Russ Tedrake

6.003 Final Review
(Part I)

Goal for today:

- Do practice problems.
- Find the "cobwebs".

$$y[n] = -3y[n-1] + x[n] + 2x[n-1].$$

what is the step response?

$s[n]$?

A:

$s[0] = 1.$

$s[1] = -3+3 = 0.$

$s[k>1] = -3s[k-1] + 3.$

$s[2] = 3$

$s[3] = -6.$

$s[4] = 21.$

$\vdots$

$$Y(z) = -3z^{-1}Y(z) + X(z). + 2z^{-1}X(z).$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+2z^{-1}}{1+3z^{-1}}$$

Step response: $\frac{1}{1-z^{-1}}H(z).$ because: $s[n] = s[n-1] + x[n].$

$$S(z) = \frac{1+2z^{-1}}{(1-z^{-1})(1+3z^{-1})} = \frac{A}{1-z^{-1}} + \frac{B}{1+3z^{-1}}.$$

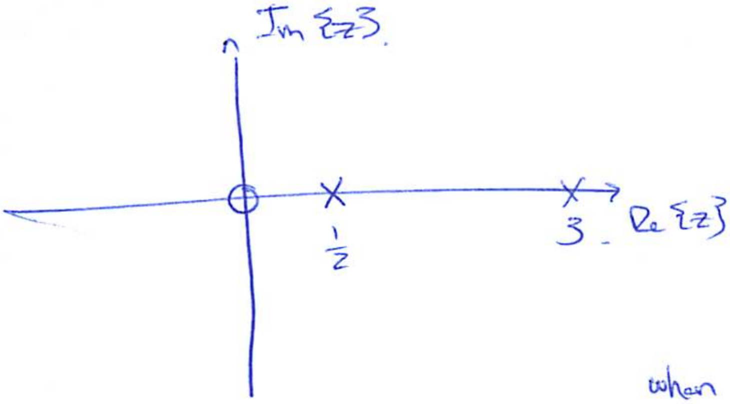
$$A(1+3z^{-1}) + B(1-z^{-1}) = 1+2z^{-1}.$$

$z^{-1} = 1 \Rightarrow 4A = 3.$ $\quad A = \frac{3}{4}.$

$z^{-1} = -\frac{1}{3} \Rightarrow \frac{4}{3}B = \frac{1}{3}$ $\quad B = \frac{1}{4}.$

$$S(z) = \frac{3/4}{1-z^{-1}} + \frac{1/4}{1+3z^{-1}}.$$

$$s[n] = \frac{3}{4}u[n]. + \frac{1}{4}(-3)^n u[n].$$



when $x[n] = 1 \quad \forall n$

$y[n] = -1 \quad \forall n.$

find $H(z)$.

$$H(z) = \frac{k \cdot z}{(z-\frac{1}{2})(z-3)}$$

when $x[n] = 1$, or 1^n, or z^n, $z = 1$

$y[n] = -1$, or $-(1)^n$ or $-1 \cdot z^n$, $z = 1$.

$$\Rightarrow H(z)\Big|_{z=1} = -1.$$

$$\Rightarrow \frac{k}{\frac{1}{2} \cdot (-2)} = -1, \quad k = 1.$$

$$H(z) = \frac{z}{(z-\frac{1}{2})(z-3)} \quad \text{or} \quad \frac{z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}.$$

$$x \longrightarrow \boxed{\text{System}} \longrightarrow y.$$

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a) $x[n] = \cos\left(\frac{\pi}{2}n\right) + 2\sin\left(\frac{\pi}{4}n\right)$

$y[n] = \cos\left(\frac{\pi}{2}n\right) + 2\sin\left(\frac{\pi}{8}n\right).$

what is $h_1[n]$?

A: not LTI.

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b) $x[n]$ as above.

$y[n] = \frac{1}{2}\cos\left(\frac{\pi}{2}n\right).$

what is $h_2[n]$?

B: Not fully specified.

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c). $x(t) = e^{2t}u(t)$

$y(t) = e^{t}u(t).$

what is $h_3(t)$?

A: $X(s) = \frac{1}{s-2}$ $\quad Re\{s\} > 2$

$Y(s) = \frac{1}{s-1}$ $\quad Re\{s\} > 1$

$H(s) = \frac{Y(s)}{X(s)} = \frac{s-2}{s-1} = 1 - \frac{1}{s-1}$. $\quad Re\{s\} > 1$.

