6.003 9/28/2011

"Tutor overload"

email class

6.003-staff

"Q4" not parsed by row.

Rec 6

Ann:

Recap: * $\mathcal{L}: X(s) = \int_{-\infty}^{\infty} x(t) e^{-st}\, dt$

* Relate CT system descriptions

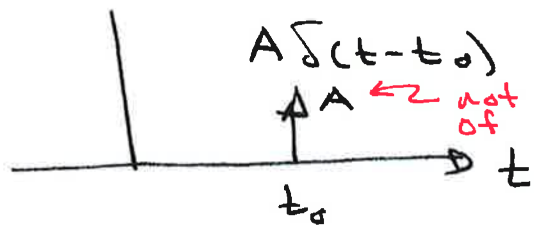
* Solve DE

Today: $\left\{ \begin{array}{l} * \ \delta(t) \\ * \ \mathcal{L} \end{array} \right.$

$\delta(t)$ – Impulse

symbol: $\delta(t)$

graph:



NOTE: ← not the "height" of peak, but area under impulse

$$\int_{-\infty}^{\infty} \delta(t-t_0)\,dt = 1$$

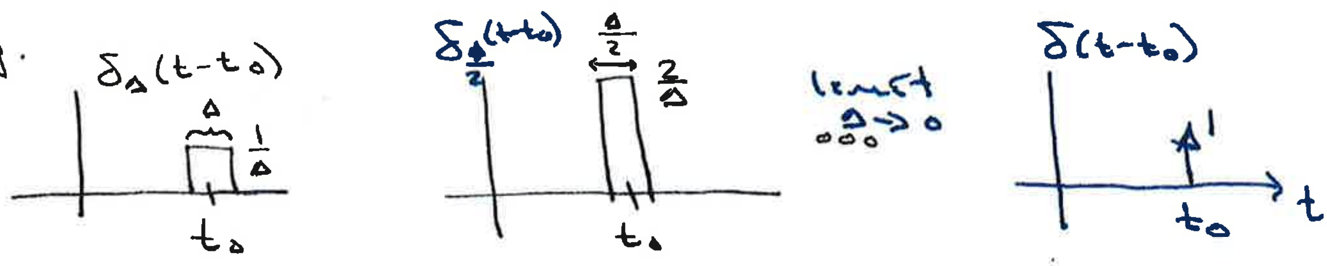
$$\delta(t-t_0) = 0 \,, \; t \neq t_0$$

Note: The height & width of impulse are undefined.

One approach:

$\delta(t)$ limit of a family of functions w/ const area:

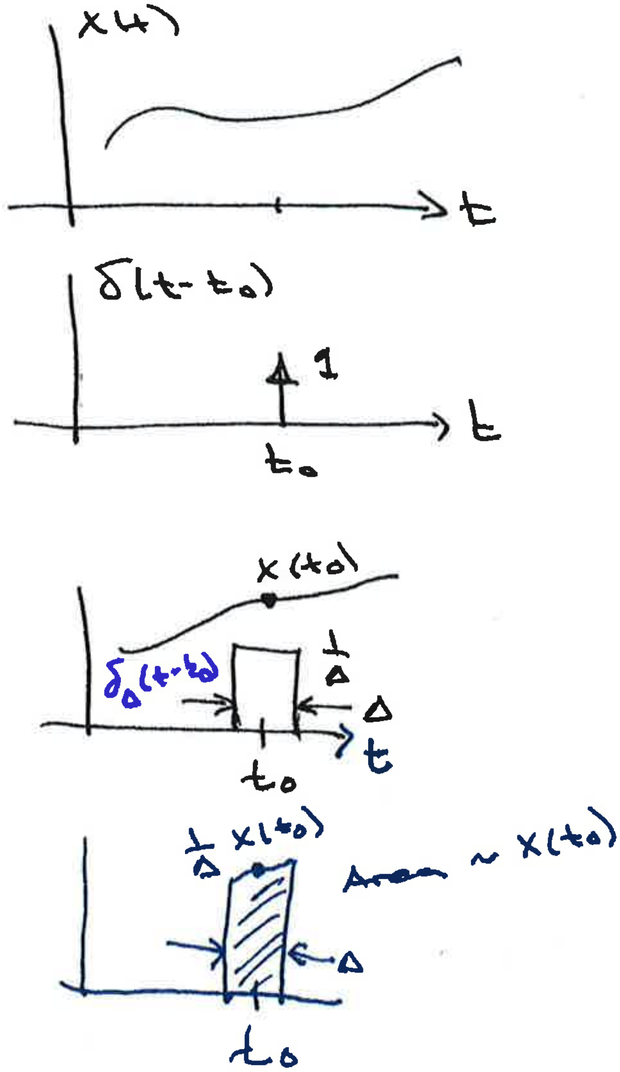
e.g.



we care about properties of $\delta(t)$.

think about how $\delta(t)$ behaves inside an integral.

