6.003 (Spring 2010)

Final Examination *May 20, 2010*

Name:

Kerberos Username:

Please circle your section number:

Section	*Instructor*	*Time*
1	Peter Hagelstein	10 am
2	Peter Hagelstein	11 am
3	Rahul Sarpeshkar	1 pm
4	Rahul Sarpeshkar	2 pm

Grades will be determined by the correctness of your answers (explanations are not required).

Partial credit will be given for ANSWERS that demonstrate some but not all of the important conceptual issues.

You have **three hours.**

Please put your initials on all subsequent sheets.

Enter your answers in the boxes.

This quiz is closed book, but you may use four 8.5×11 sheets of paper (eight sides total).

No calculators, computers, cell phones, music players, or other aids.

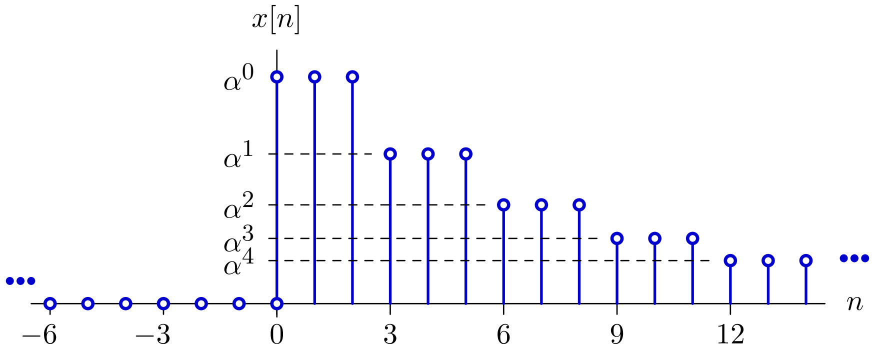
1	/10
2	/12
3	/18
4	/15
5	/15
6	/15
7	/15
Total	/100

1. Z Transform *[10 points]*

Find the Z transform of $x[n]$ defined as

$$x[n] = \alpha^{\lfloor n/3 \rfloor}\, u[n]$$

where α represents a positive real number, $u[n]$ represents the unit-step function, and $\lfloor n/3 \rfloor$ represents the largest integer that is $\leq n/3$. The signal $x[n]$ is plotted below.



Enter a closed-form expression for $X(z)$ in the box below.

$X(z) =$

Enter the region of convergence of $X(z)$ in the box below.

ROC =

Worksheet (intentionally blank)

2. CT System Design *[12 points]*

We wish to design a linear, time-invariant, continuous-time system that is causal and stable. For asymptotically low frequencies, the magnitude of the system's frequency response should be 4ω. For asymptotically high frequencies, the magnitude of the system's frequency response should be $100/\omega$.

Is it possible to design such a system so that the magnitude of its frequency response is 50 at $\omega = 5$?

Yes or **No**:

If **Yes**, determine the poles of the resulting system.

poles:

If **No**, briefly explain why not.

Worksheet (intentionally blank)

3. Frequency Responses *[18 points]*

Match each system function in the left column below with the corresponding frequency response magnitude in the right column.

Enter label of corresponding frequency response magnitude (**A-F** or **none**) in box.

$$H_1(z) = \frac{z^3}{z^3 - 0.5} \quad \rightarrow$$

$$H_2(z) = \frac{z^3 - 1}{z^3 - 0.5} \quad \rightarrow$$

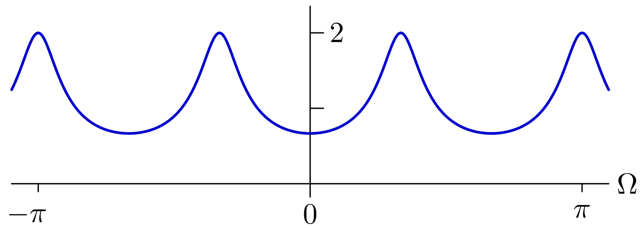
$$H_3(z) = \frac{z^3 + z^2 + z}{z^3 - 0.5} \quad \rightarrow$$

$$H_4(z) = \frac{z^3}{z^3 + 0.5} \quad \rightarrow$$

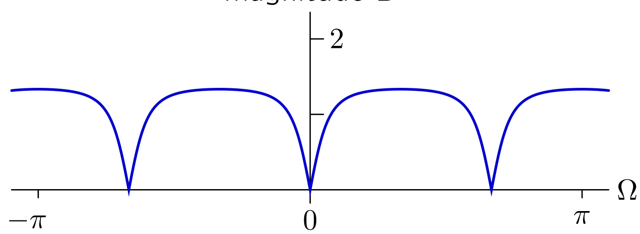
$$H_5(z) = \frac{z^3 + 1}{z^3 + 0.5} \quad \rightarrow$$

