

# 6.003: Signals and Systems

## CT Feedback and Control

*October 25, 2011*

## Mid-term Examination #2

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Tomorrow, October 26, 7:30-9:30pm, Walker (50-340)

No recitations on the day of the exam.

Coverage:     Lectures 1–12  
                  Recitations 1–12  
                  Homeworks 1–7

Homework 7 will not be collected or graded. Solutions are posted.

Closed book: 2 pages of notes ( $8\frac{1}{2} \times 11$  inches; front and back).

No calculators, computers, cell phones, music players, or other aids.

Designed as 1-hour exam; two hours to complete.

Old exams and solutions are posted on the 6.003 website.

## Feedback and Control

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Using feedback to enhance performance.

Examples:

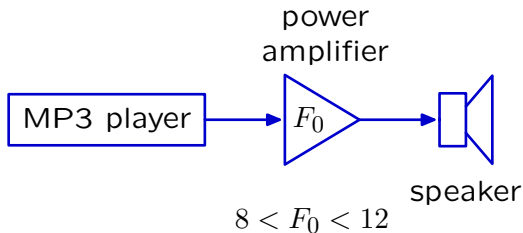
- improve performance of an op amp circuit.
- control position of a motor.
- reduce sensitivity to unwanted parameter variation.
- reduce distortions.
- stabilize unstable systems
  - magnetic levitation
  - inverted pendulum

## Feedback and Control

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Reducing sensitivity to unwanted parameter variation.

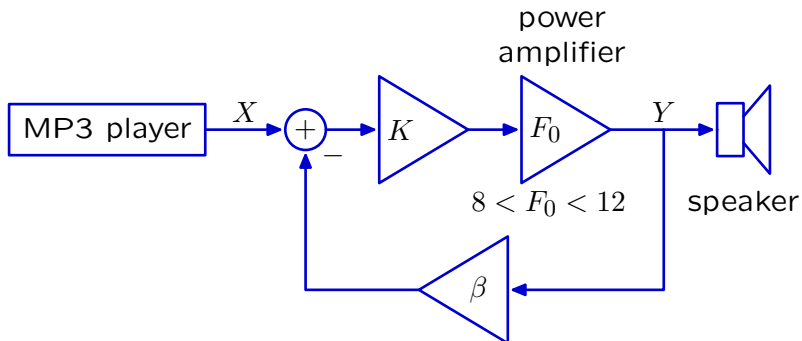
Example: power amplifier



Changes in  $F_0$  (due to changes in temperature, for example) lead to undesired changes in sound level.

## Feedback and Control

Feedback can be used to compensate for parameter variation.



$$H(s) = \frac{KF_0}{1 + \beta KF_0}$$

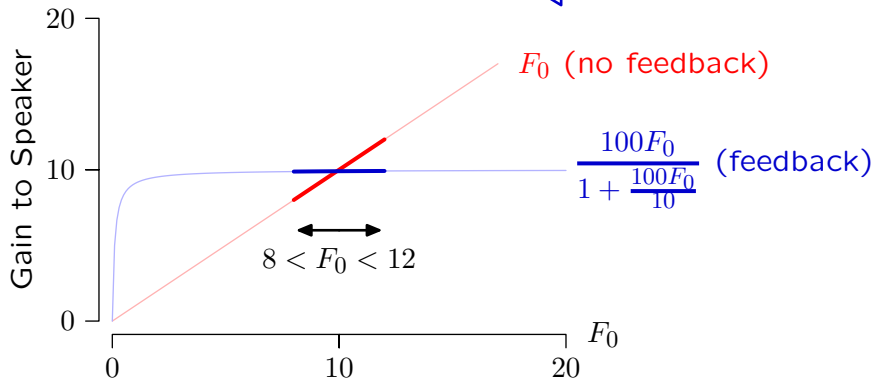
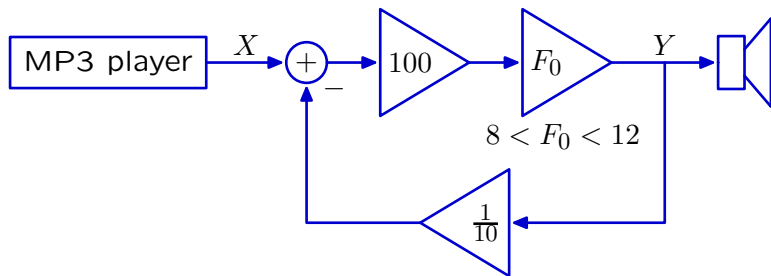
If  $K$  is made large, so that  $\beta KF_0 \gg 1$ , then

$$H(s) \approx \frac{1}{\beta}$$

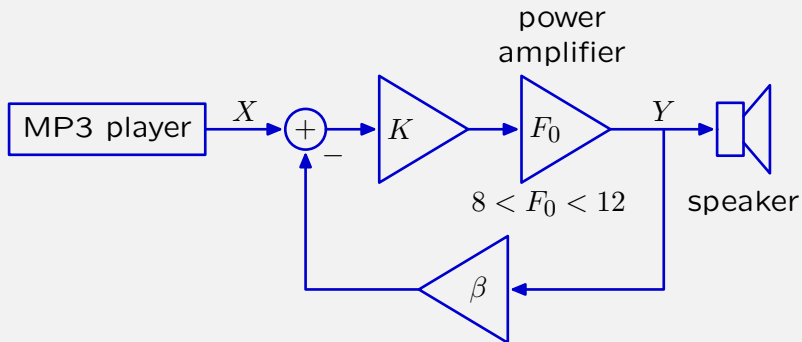
independent of  $K$  or  $F_0$ !

