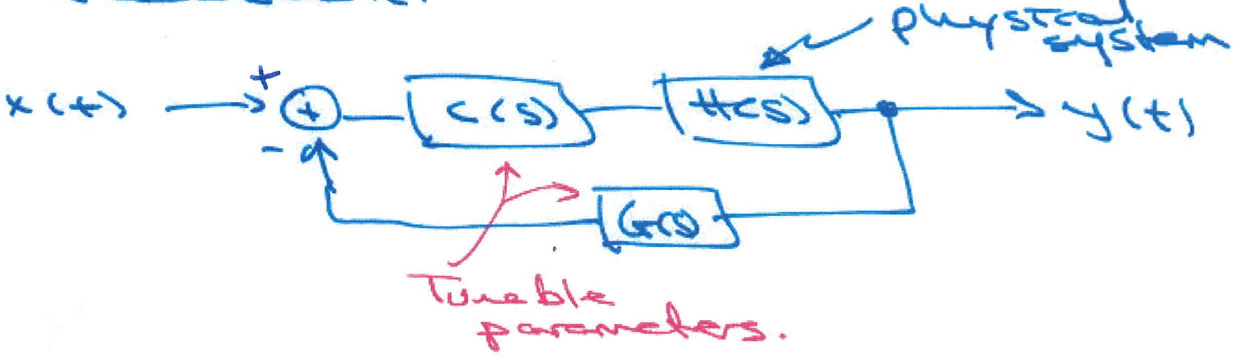
Recitation #10

Class: Aircraft, owls, and feedback control

Today: Feedback and Root-Locus

Intro:

From last time, general formulation of feedback:



H(s) – physical system

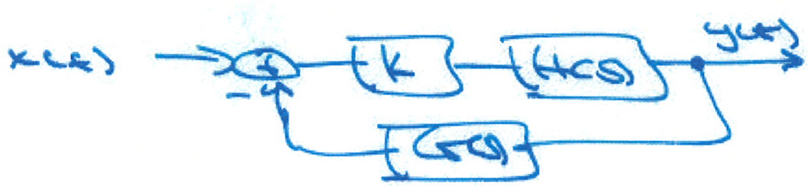
C, G – Designed w/ 1 or more free parameters.

$$Q(s) = \frac{Y(s)}{X(s)} = \frac{C \cdot H}{1 + CGH}$$

Q: How do poles of overall system behave? ← "closed-loop"

Answer: "Root locus" of $1 + C(s)G(s)H(s)$.

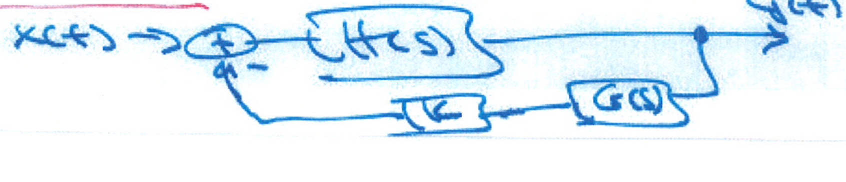
In the classical Root-locus problem, get to control 1 free parameter, k. $C(s) = k$, a gain i.e.



*closed-loop poles are the same
* only diff by gain.

$$Q(s) = \frac{kH(s)}{1 + kH(s)G(s)}$$

variant:



$$Q(s) = \frac{H(s)}{1 + kH(s)G(s)}$$

Consider

$$Q(s) = \frac{KH(s)}{1 + KH(s)G(s)} = \frac{Y(s)}{X(s)}$$

Poles of $Q(s)$ are zeros of $1+KGH$
i.e., where

$$1 + KGH = 0$$

* If G & H are rational poly in s, finding zeros of $1+KGH$ = factoring = hard. (for a specific $\underset{\text{value}}{K}$)

exception: 2nd or 1st order GH

Root locus provides alternate method

i) find roots in limiting cases, $K = 0$, $K \to \pm\infty$

ii) Connect limits by "rules."

