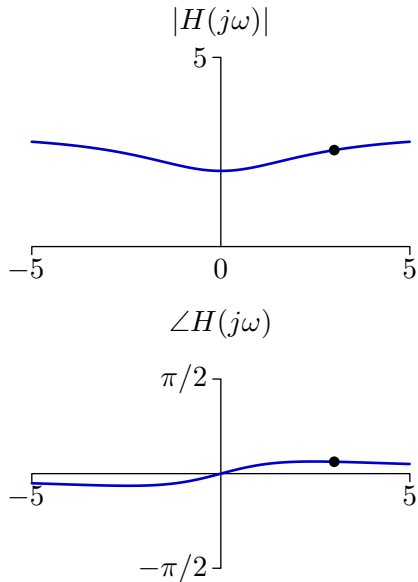
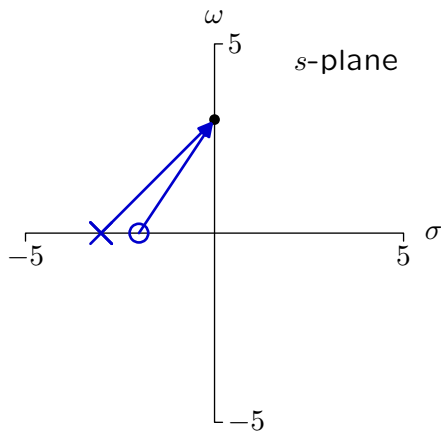
Vector Diagrams

$$H(s) = 3 \frac{s - z_1}{s - p_1}$$



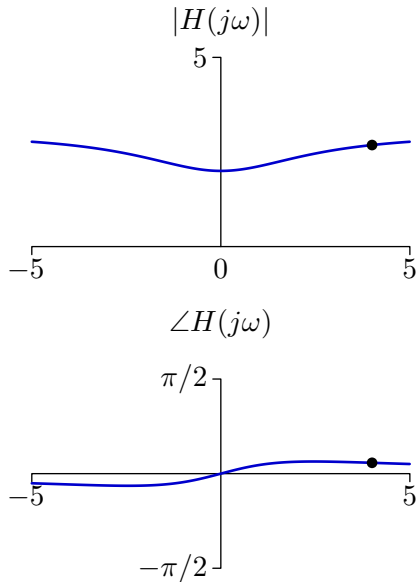
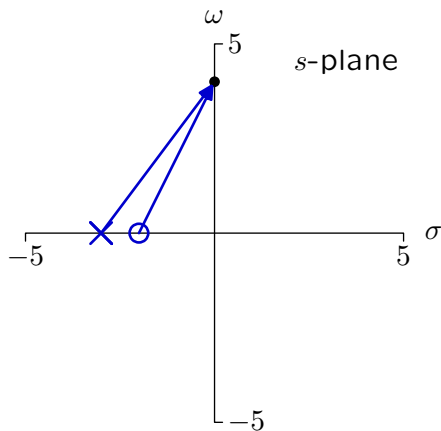
Vector Diagrams

$$H(s) = 3 \frac{s - z_1}{s - p_1}$$



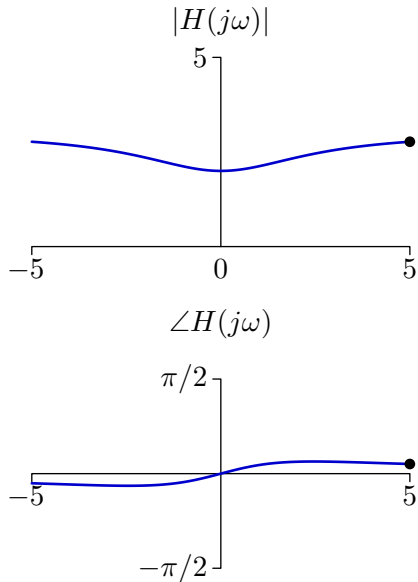
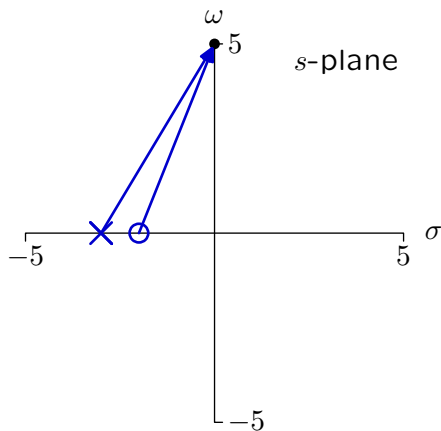
Vector Diagrams