## Feedback and Control

Feedback reduces the change in gain due to change in  $F_0$ .



## Check Yourself



Feedback greatly reduces sensitivity to variations in  $K$  or  $F_0$ .

$$\lim_{K \rightarrow \infty} H(s) = \frac{KF_0}{1 + \beta KF_0} \rightarrow \frac{1}{\beta}$$

What about variations in  $\beta$ ? Aren't those important?

## Check Yourself

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What about variations in  $\beta$ ? Aren't those important?

The value of  $\beta$  is typically determined with resistors, whose values are quite stable (compared to semiconductor devices).

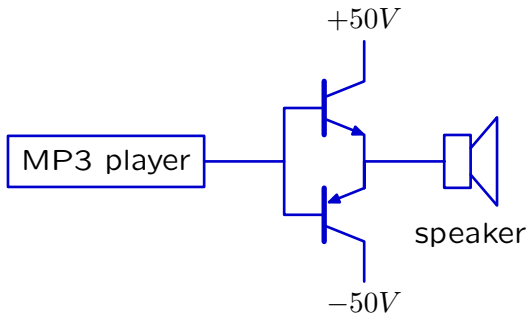


## Crossover Distortion

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Feedback can compensate for parameter variation even when the variation occurs rapidly.

Example: using transistors to amplify power.



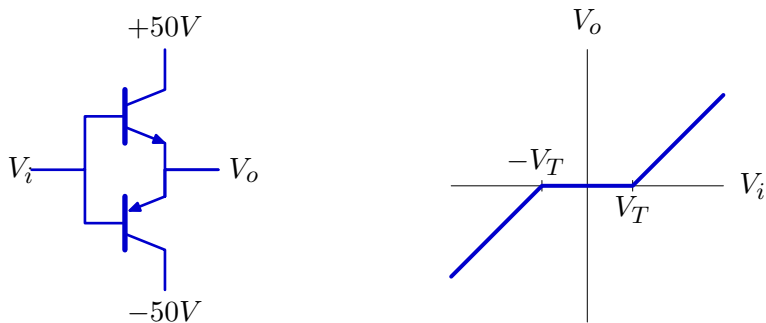
## Crossover Distortion

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This circuit introduces “crossover distortion.”

For the upper transistor to conduct,  $V_i - V_o > V_T$ .

For the lower transistor to conduct,  $V_i - V_o < -V_T$ .

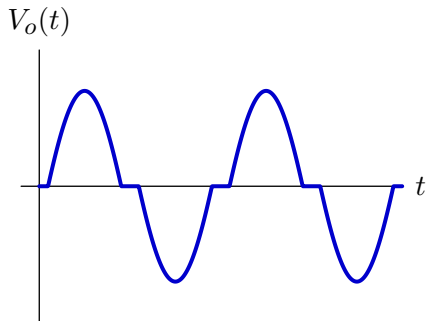
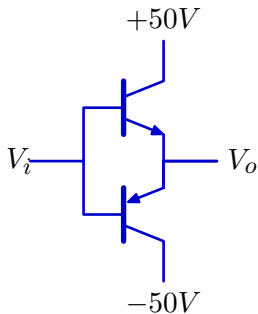


## Crossover Distortion

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Crossover distortion changes the shapes of signals.

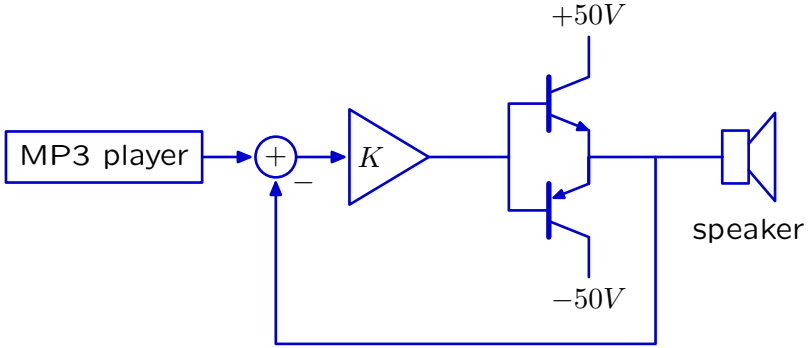
Example: crossover distortion when the input is  $V_i(t) = B \sin(\omega_0 t)$ .



