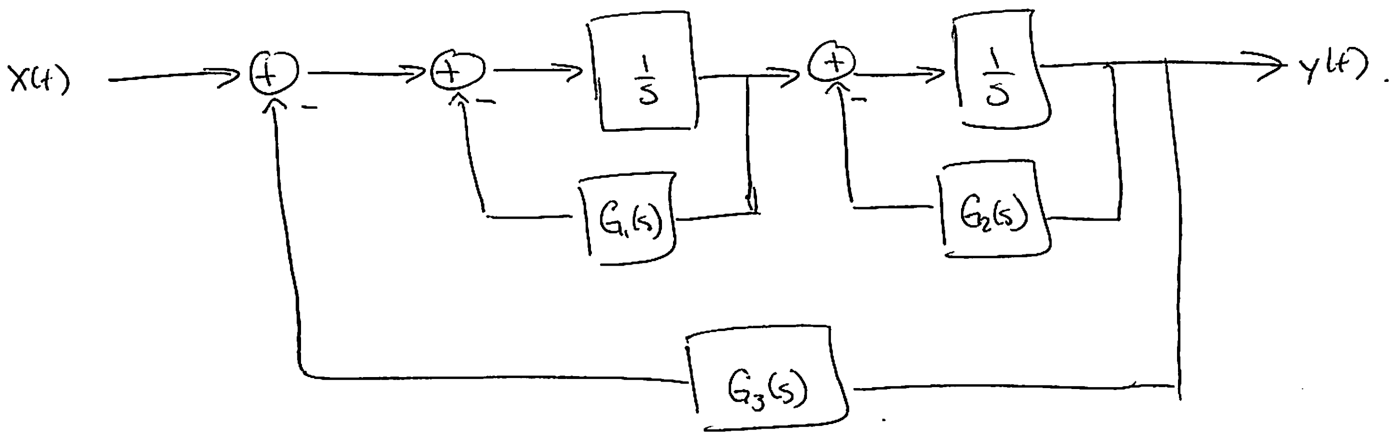
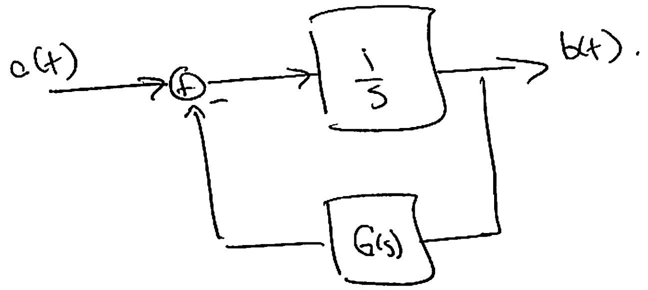
Feedback.



What's the overall system function?



$$B(s) = \frac{1}{s}\left(A(s) - G(s)B(s)\right).$$

$$\frac{B(s)}{A(s)} = \frac{\frac{1}{s}}{1 + \frac{1}{s}G(s).} = \frac{1}{s+G_1(s).}$$

$$Y(s) = \left(\frac{1}{s+G_2(s)}\right)\left(\frac{1}{s+G_1(s)}\right)\left(X(s) - G_3(s)Y(s)\right).$$

$$\frac{Y(s)}{X(s)} = \frac{\frac{1}{s+G_2}\frac{1}{s+G_1}}{1 + \frac{1}{s+G_2}\frac{1}{s+G_1}G_3.} = \frac{1}{(s+G_1)(s+G_2) + G_3.}$$

Suppose $G_1 = G_2 = 0$. Design simple G_3 so overall system is stable:

$$H(s) = \frac{1}{s^2 + G_3.}$$

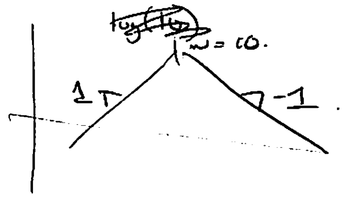
$G_3 = c$ doesn't work.