$$H(s) = 3 \frac{s - z_1}{s - p_1}$$

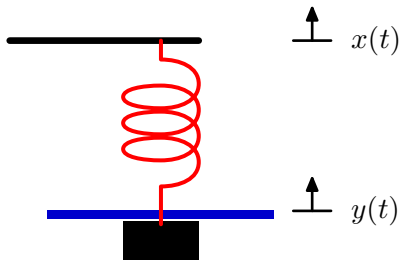


Vector Diagrams

$$H(s) = 3 \frac{s - z_1}{s - p_1}$$



Example: Mass, Spring, and Dashpot



$$F = Ma = M\ddot{y}(t) = K(x(t) - y(t)) - B\dot{y}(t)$$

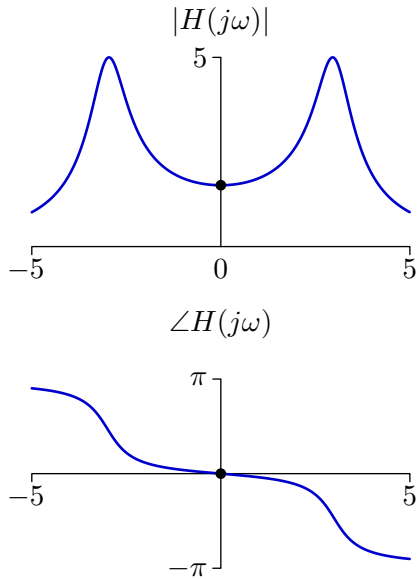
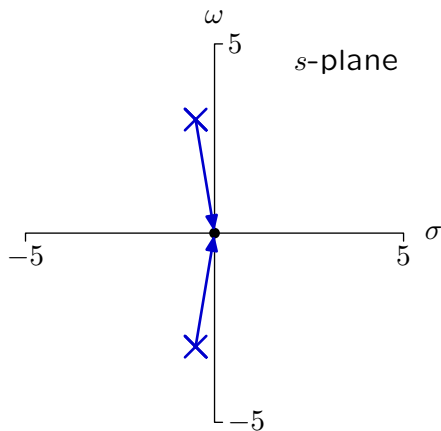
$$M\ddot{y}(t) + B\dot{y}(t) + Ky(t) = Kx(t)$$

$$(s^2M + sB + K) Y(s) = KX(s)$$

$$H(s) = \frac{K}{s^2M + sB + K}$$

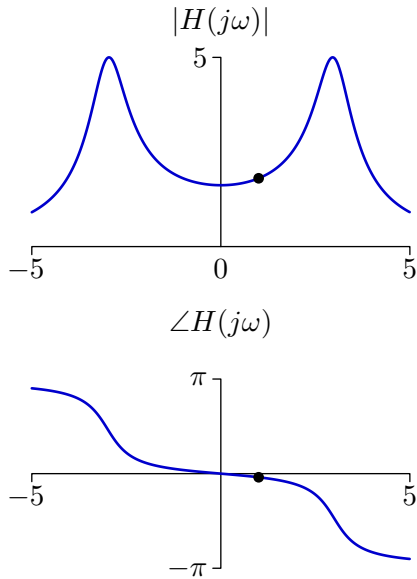
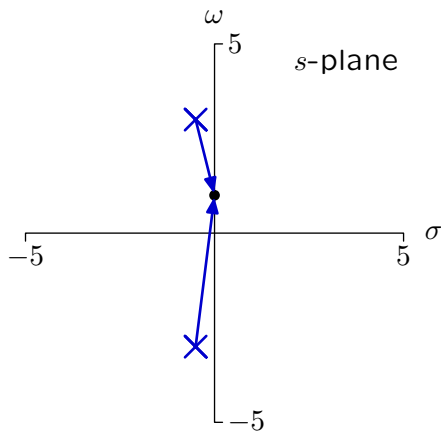
Vector Diagrams