Ex: what's $x(t)\,\delta(t-t_0)$?



$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$$

"sifting property"

i.e. write

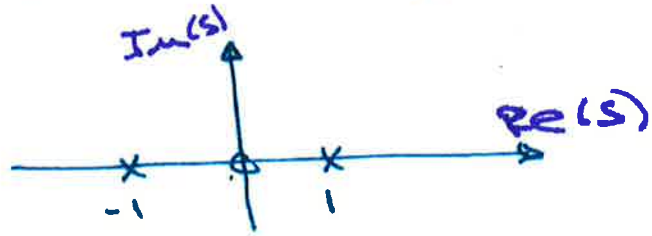
$$x(t)\,\delta(t-t_0) = \underbrace{x(t_0)\,\delta(t-t_0)}_{\text{impulse w/ area } x(t_0).}$$

e.g.

$$\mathcal{L}(\delta(t)) = \int_{-\infty}^{\infty} \delta(t)\, e^{-st}\, dt$$

$$\overset{\text{"sift"}}{=} \int_{-\infty}^{\infty} 1 \cdot \delta(t)\, dt = 1$$

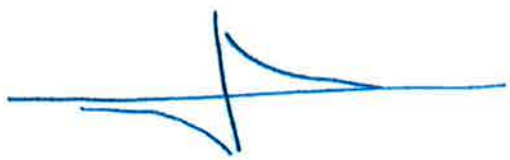
Ex: The following pole-zero diag maps to how many of the following time funcs?



(1)

$e^{-|t|}$

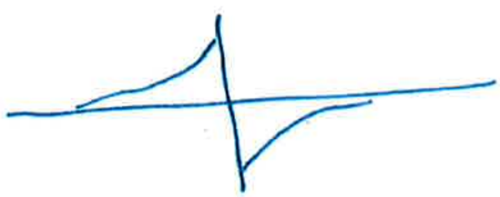
(2)



$\mathrm{sgn}(t)\, e^{-|t|}$

$\begin{cases} -1 & t<0 \\ 1 & t>0 \end{cases}$

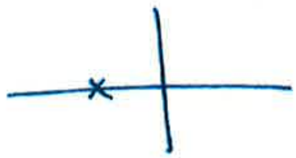
(3)



$-\mathrm{sgn}(t)\, e^{-|t|}$

Look at poles:

i)



$$e^{-t}u(t) \leftrightarrow \int_0^\infty e^{-t} e^{-st}\, dt = -\frac{1}{s+1} e^{-(s+1)t}\Big|_0^\infty$$

$$= \frac{1}{s+1} \qquad Re(s+1) > 0,\ Re(s) > -1$$

Also have "the other" solution:

$$e^{-t}u(-t) \leftrightarrow \int_{-\infty}^0 e^{-(s+1)t}\, dt$$

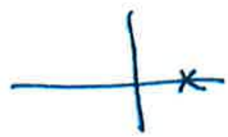
$$= -\frac{1}{s+1} e^{-(s+1)t}\Big|_{-\infty}^0$$

$$e^{-t}u(-t) \leftrightarrow -\frac{1}{s+1} \qquad Re(s+1) < 0$$

$$Re(s) < -1$$

or $$-e^{-t}u(-t) \leftrightarrow \frac{1}{s+1} \qquad Re(s) < -1$$

ii) similarly, for pole @ $s=1$;



$$e^{t} u(t) \Leftrightarrow \int_{0}^{\infty} e^{t} e^{-st}\, dt = \frac{1}{s-1} \quad Re(s)>1$$

$$e^{t} u(-t) \Leftrightarrow \int_{-\infty}^{0} e^{t} e^{-st}\, dt = \frac{e^{-(s-1)t}}{-(s-1)}\Big|_{-\infty}^{0} = \frac{-1}{s-1} \quad Re(s)<1$$

i.e. $-e^{t} u(-t) \leftrightarrow \frac{1}{s-1}$, $Re(s) < 1$

combine:

From p/z plot

gain, not specified on p/z plot

$$H(s) = K \cdot \frac{s}{(s-1)(s+1)}$$

$$H(s) = \frac{A}{s-1} + \frac{B}{s+1}$$

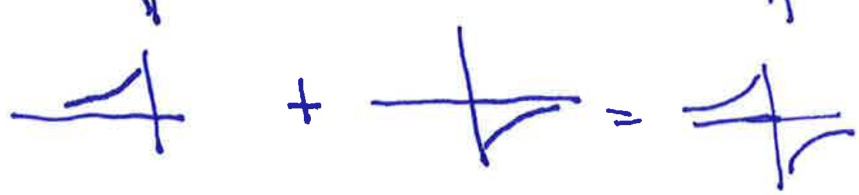
$$A = (s-1)H(s)\Big|_{s=1} = \frac{K}{2}$$

$$B = (s+1)H(s)\Big|_{s=-1} = \frac{K}{2}$$

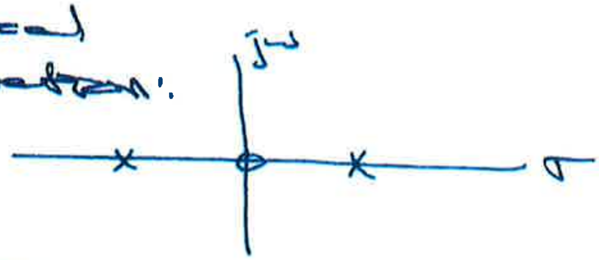
$$\therefore \quad H(s) = \frac{K/2}{s-1} + \frac{K/2}{s+1}$$

$K>0$:

$K<0$:



graphical evaluation:



ROC?

$\frac{1}{s-1} \leftrightarrow$ 

possible components to $h(t)$ from pole @ $s=1$

$\frac{1}{s+1} \leftrightarrow$ 

... pole @ $s=-1$

i.e.

a  $\leftrightarrow \quad \frac{1}{s+1} \quad Re(s) > -1$

(+)

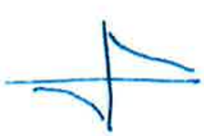
b  $\leftrightarrow \quad \frac{-1}{s-1} \quad Re(s) < +1$

=

note, missing zero

①  $\leftrightarrow \quad \frac{1}{s+1} - \frac{1}{s-1} = \frac{(s-1)-(s+1)}{(s+1)(s-1)} = \frac{-2}{(s+1)(s-1)}$

② But $a-b=$

 $\leftrightarrow \quad \frac{1}{s+1} - \frac{-1}{s-1} = \frac{(s-1)+(s+1)}{(s+1)(s-1)} = \frac{2s}{(s+1)(s-1)}$

③ & $-a+b=$

 $\leftrightarrow \quad \frac{-1}{s+1} + \frac{-1}{s-1} = \frac{-2s}{(s+1)(s-1)}$ (negat. k)

Ex: Complex poles.

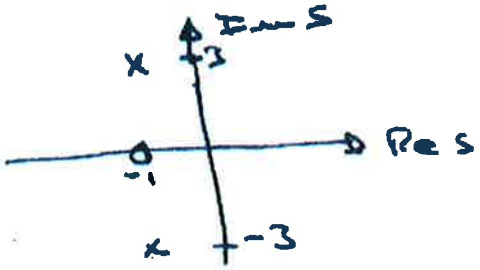
$x(t) = \delta(t) \rightarrow \quad \rightarrow y(t) = h(t)$

$$x(t) \longrightarrow \boxed{H(s) = \frac{8(s+1)}{s^2+2s+10}} \longrightarrow y(t)$$

Use $\mathcal{L}^{-1}$ via PFE:

$$H(s) = \underbrace{\frac{8(s+1)}{s^2+2s+10}}_{P/Z} = \underbrace{\frac{4}{s-(-1-3j)} + \frac{4}{s-(-1+3j)}}_{PFE}$$

$$\boxed{e^{\alpha t}u(t) \leftrightarrow \frac{1}{s-\alpha} \quad Re(s)>\alpha}$$



$$\boxed{\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{4-40}}{2} = -1 \pm j3}$$

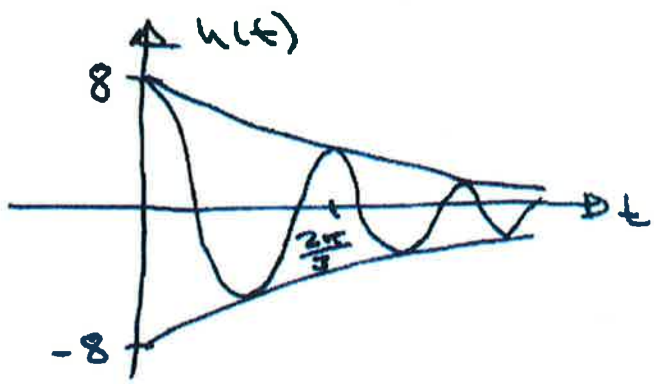
$\longleftrightarrow$ ROC for causal $h(t)$