$\underline{K=0}$: At low gain, $K \to 0$,

$1 + KGH = 0$ implies $|GH| \to \infty$
$KGH = -1$

$\therefore$ @ low gain closed-loop poles are poles of $G(s) \cdot H(s)$ [= "open-loop" system function]

$\underline{K \to \infty}$: At high gain, $K \to \infty$, so

$$1 + KGH = 0 \text{ implies } |G \cdot H| \to 0$$
$$KGH = -1$$

$\therefore$ asymptotic high-gain closed loop poles are zeros of $G(s) \cdot H(s)$.

Rule $\emptyset$

Start @ poles of $G(s)H(s)$, End at zeros $G \cdot H$.
↳ could be at ∞

To fill in the gap, $\underbrace{0 < K < \infty}_{\text{start w/ this range}}$, use

$$KGH = -1$$

i.e.

magnitude cond. $|KGH| = 1$

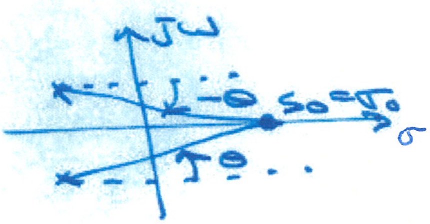
angle cond. $\measuredangle KGH = \pi + 2\pi n$, n integ.

more practical.

Rules: Real axis in s-plane / How about z-plane?

→ $h(t)$ (or $h[n]$) real $\Rightarrow$ P/Z conj pairs

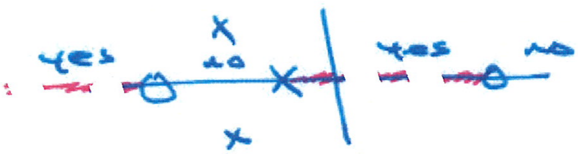
→ Conjugate pairs add $\emptyset$ to $\measuredangle$ cond for s = real (or z = real)



→ root locus on real axis in s-plane determined by real poles & zeros of $G \cdot H$.

For positive K,

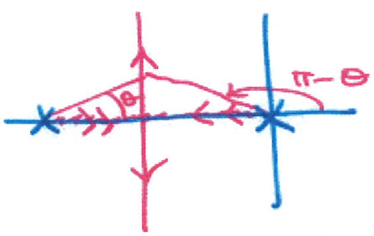
Rule: Need odd # of p/z to right of root-locus on real axis.



Break-Away Pts:

i.e. multiple roots

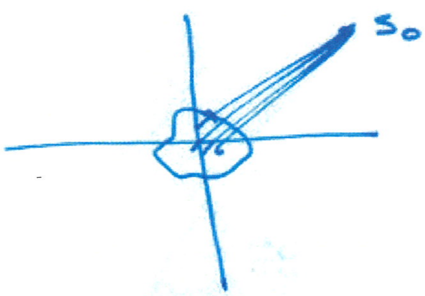
If 2 closed-loop poles collide on real axis => become complex



At point of collision: $\measuredangle = \pi + n2\pi$

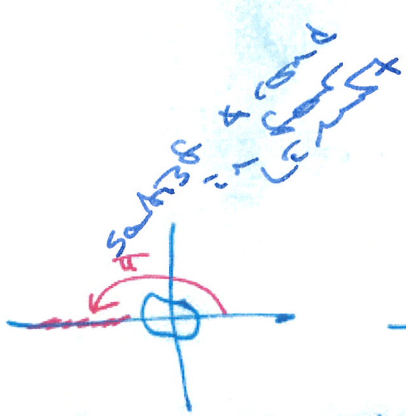
$(\pi - \theta) + (\theta) = \pi$

Asymptotes: #p, #z

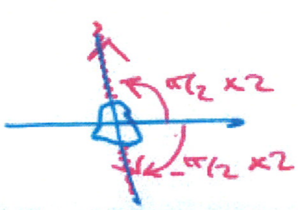


* In the far limit all angles ~ equal
* what matters is how many ∡ pole cancels ∡ zeros
* Only "excess" poles matter