$$H(s) = \frac{15}{(s - p_1)(s - p_2)}$$



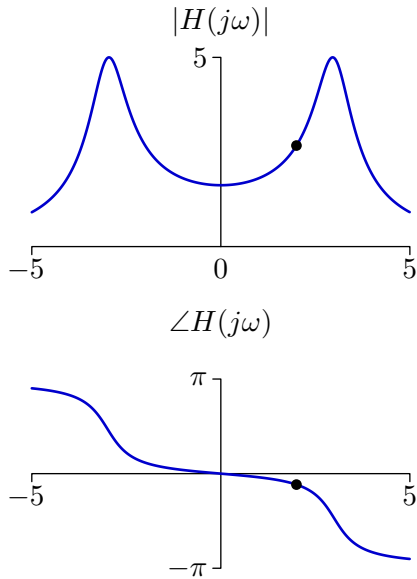
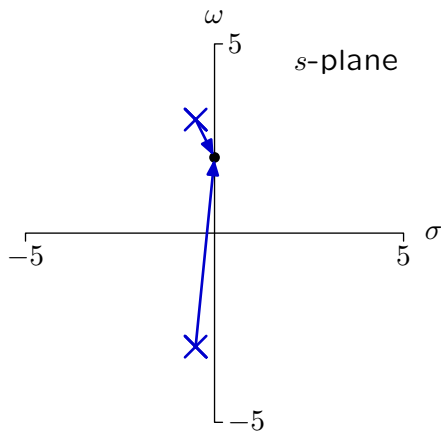
Vector Diagrams

$$H(s) = \frac{15}{(s - p_1)(s - p_2)}$$



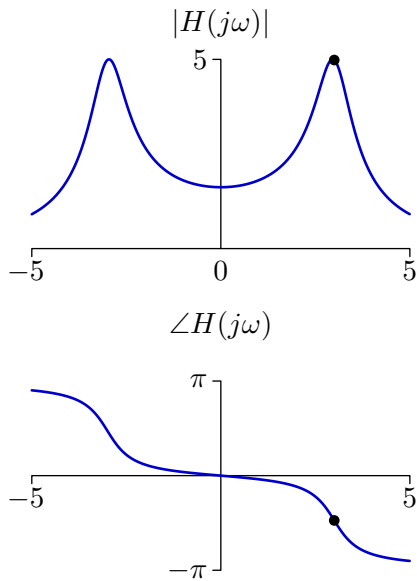
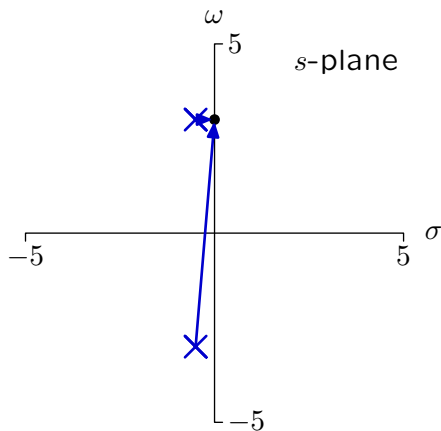
Vector Diagrams