$G_3 = \cancel{as+b}$
$2s+1.$ $\Rightarrow H(s) = \frac{1}{(s+1)^2.}$

Bode.

$$H(s) = \frac{s}{(s+10)^2}.$$

$$\log|H(j\omega)| = \log|j\omega| - 2\log|j\omega+10|.$$



$\omega < 10 \sim \log|10|.$

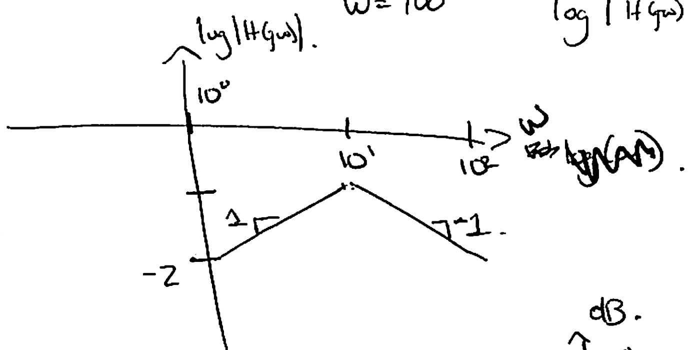
$\omega > 10 \sim \log|\omega|.$

a few points:

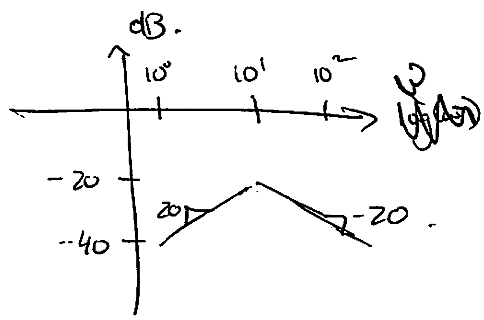
$\omega = 10.\quad \log|H(j\omega)| \sim 1 - 2\cdot 1 = -1.$

$\omega = 1\quad \log|H(j\omega)| \sim 0 - 2\cdot 1 = -2.$

$\omega = 100\quad \log|H(j\omega)| \sim 2 - 2\cdot 2 = -2.$

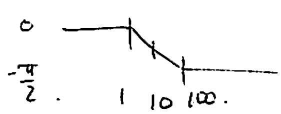


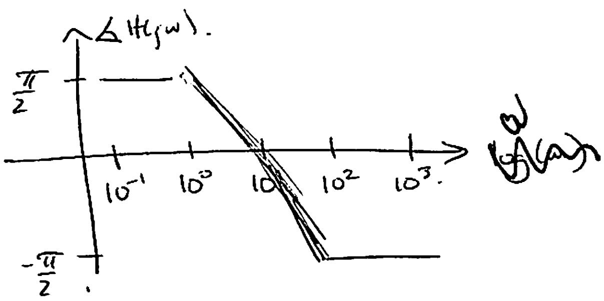
or in dB.



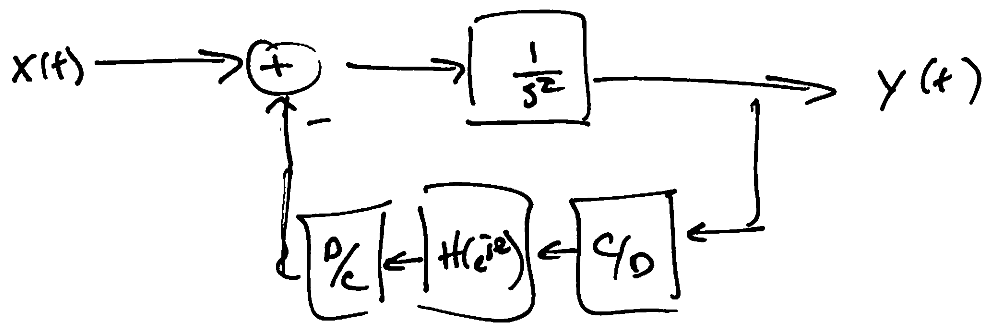
$$\measuredangle H(j\omega) = \measuredangle j\omega - 2\,\underbrace{\measuredangle(j\omega - 10)}$$

$= \frac{\pi}{2}.$





Consider this:



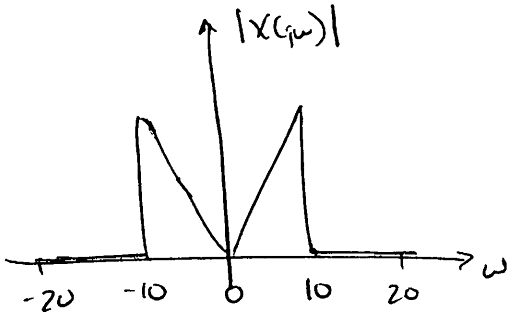
what's difficult here?

