# 6.003 Homework 12

*Due at the beginning of recitation on* **Wednesday, May 5, 2010**.

## Problems

1. **Sampling CT sinusoids**

   Consider 3 CT signals:

   $$x_1(t) = \cos(3000t) \ ,$$
   $$x_2(t) = \cos(4000t) \ , \text{and}$$
   $$x_3(t) = \cos(5000t) \ .$$

   Each of these is sampled as follows

   $$x_1[n] = x_1(nT) \ ,$$
   $$x_2[n] = x_2(nT) \ , \text{ and}$$
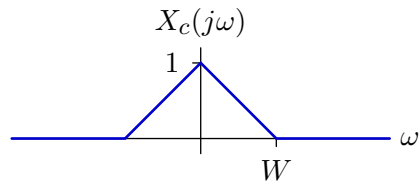   $$x_3[n] = x_3(nT) \ ,$$

   where $T = 0.001$. Which of the resulting DT signals has the highest DT frequency? Which has the lowest DT frequency?

2. **Sampling with alternating impulses**

   A CT signal $x_c(t)$ is converted to a DT signal $x_d[n]$ as follows:

   $$x_d[n] = \begin{cases} x_c(nT) & n \text{ even} \\ -x_c(nT) & n \text{ odd} \end{cases}$$

   **a.** Assume that the Fourier transform of $x_c(t)$ is $X_c(j\omega)$ shown below.

   

   Determine the DT Fourier transform $X_d(e^{j\Omega})$ of $x_d[n]$.

   **b.** Assume that $x_c(t)$ is bandlimited to $-W \le \omega \le W$. Determine the maximum value of $W$ for which the original signal $x_c(t)$ can be reconstructed from the samples $x_d[n]$.

   **c.** Make a diagram of a system to reconstruct $x_c(t)$ from $x_d[n]$.

## 3. Boxcar sampling

A digital camera focuses light from the environment onto an imaging chip that converts the incident image into a discrete representation composed of pixels. Each pixel represents the total light collected from a region of space

$$x_d[n,m] = \int_{mD-\frac{\Delta}{2}}^{mD+\frac{\Delta}{2}} \int_{nD-\frac{\Delta}{2}}^{nD+\frac{\Delta}{2}} x_c(x,y)\,dx\,dy$$

where $\Delta$ is a large fraction of the distance $D$ between pixels. This kind of sampling is often called "boxcar" sampling to distinquish it from the ideal "impulse" sampling that we described in lecture. Assume that boxcar sampling is defined in one dimension as

$$x_d[n] = \int_{nT-\frac{\Delta}{2}}^{nT+\frac{\Delta}{2}} x_c(t)\,dt$$

where $T$ is the intersample "time."

**a.** Let $X_c(j\omega)$ represent the continuous-time Fourier transform of $x_c(t)$. Determine the discrete-time Fourier transform $X_d(e^{j\Omega})$ of $x_d[n]$ in terms of $X_c(j\omega)$, $\Delta$, and $T$.

**b.** Assume that $x_c(t)$ is bandlimited to $-W \le \omega \le W$. Determine the the maximum value of $W$ for which the original signal $x_c(t)$ can be reconstructed from the samples $x_d[n]$. Compare your answer to the answer for an ideal "impulse" sampler.

**c.** Describe the effect of boxcar sampling on the resulting samples $x_d[n]$. How are the samples that result from boxcar sampling different from those that result from impulse sampling?
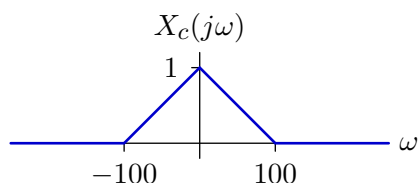
## 4. DT processing of CT signals

Sampling and reconstruction allow us to process CT signals using digital electronics as shown in the following figure.
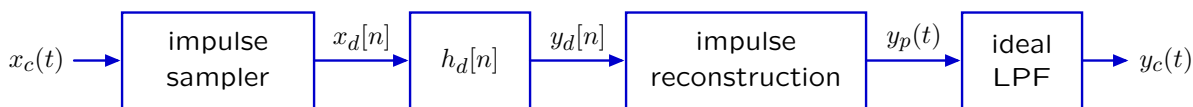


The "impulse sampler" and "impulse reconstuction" use sampling interval $T = \pi/100$. The unit-sample function $h_d[n]$ represents the unit-sample response of an ideal DT low-pass filter with gain of 1 for frequencies in the range $-\frac{\pi}{2} < \Omega < \frac{\pi}{2}$. The "ideal LPF" passes frequencies in the range $-100 < \omega < 100$. It also has a gain of $T$ throughout its pass band.

Assume that the Fourier transform of the input $x_c(t)$ is $X(j\omega)$ shown below.



Determine $Y_c(j\omega)$.