$$H(s) = \frac{15}{(s - p_1)(s - p_2)}$$



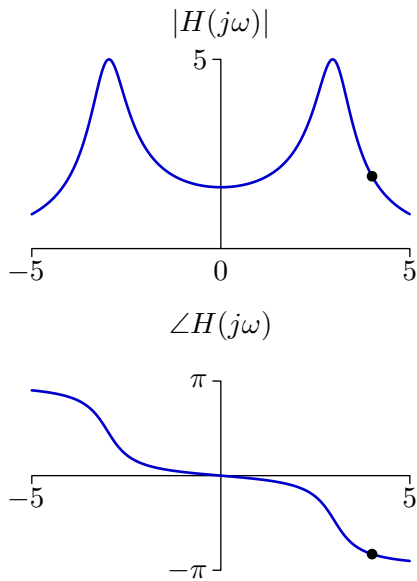
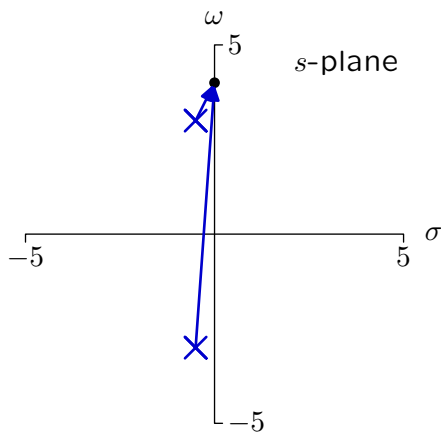
Vector Diagrams

$$H(s) = \frac{15}{(s - p_1)(s - p_2)}$$



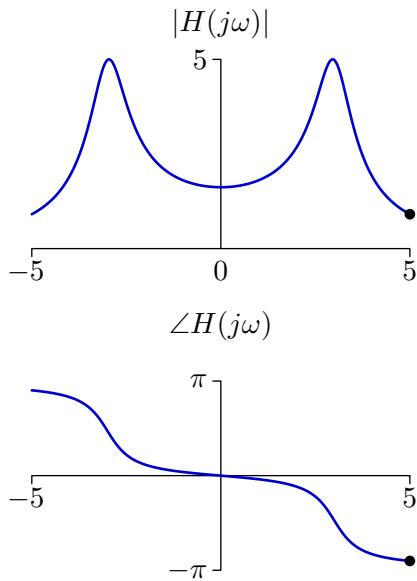
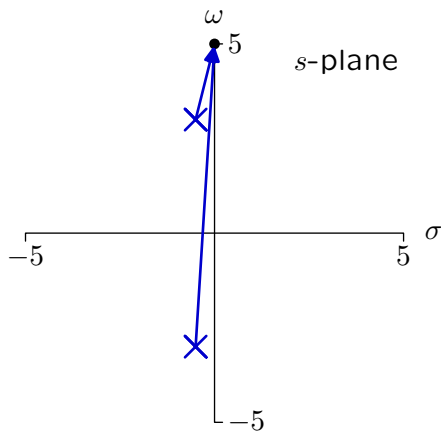
Vector Diagrams

$$H(s) = \frac{15}{(s - p_1)(s - p_2)}$$



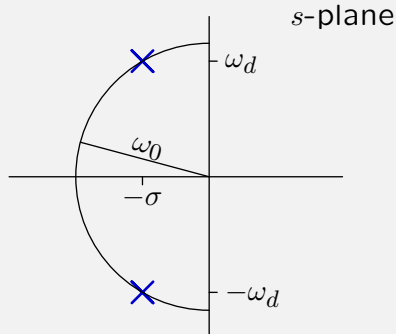
Vector Diagrams

$$H(s) = \frac{15}{(s - p_1)(s - p_2)}$$



Check Yourself

Consider the system represented by the following poles.

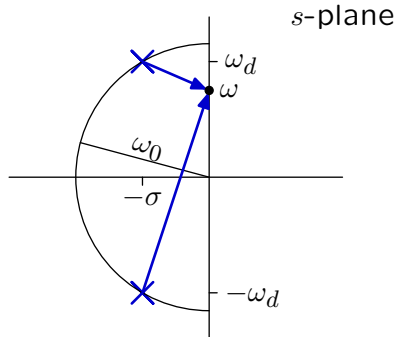


Find the frequency ω at which the magnitude of the response $y(t)$ is greatest if $x(t) = \cos \omega t$.

