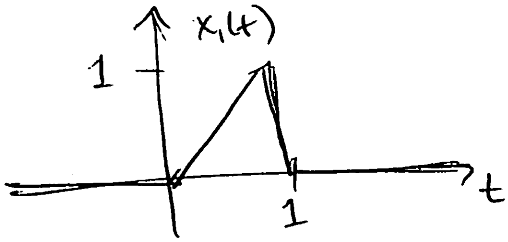
6.003 Recitation 9 10-12-2011

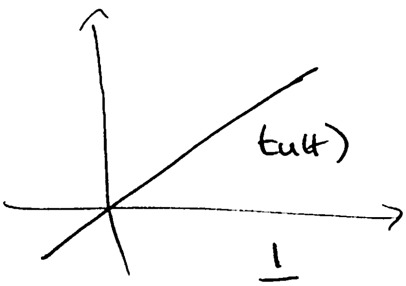
Today: More frequency response. Russ Tedrake

Quick aside from exam: Where did the extra term come from in Prob 3a.

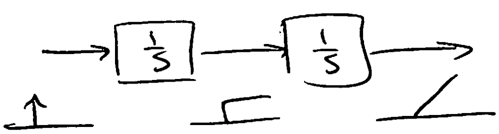


$$X_1(s) = \frac{1-e^{-s}-se^{-s}}{s^2}.$$

(many of you got $\frac{1-e^{-s}}{s^2}$)

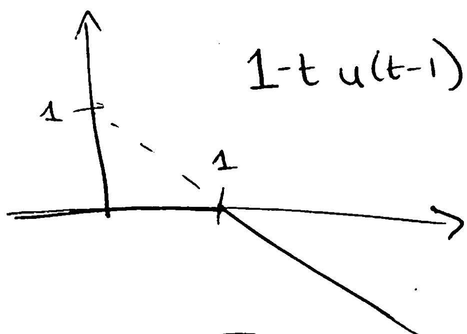


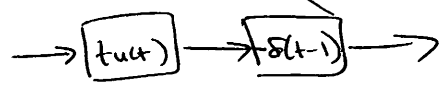
$\frac{1}{s^2}$



+

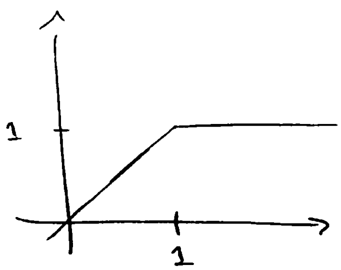
$1 - t\,u(t-1)$.





$-\frac{1}{s^2}e^{-s}$

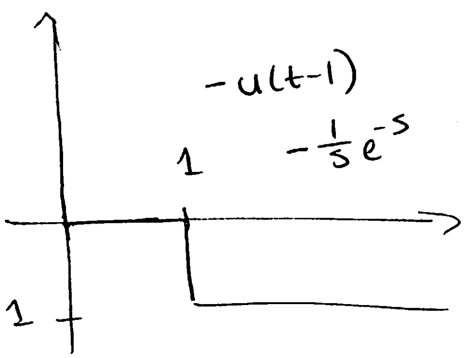
=



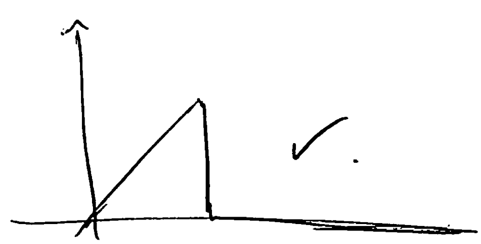
+

$-u(t-1)$

$-\frac{1}{s}e^{-s}$



=



✓.

$$X_1(s) = \frac{1}{s^2} + -\frac{1}{s^2}e^{-s} - \frac{1}{s}e^{-s} = \frac{1-e^{-s}-se^{-s}}{s^2}.$$

or take derivative, then integrate.

$u(t) - u(t-1) - \delta(t-1)$.

$\frac{1}{s} \quad -\frac{1}{s}e^{-s} \quad -e^{-s}$.

DT Frequency Response

Consider The system $h[n] = p^n u[n]$.

Q: Sketch the frequency response. (mag & phase) for

a) $p_1 = \frac{1}{2}$.

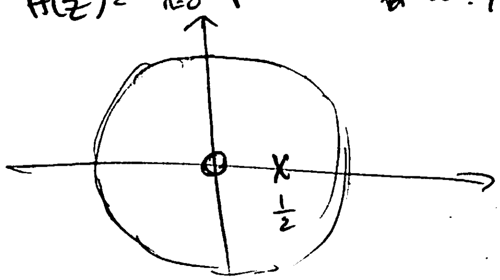
(Ask only one at a time).