$\therefore \; h(t) = 4e^{(-1+3j)t}u(t) + 4e^{(-1-3j)t}u(t)$

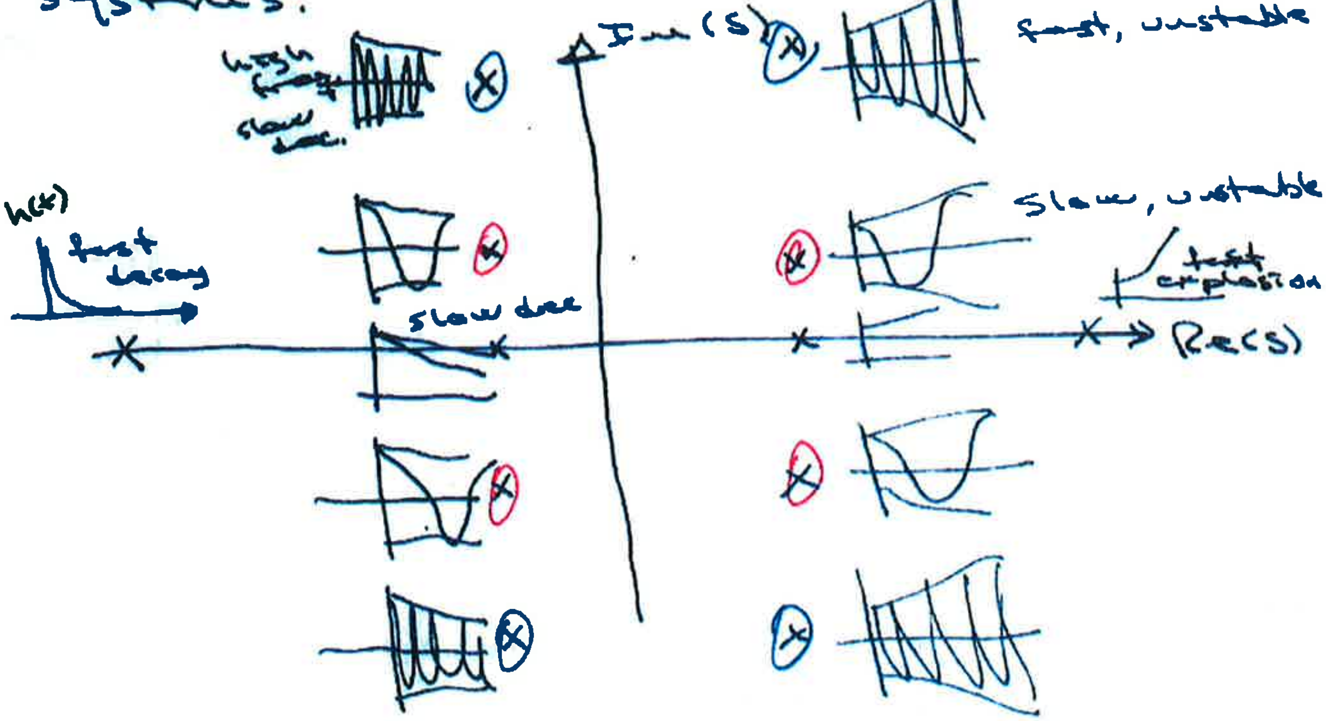
$h(t) = 8 \cdot e^{-t} \cdot \cos 3t \cdot u(t)$

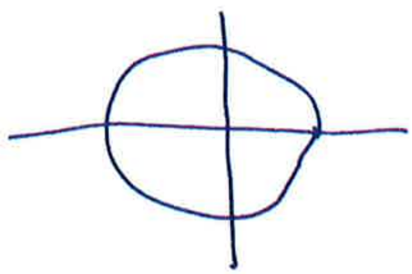
real part, decay

imaginary part of pole $\therefore$ oscillation



In general, for physical, (causal) systems:





$u(t) \leftrightarrow \frac{1}{s}$

$$\left(s-(-1-3j)\right)H(s)\Big|_{s=-1-3j} =$$

$$= \frac{8(-1-3j+1)}{(-1-3j)-(-1+3j)} = \frac{-24j}{-6j} = 4$$

$$\left(s-(-1+3j)\right)H(s)\Big|_{s=-1+3j}$$

$$= \frac{8(-1+3j+1)}{-1+3j-(-1-3j)} = \frac{24j}{6j} = 4$$