1. $\omega = \omega_d$
2. $\omega_d < \omega < \omega_0$
3. $0 < \omega < \omega_d$
4. none of the above

Check Yourself: Frequency Response

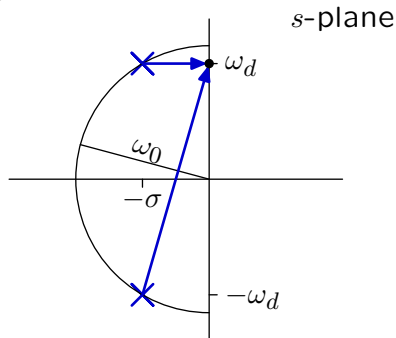
Analyze with vectors.



The product of the lengths is $\left(\sqrt{(\omega + \omega_d)^2 + \sigma^2}\right) \left(\sqrt{(\omega - \omega_d)^2 + \sigma^2}\right)$.

Check Yourself: Frequency Response

Analyze with vectors.

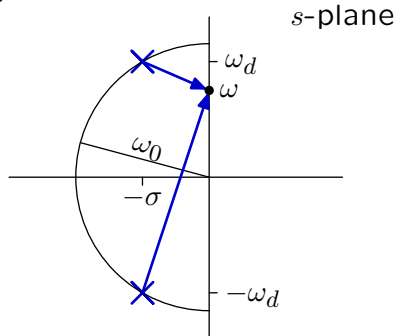


The product of the lengths is $\left(\sqrt{(\omega + \omega_d)^2 + \sigma^2}\right) \left(\sqrt{(\omega - \omega_d)^2 + \sigma^2}\right)$.

Decreasing ω from ω_d to $\omega_d - \epsilon$ decreases the product since length of bottom vector decreases as ϵ while length of top vector increases only as ϵ^2 .

Check Yourself: Frequency Response

More mathematically ...



The product of the lengths is $\left(\sqrt{(\omega + \omega_d)^2 + \sigma^2}\right) \left(\sqrt{(\omega - \omega_d)^2 + \sigma^2}\right)$.

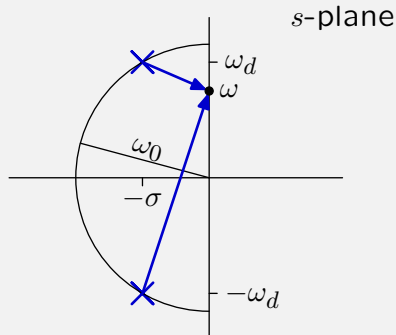
Maximum occurs where derivative of squared lengths is zero.

$$\frac{d}{d\omega} \left((\omega + \omega_d)^2 + \sigma^2 \right) \left((\omega - \omega_d)^2 + \sigma^2 \right) = 0$$

$$\rightarrow \omega^2 = \omega_d^2 - \sigma^2 = \omega_0^2 - 2\sigma^2.$$

Check Yourself

Consider the system represented by the following poles.



Find the frequency ω at which the magnitude of the response $y(t)$ is greatest if $x(t) = \cos \omega t$. **3**

1. $\omega = \omega_d$

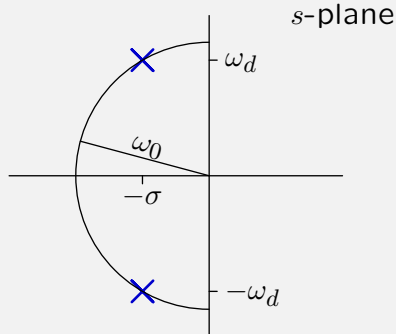
3. $0 < \omega < \omega_d$

2. $\omega_d < \omega < \omega_0$

4. none of the above

Check Yourself

Consider the system represented by the following poles.