$$H_6(z) = \frac{z^3 - z^2 + z}{z^3 + 0.5} \quad \rightarrow$$

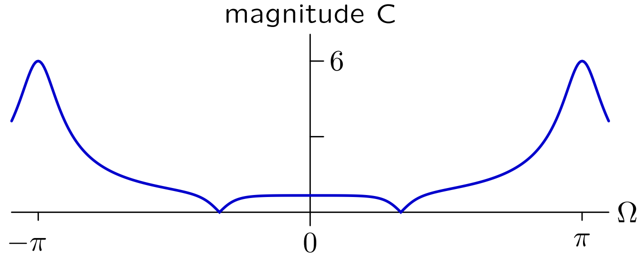
magnitude A



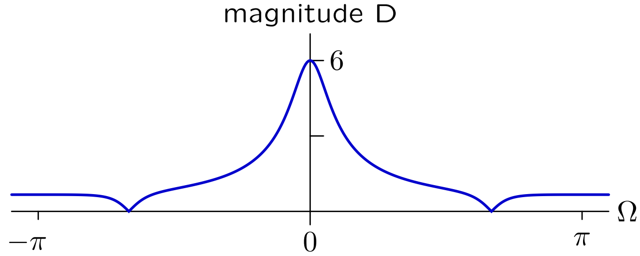
magnitude B



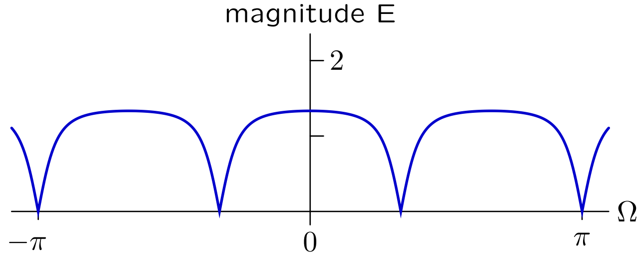
magnitude C



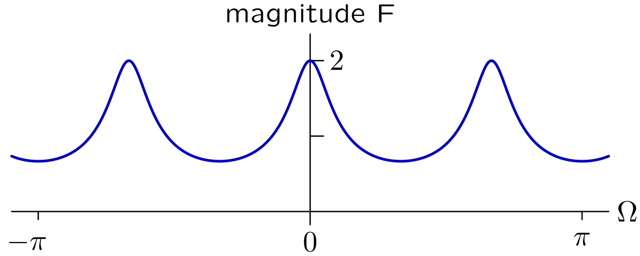
magnitude D



magnitude E



magnitude F



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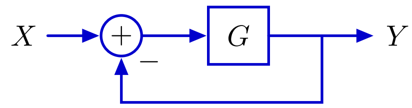
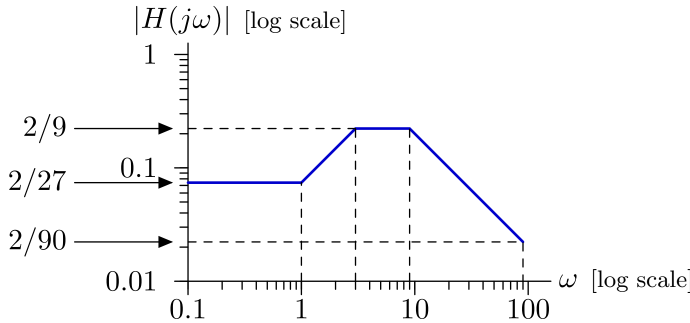
4. Feedback Design *[15 points]*

Consider a feedback system of the following form



where G represents a causal, linear, time-invariant, continuous-time system. The magnitude of the frequency response of $H = \frac{Y}{X}$ is specified by the straight-line approximation shown below.



The magnitude for asymptotically low frequencies is $\dfrac{2}{27} \approx 0.074$.