# Crossover Distortion

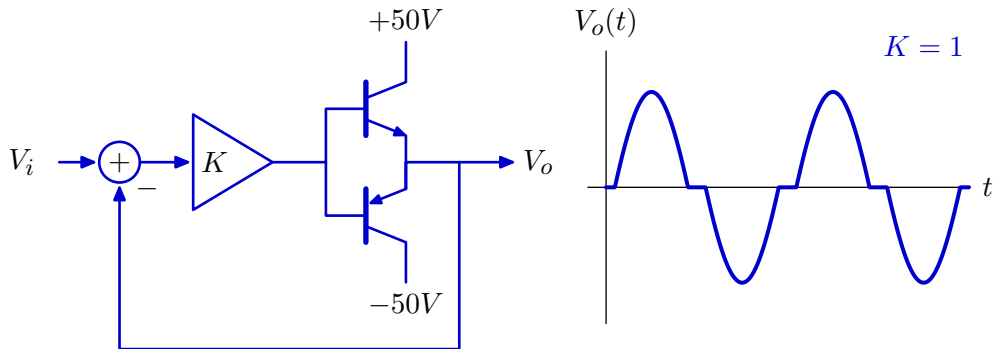
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Feedback can reduce the effects of crossover distortion.



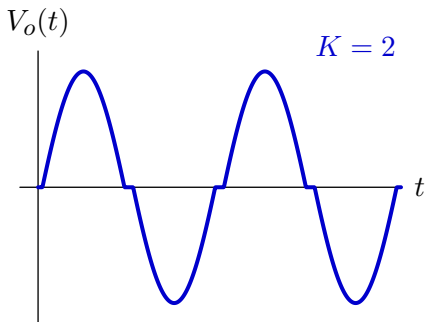
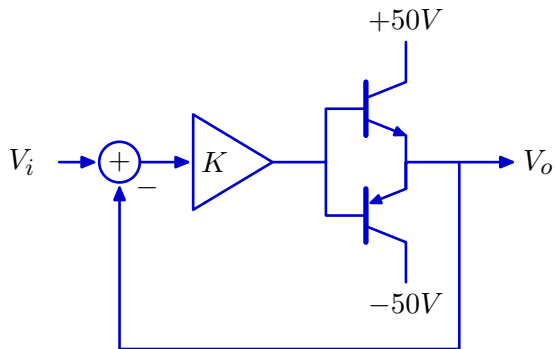
## Crossover Distortion

When  $K$  is small, feedback has little effect on crossover distortion.



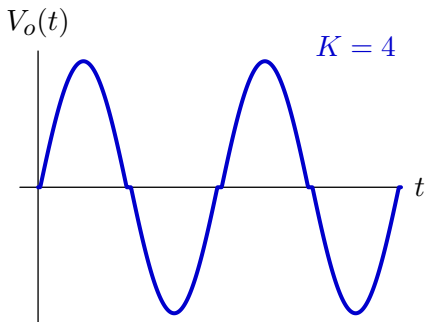
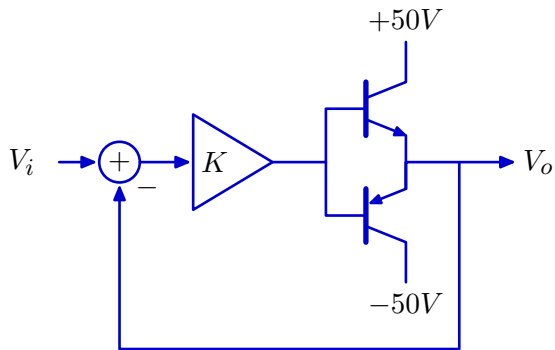
## Crossover Distortion

Feedback reduces crossover distortion.



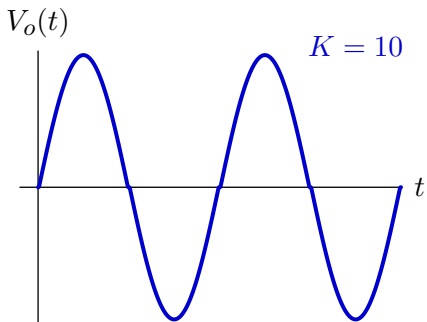
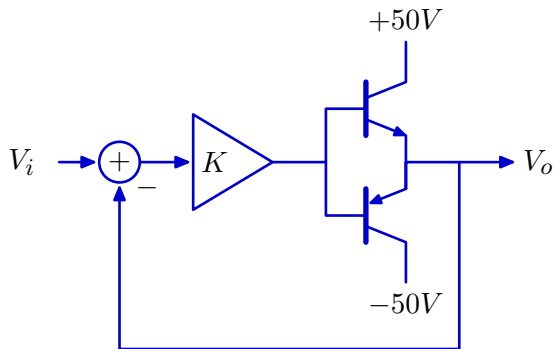
## Crossover Distortion

Feedback reduces crossover distortion.



## Crossover Distortion

Feedback reduces crossover distortion.

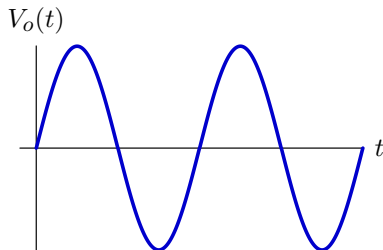
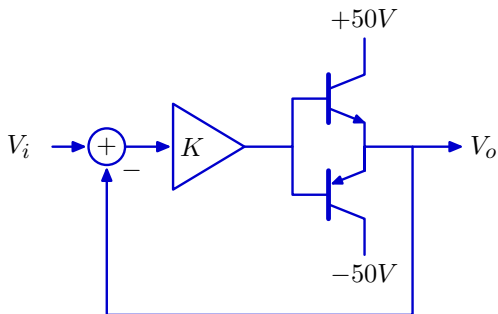




# Crossover Distortion

## Demo

- original
- no feedback
- $K = 2$
- $K = 4$
- $K = 8$
- $K = 16$
- original



J.S. Bach, Sonata No. 1 in G minor Mvmt. IV. Presto  
Nathan Milstein, violin

# Feedback and Control

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Using feedback to enhance performance.