$h_3(t) = \delta(t) - e^{t}u(t).$

$$H(s) = \frac{1}{(s-3)(s+2)}.$$ system is causal.

what is the step response?

A. $Y(s) = \frac{1}{s} H(s) = \frac{1}{s(s-3)(s+2)} = \frac{A}{s} + \frac{B}{s-3} + \frac{C}{s+2}.$

$$1 = A(s-3)(s+2) + Bs(s+2) + Cs(s-3).$$

$s=0 \Rightarrow \cancel{A} \; -6A = 1.$ $\quad A = -\frac{1}{6}.$

$s=3 \Rightarrow 15B = 1.$ $\quad B = \frac{1}{15}.$

$s=-2 \Rightarrow +10C = 1$ $\quad C = +\frac{1}{10}$

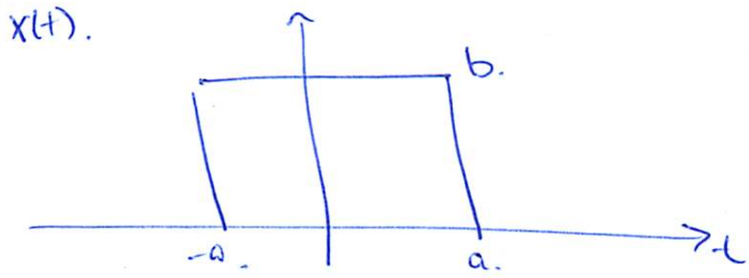
$$Y(s) = \frac{-1/6}{s} + \frac{1/15}{s-3} + \frac{1/10}{s+2}.$$

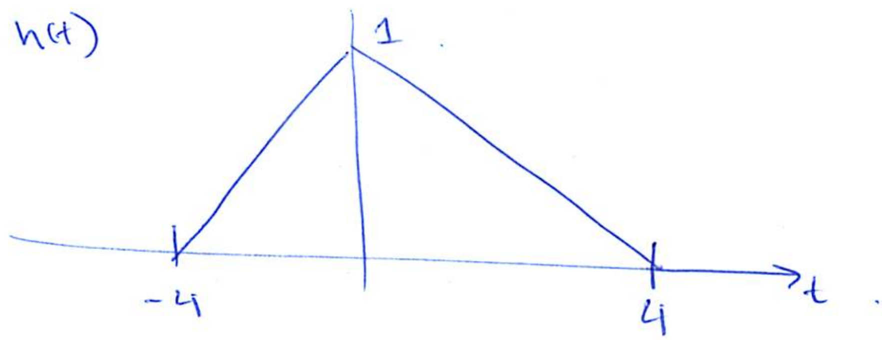
$$y(t) = \left[-\frac{1}{6} + \frac{1}{15}e^{3t} + \frac{1}{10}e^{-2t}\right]u(t).$$

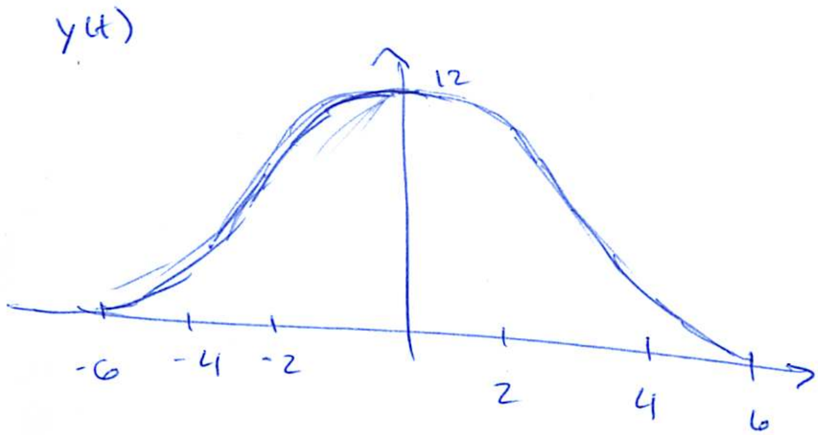
$$H(s) = \frac{s}{(s+3)(s-2)}$$ system is causal.

what is $H(j\omega)$?

A: ROC doesn't include $j\omega$-axis.







What are a and b?

A:

$a \cancel{b} = 2$.

$b = 4$.

Easy w/ convolution. (harder in frequency domain!)

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau.$$

first non-zero at $t = -6$.

$\Rightarrow a = 2$.

$$b \int_{-2}^{2} h(\cancel{t} - \tau)\, d\tau = 12.$$

$b = 4$.