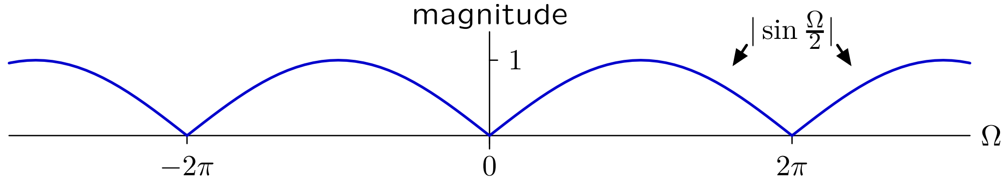
Determine $G(s)$. [You only need to find one solution, even if others exist.]

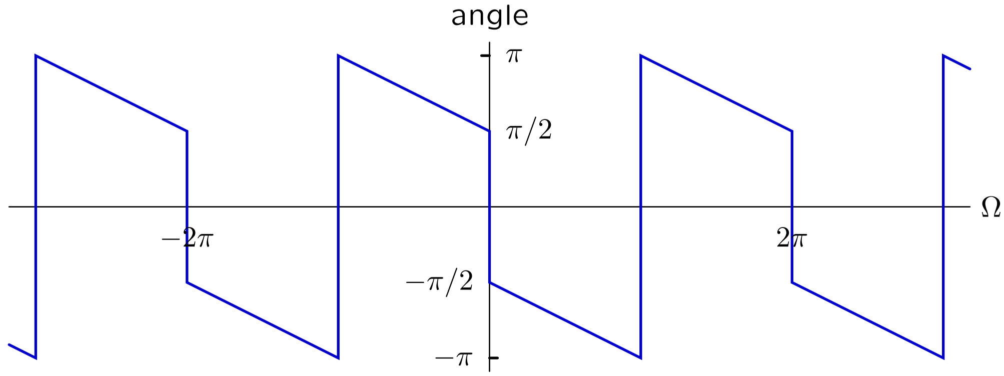
$G(s) =$ ☐

Worksheet (intentionally blank)

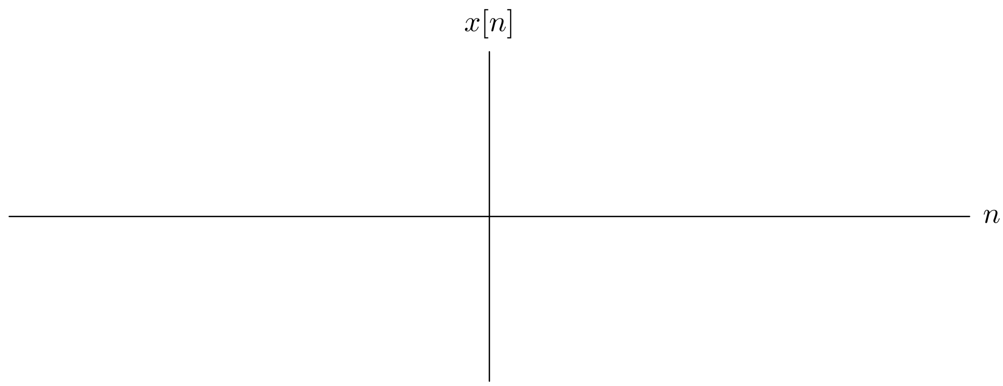
5. Inverse Fourier *[15 points]*

The magnitude and angle of the Fourier transform of $x[n]$ are shown below.





Sketch and fully label $x[n]$ on the axes below.



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6. Taylor series *[15 points]*

Let $x_c(t)$ represent the convolution of $x_a(t)$ with $x_b(t)$ where

$$x_a(t) = \begin{cases} a_0 + a_1 t + \frac{a_2}{2!}\,t^2 + \frac{a_3}{3!}\,t^3 + \cdots + \frac{a_n}{n!}\,t^n + \cdots & ;\ t > 0 \\ 0 & ;\ \text{otherwise} \end{cases}$$

$$x_b(t) = \begin{cases} b_0 + b_1 t + \frac{b_2}{2!}\,t^2 + \frac{b_3}{3!}\,t^3 + \cdots + \frac{b_n}{n!}\,t^n + \cdots & ;\ t > 0 \\ 0 & ;\ \text{otherwise} \end{cases}$$

$$x_c(t) = \begin{cases} c_0 + c_1 t + \frac{c_2}{2!}\,t^2 + \frac{c_3}{3!}\,t^3 + \cdots + \frac{c_n}{n!}\,t^n + \cdots & ;\ t > 0 \\ 0 & ;\ \text{otherwise} \end{cases}$$

Determine expressions for the c coefficients (c_0, c_1, ... c_n) in terms of the a coefficients (a_0, a_1, ... a_n) and b coefficients (b_0, b_1, ... b_n).