Examples:

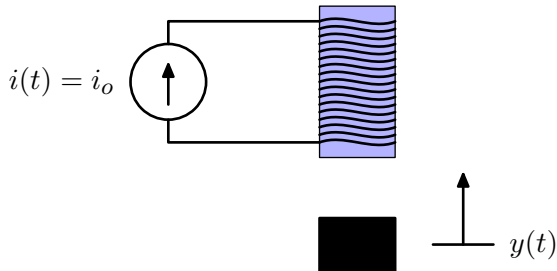
- improve performance of an op amp circuit.
- control position of a motor.
- reduce sensitivity to unwanted parameter variation.
- reduce distortions.
- stabilize unstable systems
  - magnetic levitation
  - inverted pendulum

# Control of Unstable Systems

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Feedback is useful for controlling **unstable** systems.

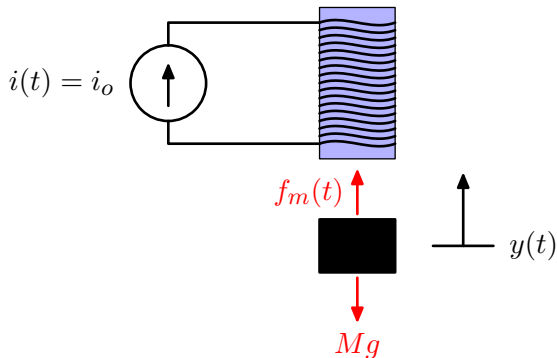
Example: Magnetic levitation.



## Control of Unstable Systems

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Magnetic levitation is unstable.



Equilibrium ( $y = 0$ ): magnetic force  $f_m(t)$  is equal to the weight  $Mg$ .

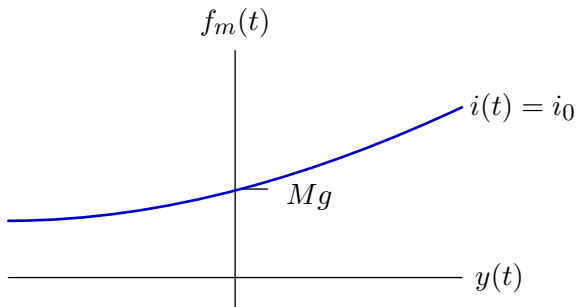
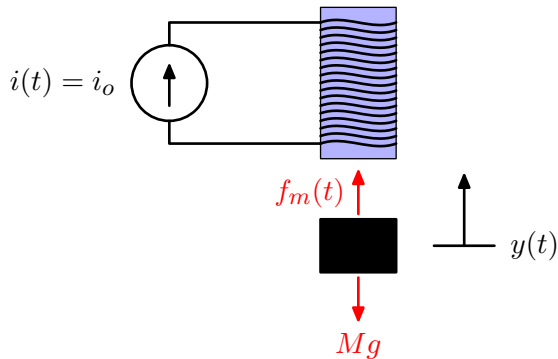
Increase  $y \rightarrow$  increased force  $\rightarrow$  further increases  $y$ .

Decrease  $y \rightarrow$  decreased force  $\rightarrow$  further decreases  $y$ .

Positive feedback!

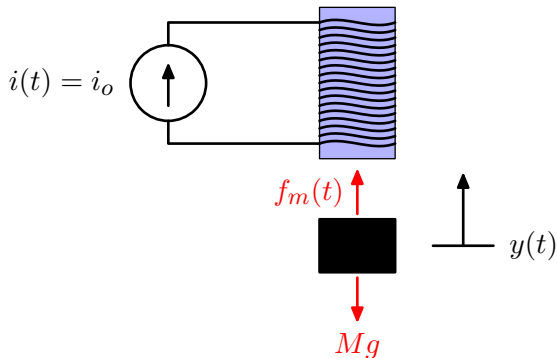
## Modeling Magnetic Levitation

The magnet generates a force that depends on the distance  $y(t)$ .

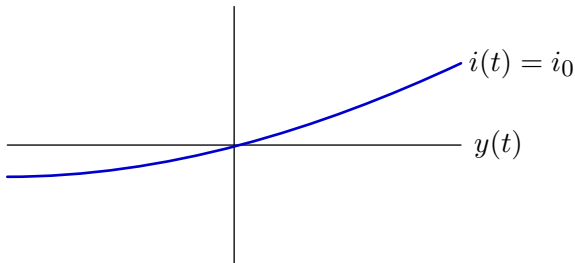


## Modeling Magnetic Levitation

The net force  $f(t) = f_m(t) - Mg$  accelerates the mass.