↑ i.e. zero @ infinity



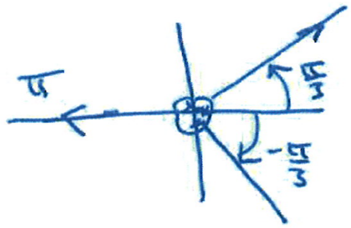
1 excess pole
zero @ ∞



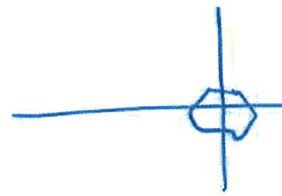
2 excess poles
2 zeros @ ∞

...

3 excess poles
(3 zeros @ ∞)



no excess poles (no zeros @ ∞)



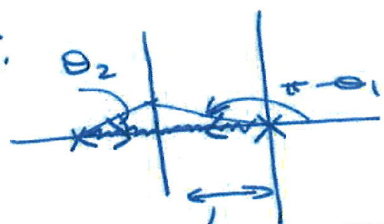
root locus cannot get far from ~~open loop~~ G·H poles/zeros

next

<u>point</u> ≠ know rules by heart

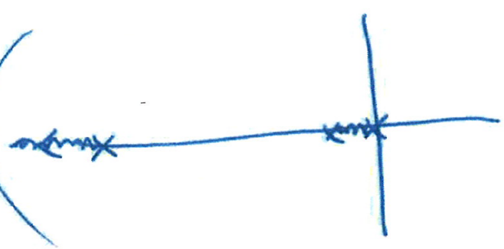
<u>point</u> = a new way to analyze & think about systems
- will let you <u>derive</u> new rules.

<u>Eg</u>:



where will break-away occur?

need $\theta_1 = \theta_2$ ∴ ½-way btwn.



$\pi-\theta_1 + \theta_2 + \theta_3 = \pi + n \cdot 2\pi$

i.e. θ

point: ≠ know 10^6 rules

point: = Lets you derive new "rules"
~ new way to think about systems.

#

E.g. So far assumed $K \geq 0$, $KGH = -1$

But, when $K < 0$, $\measuredangle GH = 2n\pi$

$G \cdot H = \frac{-1}{K}$ $(K<0)$? $\Rightarrow \measuredangle GH = 2\pi \cdot n$

so root locus is on "the other" part of real axis.

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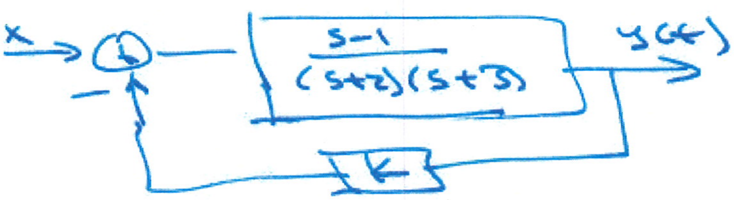
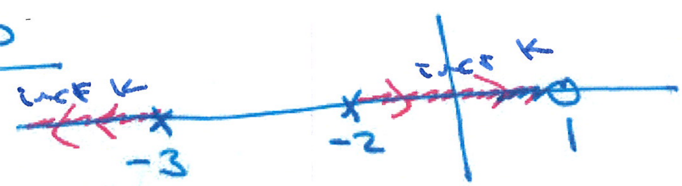
Ex: $G(s) \cdot H(s) = \dfrac{s-1}{(s+2)(s+3)}$



* poles of GH @ $s = -2, -3$
* zeros of GH @ $s = 1$ & $(|s| = \infty)$

$K \geq 0$



For which value of K does it become unstable?

$$\frac{Y(s)}{X(s)} = \frac{s-1/(s+2)(s+3)}{1 + K(s-1)/(s+2)(s+3)} = \frac{s-1}{(s+2)(s+3)+K(s-1)}$$

$$= \frac{s-1}{s^2 + s(5+K) + 6 - K}$$ ← pole @ $s=0$ for $K=6$ ∴ unstable