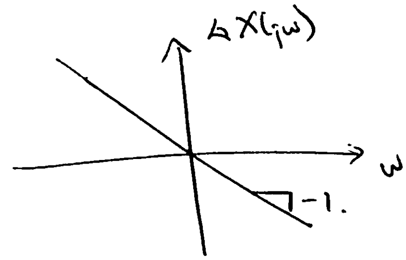
Sampling, etc, is not LTI!

Discuss what happens in practice...

Frequency Response / CTFT.

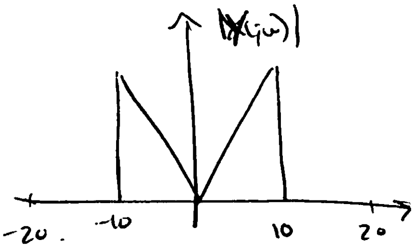
Given a signal $x(t)$ with

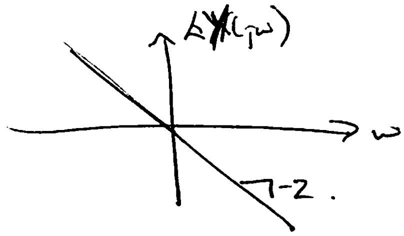




Sketch mag + phase for

i) $x(t-1) \Longleftrightarrow e^{-j\omega 1} X(j\omega)$



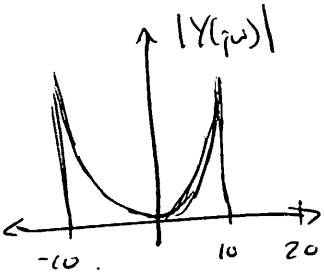


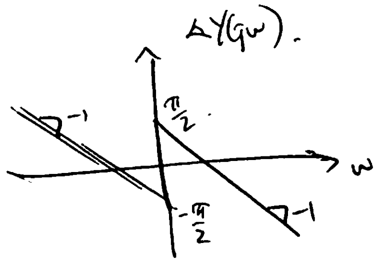
ii) $\dfrac{dx(t)}{dt} \Longleftrightarrow j\omega X(j\omega)$.

$|Y(j\omega)| = |j\omega|\,|X(j\omega)| = \omega\,|X(j\omega)|$

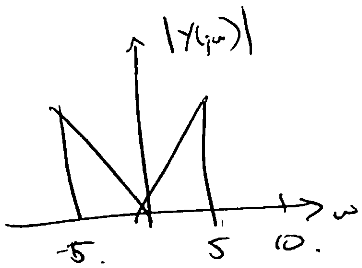
$\measuredangle Y(j\omega) = \measuredangle(j\omega) + \measuredangle X(j\omega)$.

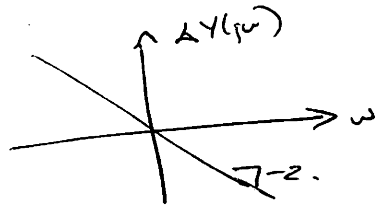
$$= \begin{cases} \frac{\pi}{2} + \measuredangle X(j\omega) & \text{for } \omega > 0. \\ -\frac{\pi}{2} + \measuredangle X(j\omega) & \omega < 0. \end{cases}$$



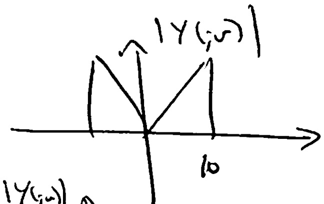


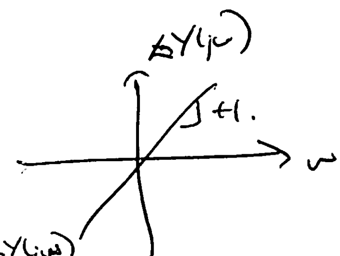
iii). $x\left(\frac{t}{2}\right) \Longleftrightarrow 2X(2j\omega)$.