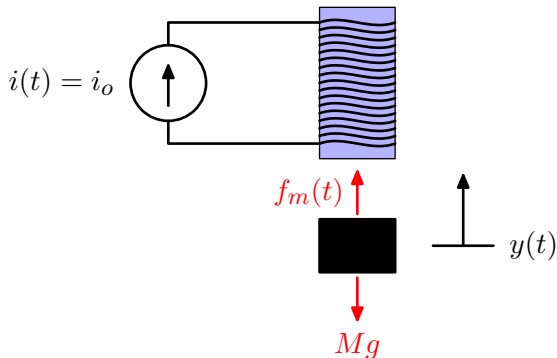


$$f(t) = f_m(t) - Mg = Ma = M\ddot{y}(t)$$

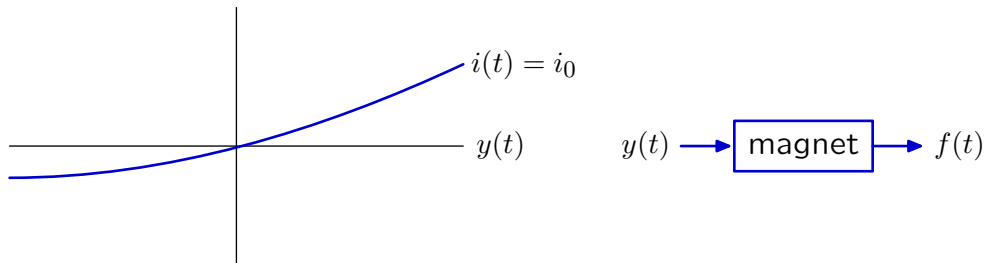


## Modeling Magnetic Levitation

Represent the magnet as a system: input  $y(t)$  and output  $f(t)$ .



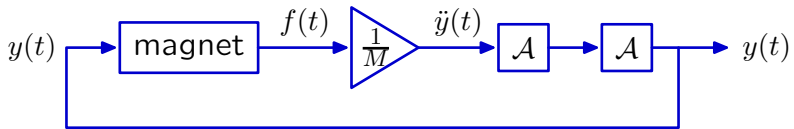
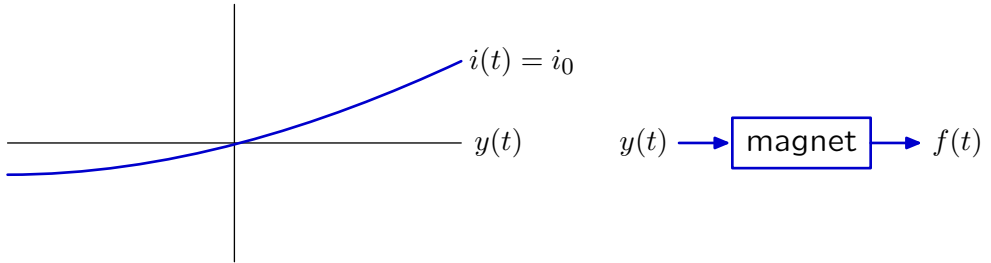
$$f(t) = f_m(t) - Mg = Ma = M\ddot{y}(t)$$



# Modeling Magnetic Levitation

The magnet system is part of a feedback system.

$$f(t) = f_m(t) - Mg = Ma = M\ddot{y}(t)$$

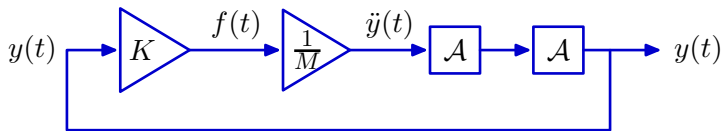
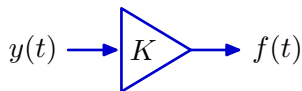
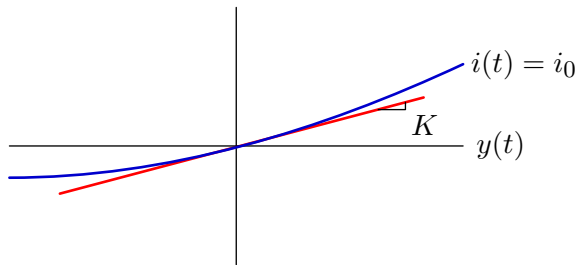




## Modeling Magnetic Levitation

For small distances, force grows approximately linearly with distance.

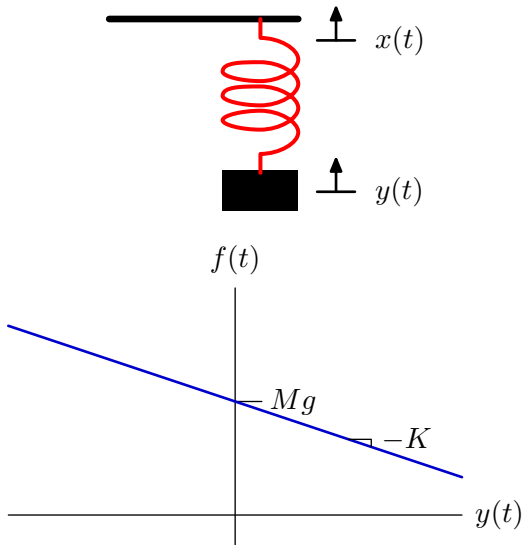
$$f(t) = f_m(t) - Mg = Ma = M\ddot{y}(t)$$