Find the frequency ω at which the phase of the response $y(t)$ is $-\pi/2$ if $x(t) = \cos \omega t$.

0. $0 < \omega < \omega_d$

1. $\omega = \omega_d$

2. $\omega_d < \omega < \omega_0$

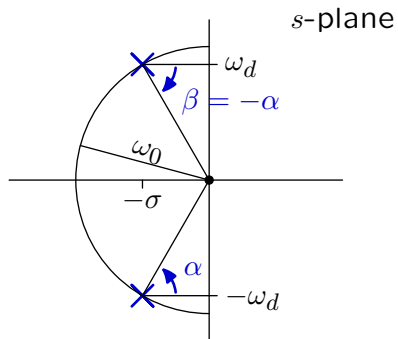
3. $\omega = \omega_0$

4. $\omega > \omega_0$

5. none

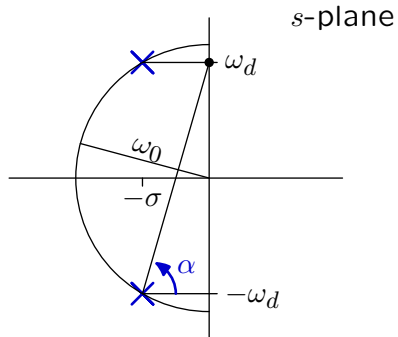
Check Yourself

The phase is 0 when $\omega = 0$.



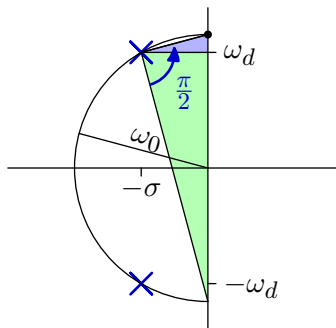
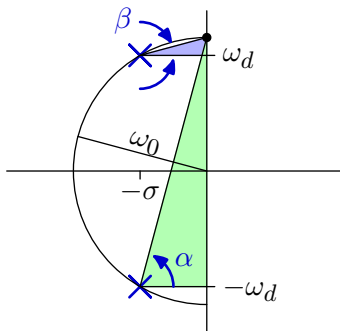
Check Yourself

The phase is less than $\pi/2$ when $\omega = \omega_d$.



Check Yourself

The phase is $-\pi/2$ at $\omega = \omega_0$.



Check Yourself

Check result by evaluating the system function.

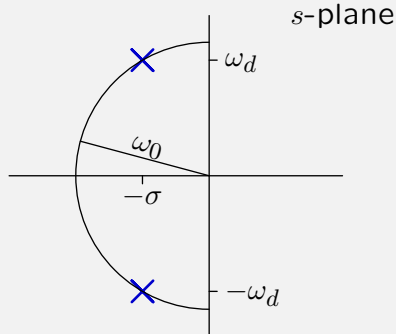
Substitute $s = j\omega_0 = j\sqrt{\frac{K}{M}}$ into

$$H(s) = \frac{K}{s^2M + sB + K} = \frac{K}{-\frac{K}{M}M + j\omega_0B + K} = \frac{K}{j\omega_0B}$$

The phase is $-\frac{\pi}{2}$.

Check Yourself

Consider the system represented by the following poles.



Find the frequency ω at which the phase of the response $y(t)$ is $-\pi/2$ if $x(t) = \cos \omega t$. **3**

0. $0 < \omega < \omega_d$

3. $\omega = \omega_0$

1. $\omega = \omega_d$

4. $\omega > \omega_0$

2. $\omega_d < \omega < \omega_0$

5. none

Frequency Response: Summary

LTI systems can be characterized by responses to eternal sinusoids.

Many systems are naturally described by their frequency response.

- audio systems
- mass, spring, dashpot system

Frequency response is easy to calculate from the system function.

Frequency response lives on the $j\omega$ axis of the Laplace transform.