6.003 Homework 3

Due at the beginning of recitation on ***Wednesday, February 24, 2010****.*

Problems

1. Laplace Transforms

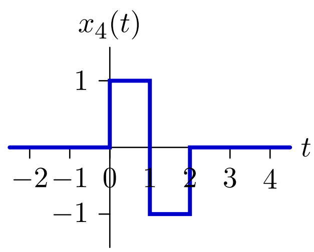
Determine the Laplace transforms (including the regions of convergence) of each of the following signals:

a. $x_1(t) = e^{-2(t-3)}u(t-3)$

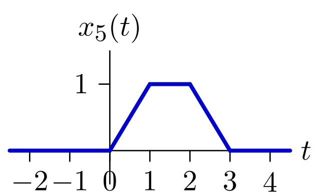
b. $x_2(t) = (1-(1-t)e^{-3t})u(t)$

c. $x_3(t) = |t|e^{-|t|}$

d.



e.



2. Inverse Laplace transforms

Determine and sketch all possible signals with Laplace transforms of the following forms. For each signal, indicate the associated region of convergence.

a. $X_1(s) = \dfrac{s+2}{(s+1)^2}$

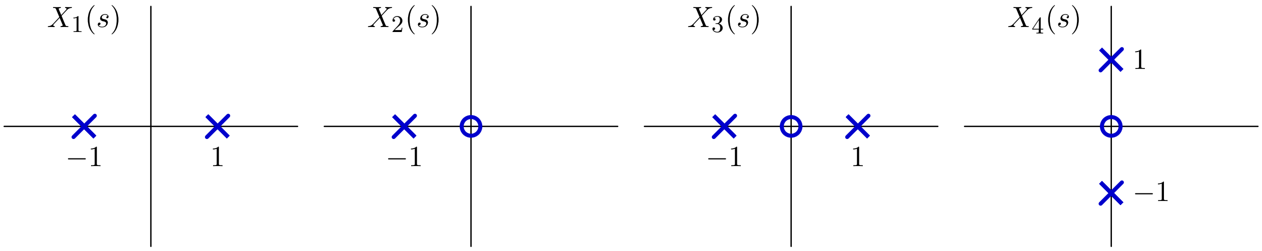
b. $X_2(s) = \dfrac{1}{s^2(s-1)}$

c. $X_3(s) = \dfrac{s+1}{s^2+2s+2}$

d. $X_4(s) = \left(\dfrac{1-e^{-s}}{s}\right)^2$

3. Symmetry

Determine which of the following pole-zero[1] diagrams could represent Laplace transforms of even functions of time. Determine expressions for the time functions of those that can represent even functions of time. For those that cannot, explain why they cannot.



How can you determine if a signal is even or not by looking at its Laplace transform and region of convergence?

4. Differential equation

A system is represented by the following differential equation

$$\dot{y}(t) + y(t) = \dot{x}(t) - x(t)$$

where $x(t)$ represents the input signal and $y(t)$ represents the output signal.

a. Determine the step response of the system with and without using Laplace transforms.

b. Determine the output $y(t)$ when $x(t) = e^{-t}u(t)$ with and without using Laplace transforms.

5. Intial and final values (from "Circuits, Signals, and Systems" by Siebert)

a. Use the initial and final value theorems (where applicable) to find $x(0)$ and $x(\infty)$ for the signals with the following Laplace transforms:

1. $\dfrac{1-e^{-sT}}{s}$
2. $\dfrac{1}{s}e^{-sT}$
3. $\dfrac{1}{s(s+1)^2}$
4. $\dfrac{1}{s^2(s+1)}$
5. $\dfrac{1}{s^2+1}$
6. $\dfrac{(s+1)^2-1}{[(s+1)^2+1]^2}$

b. Find the inverse Laplace transforms for each of the previous parts and show that the time waveforms and initial and final values agree.

6. Impulse response

Sketch a block diagram for a CT system with impulse response

$$h(t) = \left(1 - te^{-t}\right)e^{-2t}u(t)\,.$$

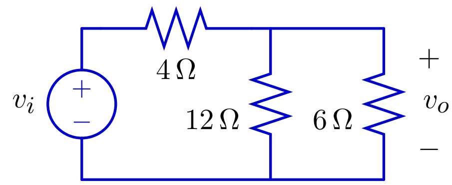
The block diagram should contain only adders, gains, and integrators.

[1] A zero is a root of the numerator of the system function.

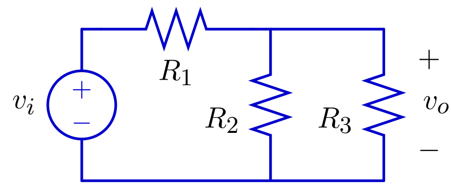
Engineering Design Problems

7. Impedance Method

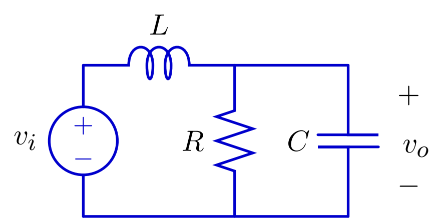
a. Determine the output voltage of the following circuit, using series or parallel resistor combinations and/or voltage or current dividers.



b. Generalize the result from part a for arbitrary resistor values, and determine an expression for the resulting ratio $\frac{v_o}{v_i}$.



c. Consider the following circuit.



Determine a differential equation that relates v_i to v_C as follows. First, determine relations among the element currents (i_R, i_L, and i_C) and element voltages (v_R, v_L, and v_C) using KVL and KCL. Second, relate each element voltage to the corresponding element current using the constitutive relation for the element: i.e., $v_R = Ri_R$, $i_C = C\frac{dv_C}{dt}$, and $v_L = L\frac{di_L}{dt}$. Finally, solve your equations to find a single equation with terms that involve v_i, v_o, and derivatives of v_i and v_o.

d. Determine the system function $H(s) = \frac{V_o(s)}{V_i(s)}$, based on the Laplace transform of your answer to the previous part.

e. Substitute $R_1 \to sL$, $R_2 \to R$, and $R_3 \to \frac{1}{sC}$ into your result from part b. Compare this new expression to your result from part d.

f. The impedance of an electrical element is a function of s that can be analyzed using rules that are quite similar to those for resistances. Explain the basis of this method.