## “Levitation” with a Spring

Relation between force and distance for a spring is opposite in sign.

$$F = K(x(t) - y(t)) = M\ddot{y}(t)$$



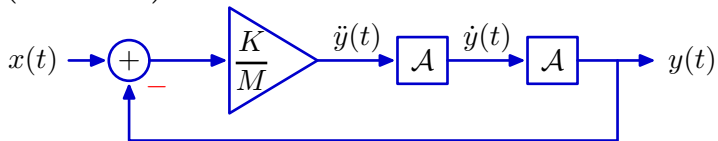
## Block Diagrams

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Block diagrams for magnetic levitation and spring/mass are similar.

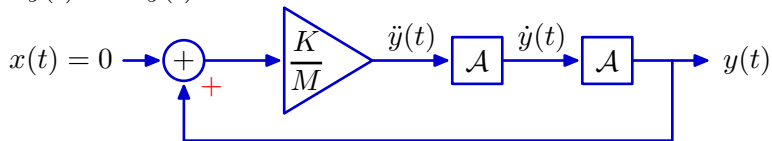
Spring and mass

$$F = K(x(t) - y(t)) = M\ddot{y}(t)$$



Magnetic levitation

$$F = Ky(t) = M\ddot{y}(t)$$

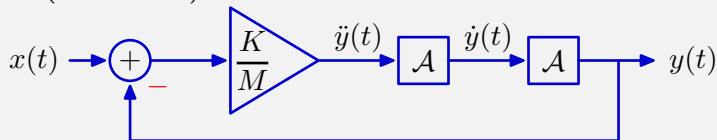


## Check Yourself

How do the poles of these two systems differ?

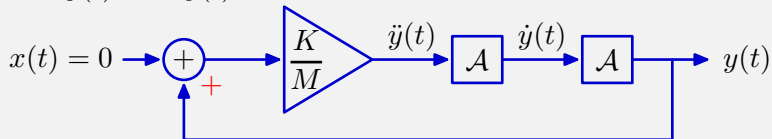
Spring and mass

$$F = K(x(t) - y(t)) = M\ddot{y}(t)$$



Magnetic levitation

$$F = Ky(t) = M\ddot{y}(t)$$



## Check Yourself

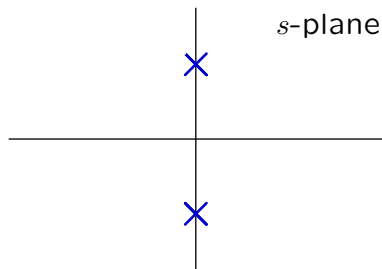
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How do the poles of the two systems differ?

Spring and mass

$$F = K(x(t) - y(t)) = M\ddot{y}(t)$$

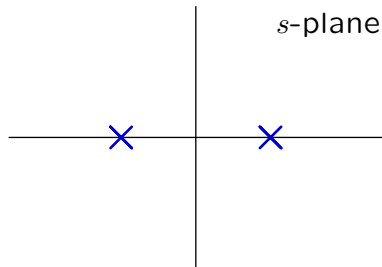
$$\frac{Y}{X} = \frac{\frac{K}{M}}{s^2 + \frac{K}{M}} \rightarrow s = \pm j\sqrt{\frac{K}{M}}$$



