Quiz I Review
Signals and Systems
6.003

Massachusetts Institute of Technology

March 1, 2010

Quiz 1 Details

- *Date:* Wednesday March 3, 2010
- *Time:* 7.30pm–9.30pm
- *Where:* 34-101
- *Content:* (boundaries inclusive)
  - Lectures 1–7
  - Recitations 1–8
  - Homeworks 1–4

Review Outline

- Preliminaries
  - Converting CT to DT
  - System modeling
- Discrete time systems
- Feedback, poles, and fundamental modes
- Continuous time systems
- Laplace transforms
- Z transforms
- Numerical methods

Preliminaries: converting CT to DT

When converting a DT signal to CT, we can use either *zero-order hold*

$$x_c(t) = \sum_{n=-\infty}^{\infty} x_d[n] b\left(\frac{t-nT}{T}\right) \tag{1}$$

where b is a unit square function. Additionally, we can also use a *piecewise linear* approximation

$$x_c(t) = \sum_{n=-\infty}^{\infty} x_d[n] a\left(\frac{t-nT}{T}\right) + \sum_{n=\infty}^{\infty} x_d[n+1] c\left(\frac{t-nT}{T}\right) \tag{2}$$

where a and c are the right- and left-sided unit triangles functions, respectively.

Preliminaries: System modeling

Know the basics: (1) *system modeling:* spring equations, LRC circuits, leaky tank models; (2) *equations solutions:* solving difference and differential equations; (3) *signals:* scaling, inverting and shifting.

- *Leaky tank modeling:* The leak rate $r(t)$ is proportional to the height of the water in the tank $h(t)$,

$$\frac{dh(t)}{dt} \propto r_{\text{in}}(t) - r_{\text{out}}(t) \quad (3)$$

$$\frac{dr(t)}{dt} = \frac{r_{\text{in}}(t)}{\tau} - \frac{r_{\text{out}}(t)}{\tau} \quad (4)$$

- *Circuit modeling:*
  - Capacitor: $V = CdV/dt$
  - Inductor: $V = LdI/dt$
  - Resistor: $V = IR$:-)

Discrete Time Systems

The unit sample is given by

$$\delta[n] = \begin{cases} 1 & n = 0\,, \\ 0 & \text{otherwise}\,. \end{cases} \quad (5)$$

The unit step is given by

$$u[n] = \begin{cases} 1 & n \geq 0\,, \\ 0 & \text{otherwise}\,. \end{cases} \quad (6)$$

- Given a system function equation $H(s) = AB$, A and B are two systems running in series
- Given a system function equation $H(s) = A + B$, A and B are two systems running in parallel

For systems with feedback, we often use Black's formula

$$H(s) = \text{feed through transmission}/(1 - \textit{looptransmission}) \quad (7)$$

Poles, and fundamental modes

- A pole p is the base of a geometric sequence
- When dealing with a system functional Y/X, use partial fractions to find poles
- $p < -1$, system does not converge, alternating sign
- $p \in [-1, 0)$, magnitude converges, alternating sign
- $p \in [0, 1]$, magnitude converges monotonically
- $p > 1$, magnitude diverges monotonically
- Complex poles cause oscillations

Continuous Time Systems

The unit sample is given by

$$\delta(t) = \lim_{\epsilon \to 0} \begin{cases} 1/2\epsilon & t \in [-\epsilon, \epsilon] \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

The unit step is given by

$$u(t) = \int_{-\infty}^{t} \delta(\lambda) d\lambda = \begin{cases} 1 & t \geq 0\,, \\ 0 & \text{otherwise}\,. \end{cases} \quad (9)$$

- The fundamental mode associated with p converges if $Re(p) < 0$ and diverges if $Re(p) > 0$
- Compared to a DT system, the fundamental mode associated with p converges if p lies within the unit circle

Laplace Transforms

- Defined by

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt \quad (10)$$

- A double-sided LT and its ROC provide a unique system function
- Left-sided signals have left-sided ROCs, and right-sided signals have right-sided ROCs
- The ROC is the intersection of each ROC generated by each pole individually
- Go over problem 3 in homework 3 to review ROCs
- The *sifting property* of $\delta(t)$

$$f(0) = \int_{-\infty}^{\infty} f(t)\delta(t)dt \quad (11)$$

Laplace Transforms: Properties

Table: Key LT properties

Property	$x(t)$	$X(s)$
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$
Delay by T	$x(t - T)$	$e^{-sT}X(s)$
Multiply by t	$tx(t)$	$\frac{-dX(s)}{ds}$
Multiply by $e^{-\alpha T}$	$x(t)e^{-\alpha T}$	$X(s + \alpha)$
Differentiate	$\frac{dx(t)}{dt}$	$sX(s)$
Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda$	$\frac{X(s)}{s}$

Initial and Final value theorems

- *Initial value theorem:* If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$ then

$$x(0^+) = \lim_{s \to \infty} sX(s) \quad (12)$$

- *Final value theorem:* If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \to \infty$ then

$$x(\infty) = \lim_{s \to 0} sX(s) \quad (13)$$

Z Transforms

- Defined by

$$X(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} \quad (14)$$

- ROCs are delimited by circles
  - Inside and outside circles are given by left- and right-sided transforms, respectively.

Z Transforms: Properties

Table: Key ZT properties

Property	$x[n]$	$X(z)$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$
Delay	$x[n-1]$	$z^{-1}X(z)$
Multiply by n	$nx[n]$	$\frac{-zdX(z)}{dz}$
Multiply by a^n	$x[n]a^n$	$X(z/a)$
Unit step	$u[n]$	$1/(1-z^{-1})$

Numerical Methods

To approximate derivatives, we have the following techniques.

- *Forward Euler:*

$$y_c(nT) = (y_d[n+1] - y_d[n])/T\,, \tag{15}$$

where T is the time difference. The pole can often shift out of the stability region!

- *Backward Euler:*

$$y_c(nT) = (y_d[n] - y_d[n-1])/T\,. \tag{16}$$

This approximation is more stable than forward Euler.

- *Trapezoidal rule:* Use centered differences.

If $$\dot{y}(t) = x(t) \Rightarrow (y[n] - y[n-1])/T = (x[n] - x[n-1])/2\,. \tag{17}$$

The entire left half plane is mapped onto the unit circle. In particular, the entire $j\omega$ axis is mapped onto the unit circle

End of Review

Good luck on Wednesday! :-)