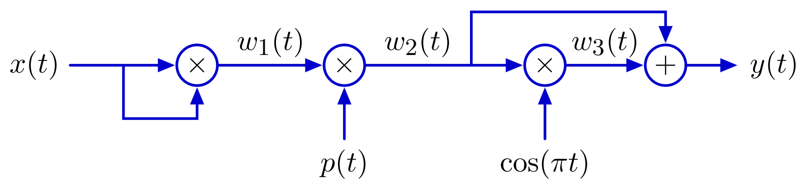
6.003 Homework 13

Please do the following problems by ***Wednesday, May 12, 2010****. You need not submit your answers: they will NOT be graded. Solutions will be posted.*

Problems

1. Transformation

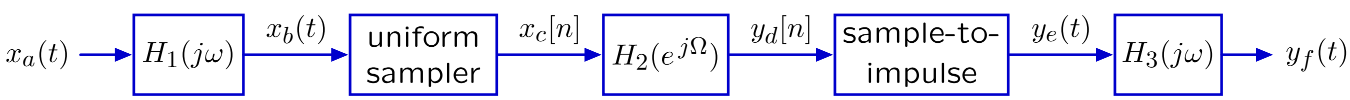
Consider the following transformation from $x(t)$ to $y(t)$:



where $p(t) = \sum_{k=-\infty}^{\infty} \delta(t-k)$. Determine an expression for $y(t)$ when $x(t) = \sin(\pi t/2)/(\pi t)$.

2. DT processing of CT signals

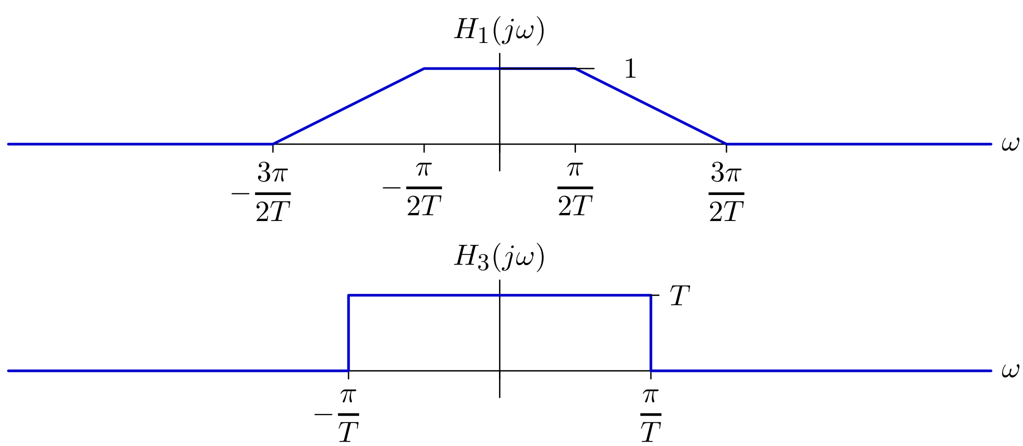
Consider the following system for DT processing of CT signals:



where $x_c[n] = x_b(nT)$ and

$$y_e(t) = \sum_{n=-\infty}^{\infty} y_d[n]\delta(t-nT)\,.$$

The frequency responses $H_1(j\omega)$ and $H_3(j\omega)$ are given below.



Part a. Assume in this part that $H_2(e^{j\Omega}) = 1$ for all frequencies Ω. Determine $y_f(t)$ when

$$x_a(t) = \cos\left(\frac{\pi}{2T}t\right) + \sin\left(\frac{5\pi}{4T}t\right)\,.$$

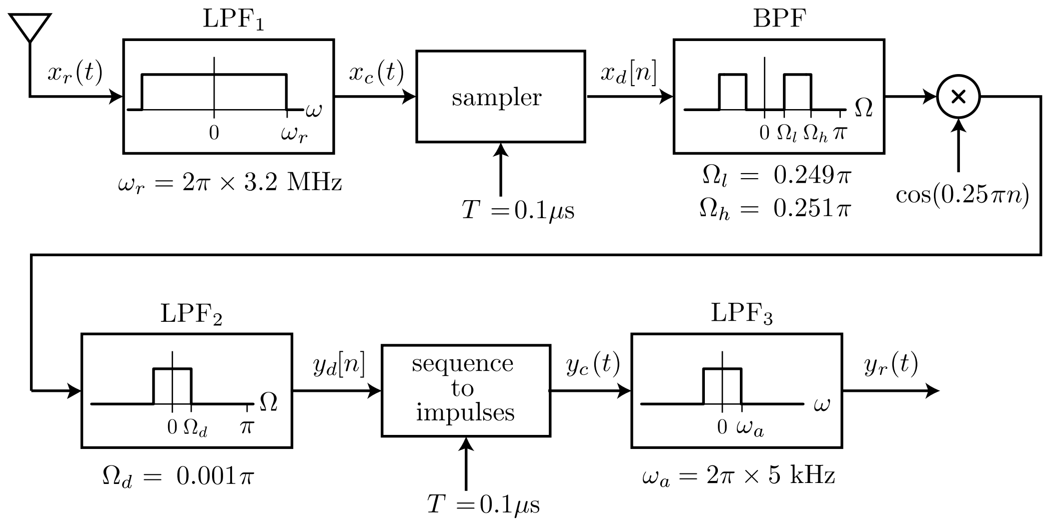
Part b. For this part, assume that

$$H_2(e^{j\Omega}) = \begin{cases} 1\,; & |\Omega| < \Omega_c \\ 0\,; & \Omega_c < |\Omega| < \pi\,. \end{cases}$$

For what values of Ω_c is the overall system from $x_a(t)$ to $y_f(t)$ linear and time-invariant?