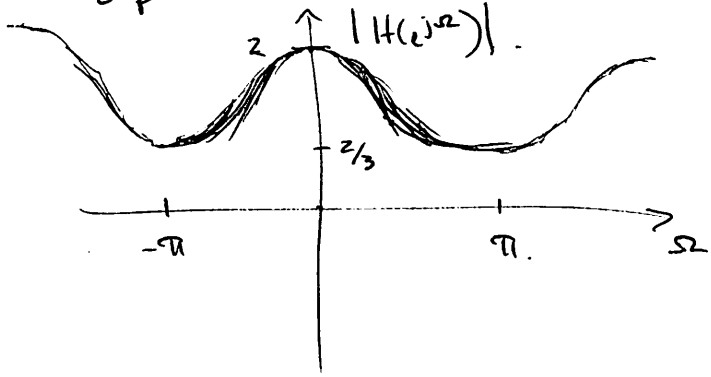
~~b) $p_1 = -\frac{1}{2}$~~

~~c) $p_1 = \frac{3}{2}$~~

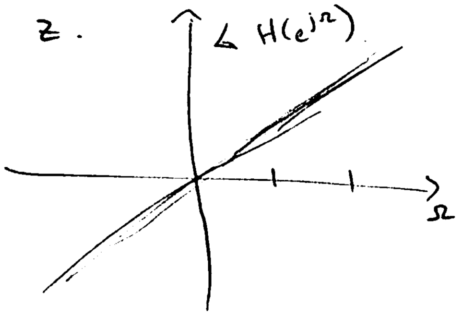
$$H(z) = \sum_{n=0}^{\infty} (p z^{-1})^n = \frac{1}{1 - p z^{-1}} = \frac{z}{z-p}.$$

a)



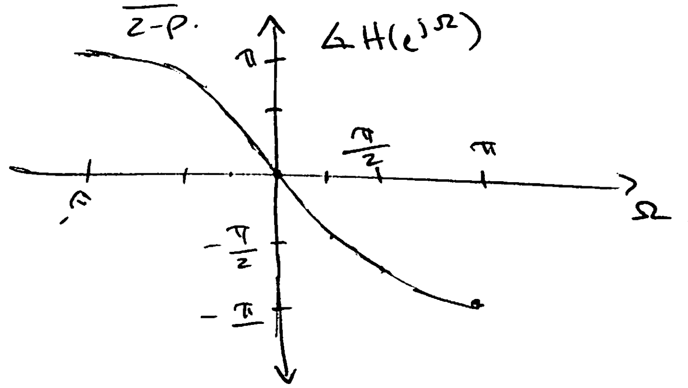


z.

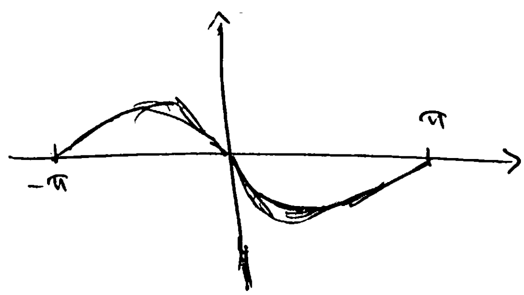


+

$\frac{1}{z-p}$.

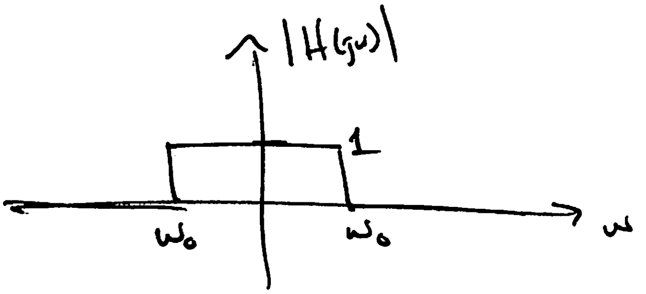


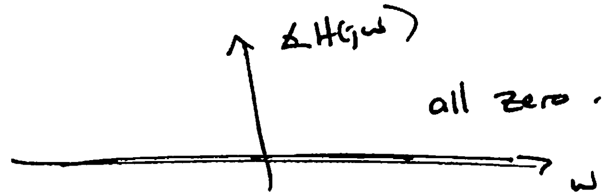
=



Let's think about this response as a "low pass filter".