$c_0 =$

$c_1 =$

$c_2 =$

$c_3 =$

$c_n =$

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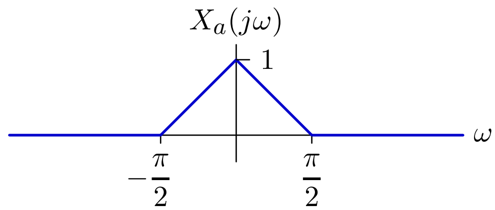
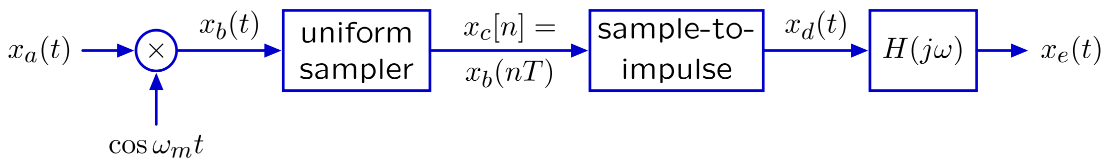
7. Modulated Sampling *[15 points]*

The Fourier transform of a signal $x_a(t)$ is given below.



This signal passes through the following system



where $x_d(t) = \sum_{n=-\infty}^{\infty} x_c[n]\delta(t-nT)$ and

$$H(j\omega) = \begin{cases} T & \text{if } |\omega| < \frac{\pi}{T} \\ 0 & \text{otherwise}. \end{cases}$$

The constants ω_m and T can be different in **Parts a-c** below.

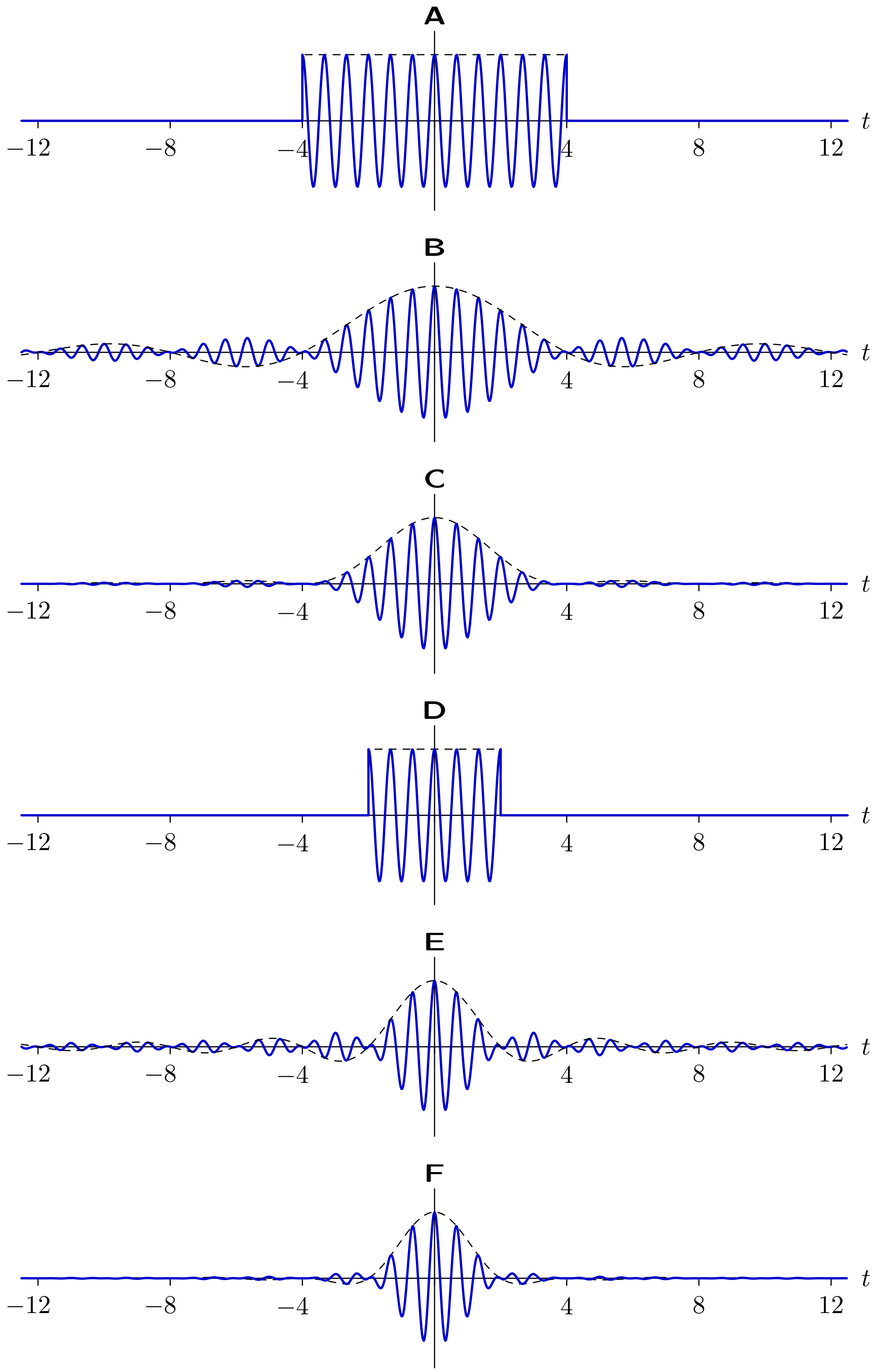
Part a. Which (if any) of the waveforms on the following page could represent $x_b(t)$?