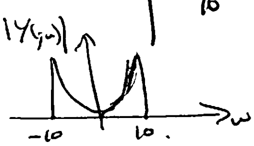


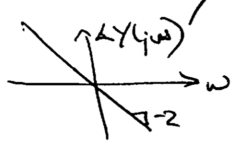
iv) $x(-t) \Longleftrightarrow X(-j\omega)$



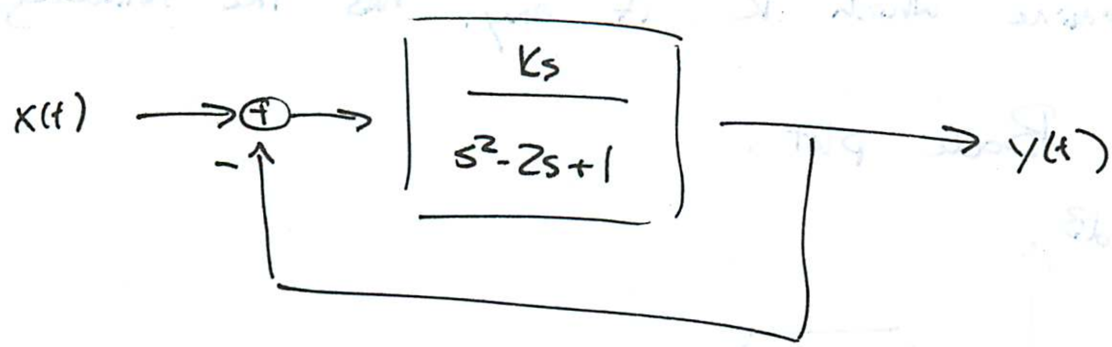


v) $x(t) * x(t) \Longleftrightarrow X(j\omega)X(j\omega)$





Feedback



For what values of K is it stable?

$$Y(s) = \frac{Ks}{s^2-2s+1}\left(X(s) - Y(s)\right).$$

$$H(s) = \frac{Ks}{s^2-2s+1+Ks.} = \frac{Ks}{s^2+(K-2)s+1.}$$

poles are $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a.} = \cancel{2-K \pm \sqrt{(K-2)^2}}$

$$\frac{-b \pm \sqrt{b^2-4}}{2.}$$

necessary
+ . sufficient for $b>0$.

because $\sqrt{b^2 - \text{pos}} < b$.

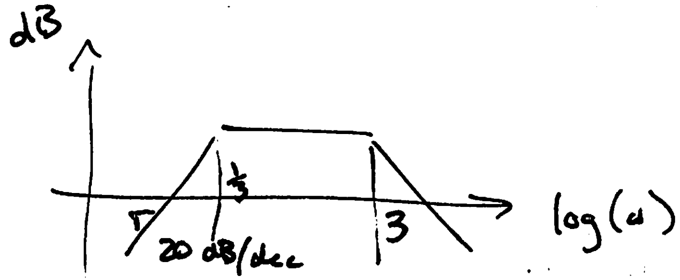
$K-2>0$

$\Rightarrow K>2.$

What ~~are the poles~~ for $K=4$?
is the step response.

$$\frac{1}{s}H(s) = \frac{4}{s^2+2s+1} = \frac{4}{(s+1)^2} \overset{\mathcal{L}}{\Longleftrightarrow} 4te^{-t}u(t).$$

Determine which K, if any, has the following Bode plot.



$$H(s) = \frac{Ks}{s^2 + (K-2)s + 1}$$

two poles: $s = -3$, $s = -\frac{1}{3}$.

$$(s+3)(s+\tfrac{1}{3}) = s^2 + (K-2)s + 1.$$

$$s^2 + \frac{10}{3}s + 1$$

$$K - 2 = \frac{10}{3}$$

$$K = \frac{16}{3}$$