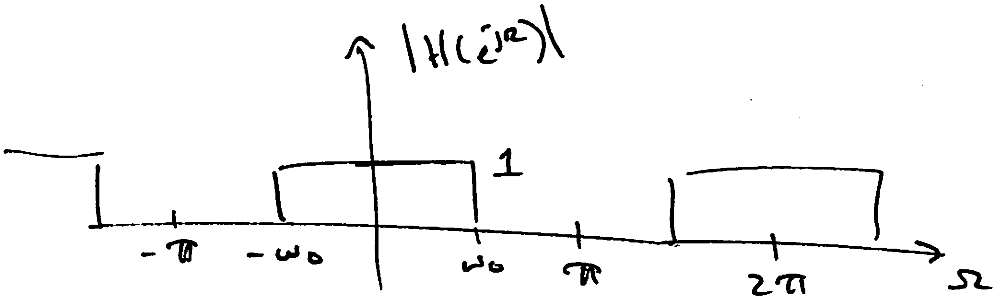
Ideal CT Low Pass Filter.





Ideal DT Low Pass Filter.

(must be periodic in 2π).



$\angle H(e^{j\Omega})$ zero.

our response : - did not start at one.
- went down slowly (not a box)
- didn't go to zero.

Q: How would you change p_1 to make it a high-pass filter.

A: e.g. $p_1 = -\frac{1}{2}$.

Moving average filter.

$$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$$

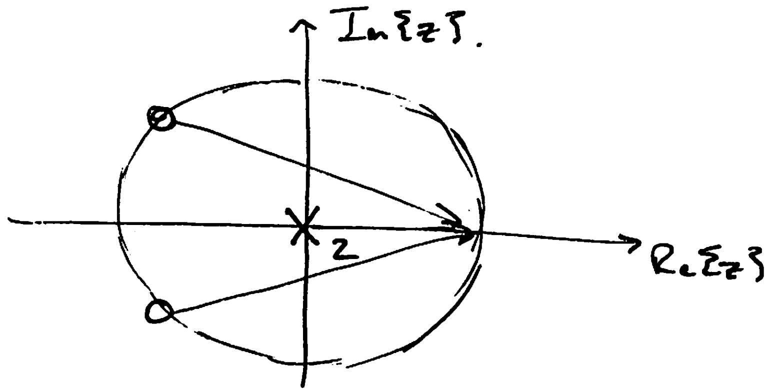
Known as a FIR (finite impulse response) filter.

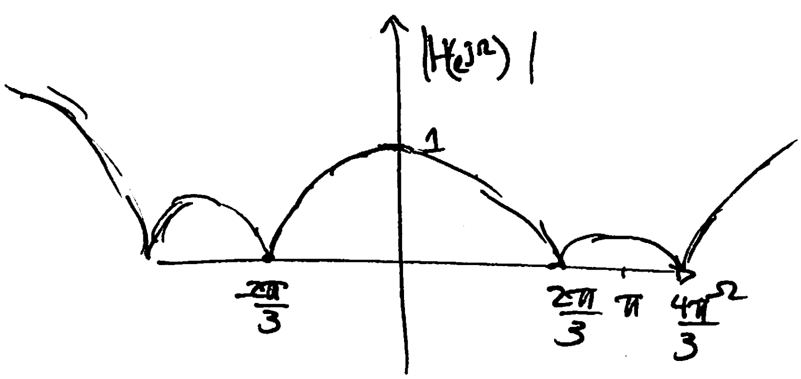
Previous example was IIR.

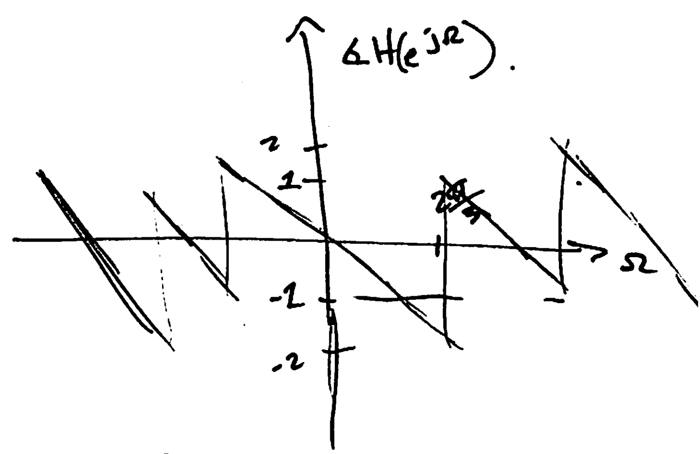
$$H(z) = \frac{1}{3}\left(1 + z^{-1} + z^{-2}\right)$$

$$= \frac{1}{3}\,\frac{z^2 + z + 1}{z^2}$$

$$z_1 = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2} = e^{\pm j\frac{2\pi}{3}}.$$







You know $\frac{1}{s-p_1}$ is a LPF in CT.

What about HPF? Try: $\frac{s}{s-p_1}$