A-F or **none**

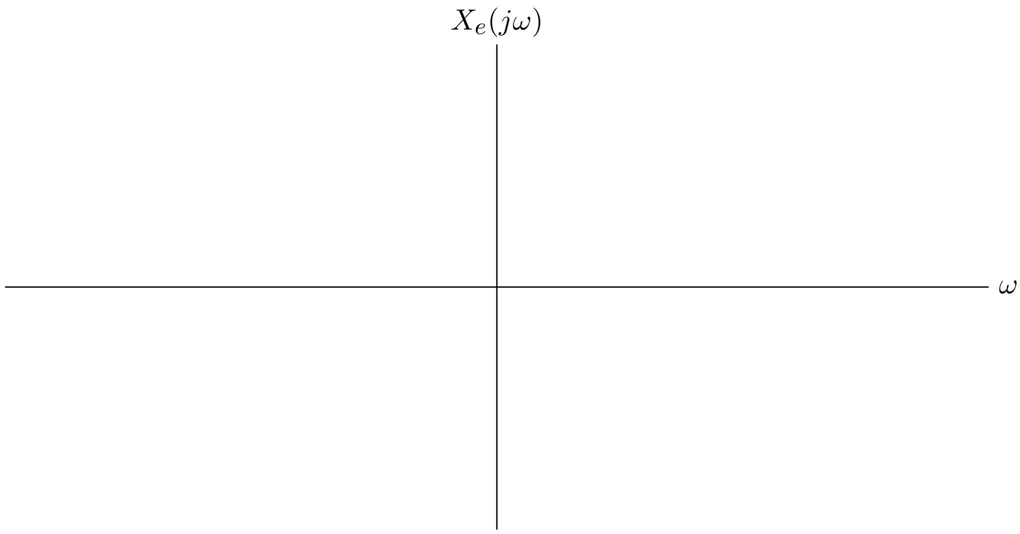
If your answer was **A-F** then specify the corresponding frequency ω_m.

$\omega_m =$

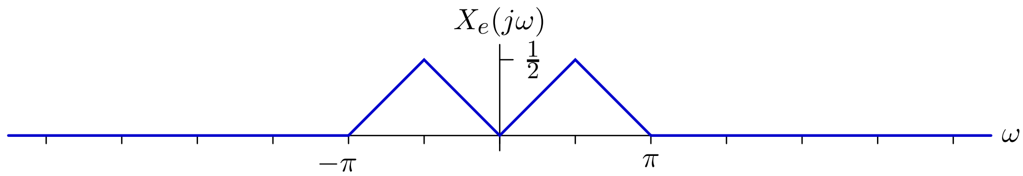
If your answer was **none** then briefly explain why.


A
B
C
D
E
F
t
−12
−8
−4
4
8
12

Part b. Sketch the Fourier transform of $x_e(t)$ for the case when $\omega_m = 5\pi$ and $T = \frac{1}{3}$. Label the important features of your plot.



Part c. Is it possible to adjust T so that $X_e(j\omega)$ has the following form when $\omega_m = 5\pi$?



yes or **no**:

If yes, determine such a value of T (there may be multiple solutions, you need only specify one of them).

$T =$

If no, briefly explain why not.

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Worksheet (intentionally blank)

Worksheet (intentionally blank)