3. DT Radio Demodulation

Commercial AM radio stations broadcast radio frequencies within a limited range: $2\pi(f_c - 5\text{ kHz}) < \omega < 2\pi(f_c + 5\text{ kHz})$, where $f_c = \omega_c/(2\pi) = n \times 10$ kHz and n is an integer between 54 and 160. The system shown below is intended to decode one of the AM radio signals using DT signal processing methods. Assume that all of the filters are ideal.



sampler: $x_d[n] = x_c(nT)$

sequence to impulses: $y_c(t) = \sum_{n=-\infty}^{\infty} y_d[n]\delta(t - nT)$

Part a. Determine the center frequency f_c for the AM station that this receiver will detect.

Part b. Which of the following statement(s) is/are correct?

b1. Increasing the cutoff frequency ω_r of LPF$_1$ by a factor of 1.5 will cause aliasing.

b2. Decreasing the cutoff frequency ω_r of LPF$_1$ by a factor of 2 will have no effect on the output $y_r(t)$.

b3. Halving the sampling interval T would have no effect on the output $y_r(t)$.

b4. Doubling the sampling interval T would have no effect on the output $y_r(t)$.

Part c. Briefly describe the effect of removing the band-pass filter BPF (i.e., set $\Omega_l = 0$ and $\Omega_h = \pi$) on the output signal $y_r(t)$.

Part d. Which of the following statement(s) is/are correct?

d1. Increasing the cutoff frequency Ω_d of LPF$_2$ will change $y_r(t)$ by adding signals from unwanted radio stations.

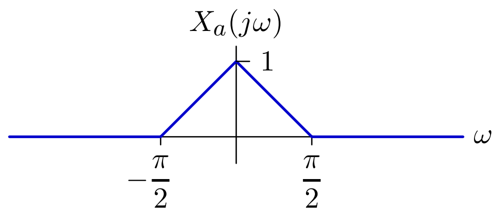
d2. Increasing the cutoff frequency Ω_d of LPF$_2$ will change $y_r(t)$ because aliasing will occur.

d3. Doubling the cutoff frequency Ω_d of LPF$_2$ will have no effect on $y_r(t)$.

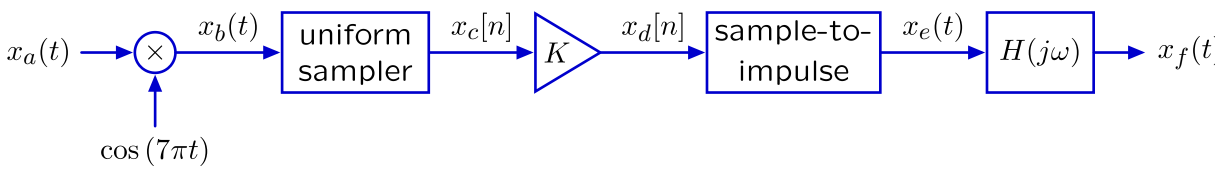
d4. Halving the cutoff frequency Ω_d of LPF$_2$ will have no effect on $y_r(t)$.

4. Multiplied Sampling

The Fourier transform of a signal $x_a(t)$ is given below.



This signal passes through the following system



where $x_c[n] = x_b(nT)$ and

$$x_e(t) = \sum_{n=-\infty}^{\infty} x_d[n]\delta(t - nT)$$

and

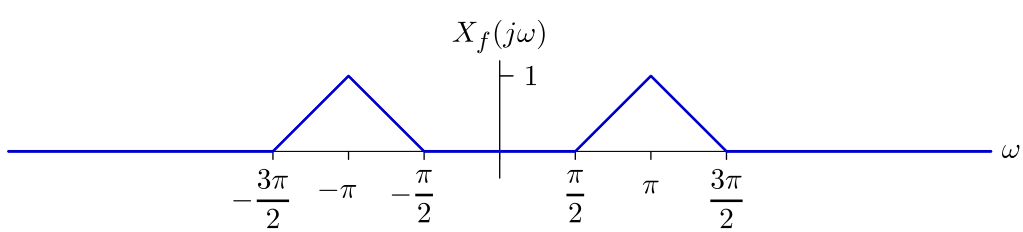
$$H(j\omega) = \begin{cases} T & \text{if } |\omega| < \frac{\pi}{T} \\ 0 & \text{otherwise}. \end{cases}$$

a. Sketch the Fourier transform of $x_f(t)$ for the case when $K = 1$ and $T = 1$. Label the important features of your plot.

b. Is it possible to adjust T and K so that $x_f(t) = x_a(t)$?

If yes, specify a value T and the corresponding value of K (there may be multiple solutions, you need only specify one of them). If no, briefly explain why not.

c. Is it possible to adjust T and K so that the Fourier transform of $x_f(t)$ is equal to the following, and is zero outside the indicated range?



If yes, specify all possible pairs of T and K that work in the table below. If there are more rows in the table than are needed, leave the remaining entries blank.

T	K

If no, briefly explain why not.