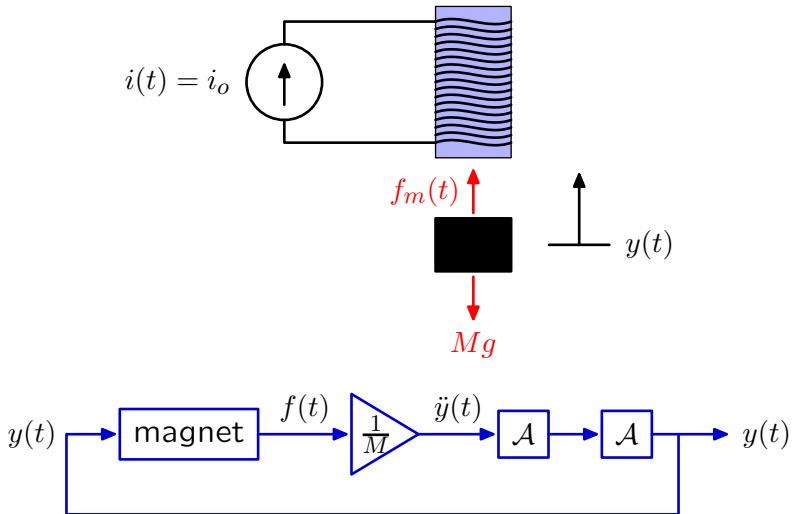
Magnetic levitation

$$F = Ky(t) = M\ddot{y}(t)$$

$$s^2 = \frac{K}{M} \rightarrow s = \pm\sqrt{\frac{K}{M}}$$



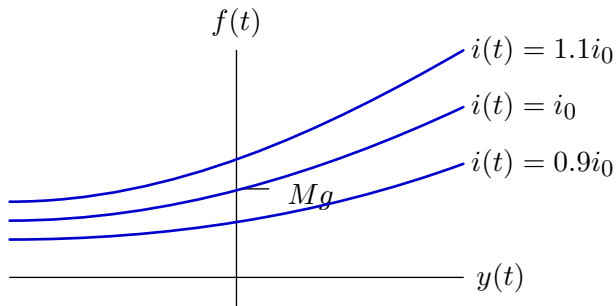
# Magnetic Levitation is Unstable



## Magnetic Levitation

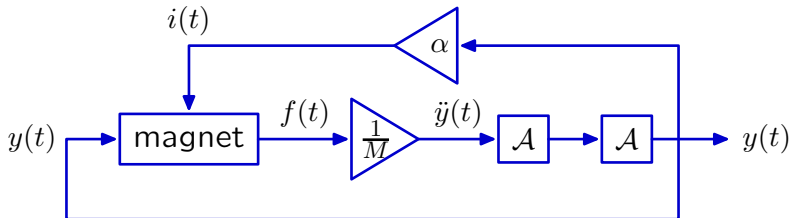
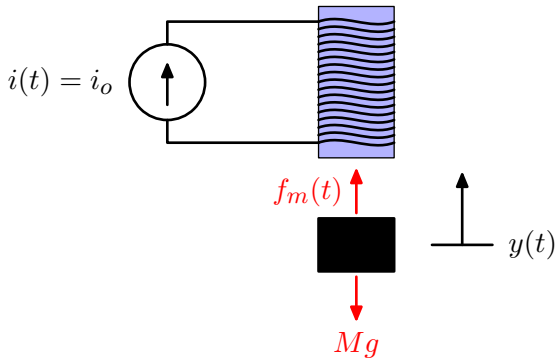
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We can stabilize this system by adding an additional feedback loop to control  $i(t)$ .



# Stabilizing Magnetic Levitation

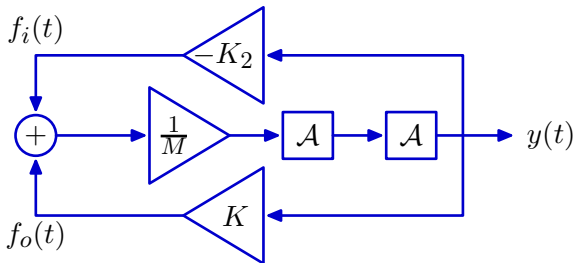
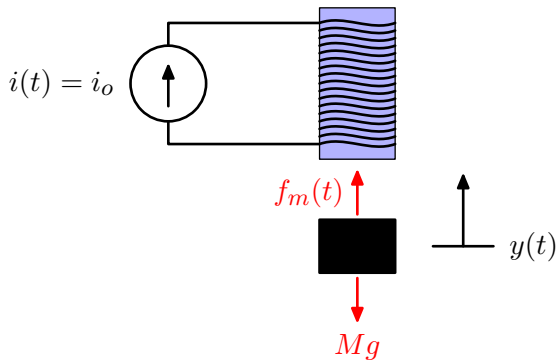
Stabilize magnetic levitation by controlling the magnet current.





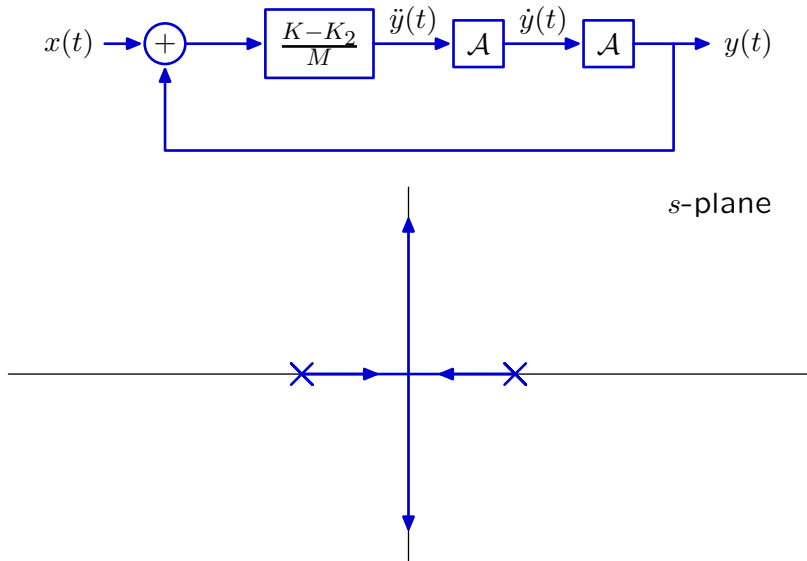
# Stabilizing Magnetic Levitation

Stabilize magnetic levitation by controlling the magnet current.



## Magnetic Levitation

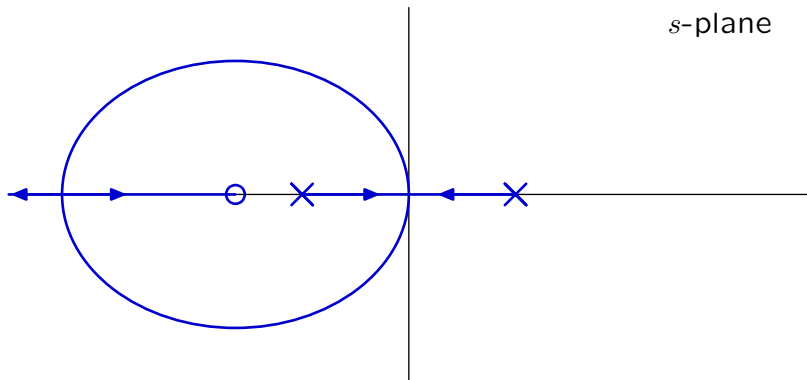
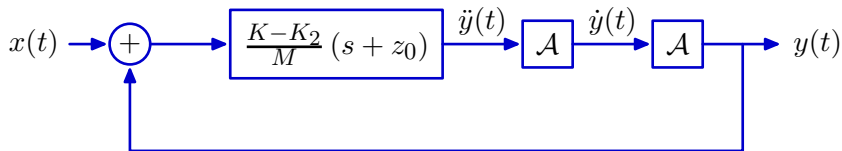
Increasing  $K_2$  moves poles toward the origin and then onto  $j\omega$  axis.



But the poles are still marginally stable.

## Magnetic Levitation

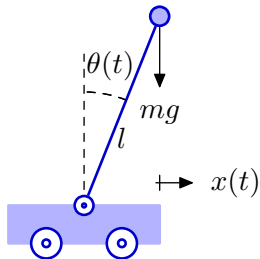
Adding a zero makes the poles stable for sufficiently large  $K_2$ .



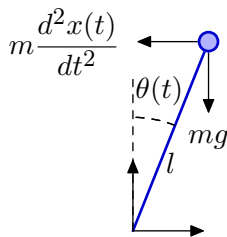
Try it: Demo [designed by Prof. James Roberge].

## Inverted Pendulum

As a final example of stabilizing an unstable system, consider an inverted pendulum.



lab frame  
(inertial)

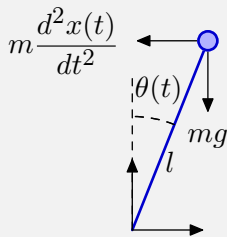
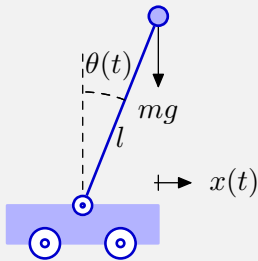


cart frame  
(non-inertial)

$$\underbrace{ml^2}_{I} \frac{d^2\theta(t)}{dt^2} = \underbrace{mg}_{\text{force}} \underbrace{l \sin \theta(t)}_{\text{distance}} - \underbrace{m \frac{d^2 x(t)}{dt^2}}_{\text{force}} \underbrace{l \cos \theta(t)}_{\text{distance}}$$

## Check Yourself: Inverted Pendulum

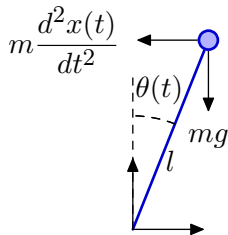
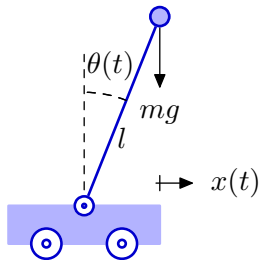
Where are the poles of this system?



$$ml^2 \frac{d^2 \theta(t)}{dt^2} = mgl \sin \theta(t) - m \frac{d^2 x(t)}{dt^2} l \cos \theta(t)$$

## Check Yourself: Inverted Pendulum

Where are the poles of this system?



$$ml^2 \frac{d^2 \theta(t)}{dt^2} = mgl \sin \theta(t) - m \frac{d^2 x(t)}{dt^2} l \cos \theta(t)$$

$$ml^2 \frac{d^2 \theta(t)}{dt^2} - mgl \theta(t) = -ml \frac{d^2 x(t)}{dt^2}$$

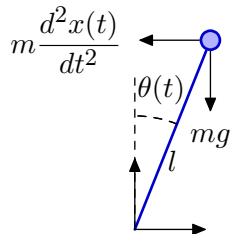
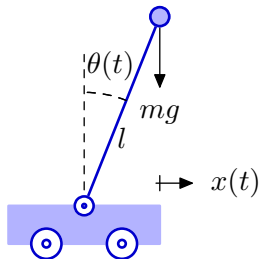
$$H(s) = \frac{\Theta}{X} = \frac{-mls^2}{ml^2 s^2 - mgl} = \frac{-s^2/l}{s^2 - g/l}$$

$$\text{poles at } s = \pm \sqrt{\frac{g}{l}}$$

## Inverted Pendulum

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This unstable system can be stabilized with feedback.



Try it. Demo. [originally designed by Marcel Gaudreau]

## Feedback and Control

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Using feedback to enhance performance.

Examples:

- improve performance of an op amp circuit.
- control position of a motor.
- reduce sensitivity to unwanted parameter variation.
- reduce distortions.
- stabilize unstable systems
  - magnetic levitation
  - inverted pendulum