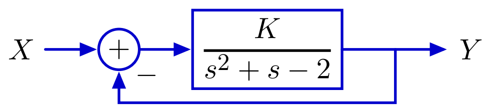
6.003 Homework 7

Due at the beginning of recitation on ***Wednesday, March 31, 2010****.*

Problems

1. CT stability

Consider the following feedback system in which the box represents a causal LTI CT system that is represented by its system function.

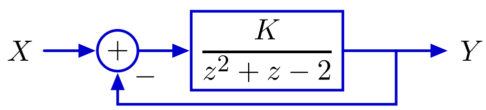


a. Determine the range of K for which this feedback system is stable.

b. Determine the range of K for which this feedback system has real-valued poles.

2. DT stability

Consider the following feedback system in which the box represents a causal LTI DT system that is represented by its system function.



a. Determine the range of K for which this feedback system is stable.

b. Determine the range of K for which this feedback system has real-valued poles.

3. BIBO stability

A signal is said to be bounded if its absolute value is less than some constant at all times.

A system is said to be stable in the bounded-input/bounded-output sense if all bounded inputs to the system generate bounded output signals.

a. Let $h[n]$ represent the unit-sample response of a DT LTI system. Determine the bounded input signal $x[n]$ ($|x[n]| < B$) that maximizes the output $y[n]$ at $n = 0$. [Hint: look at the convolution sum.]

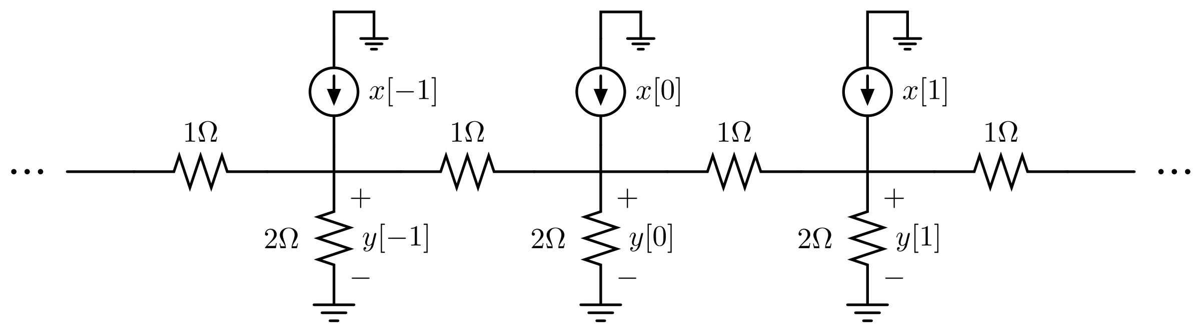
b. Determine a rule based on $h[n]$ to determine if a system is BIBO stable.

c. Let $h(t)$ represent the unit-impulse response of a CT LTI system. Determine the bounded input signal $x(t)$ ($|x(t)| < B$) that maximizes the output $y(t)$ at $t = 0$. [Hint: look at the convolution integral.]

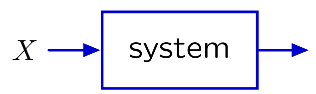
d. Determine a rule based on $h(t)$ to determine if a system is BIBO stable.

4. Ladder network

An infinite network of resistors is excited by an infinite network of current sources as shown below.



We can consider the transformation from x to y as a DT system.



a. Show that this system is linear and "time"-invariant.

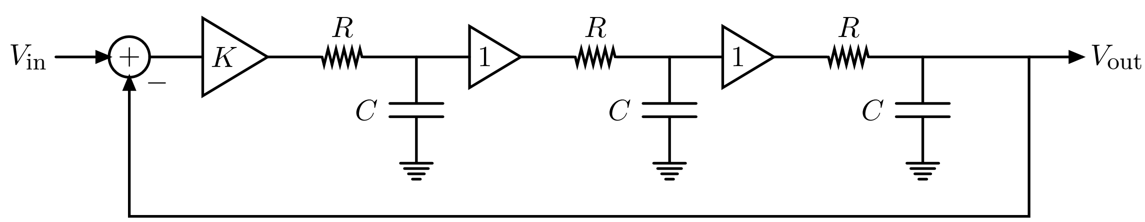
b. Determine the unit-sample response $h[n]$.

c. Determine the system function $H(z)$ and region of convergence.

d. Determine the system's pole(s) and zero(s).

5. Desired oscillations

The following feedback circuit was the basis of Hewlett and Packard's founding patent.



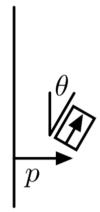
a. With $R = 1\,\text{k}\Omega$ and $C = 1\mu\text{F}$, sketch the pole locations as the gain K varies from 0 to ∞, showing the scale for the real and imaginary axes. Find the K for which the system is barely stable and label your sketch with that information. What is the system's oscillation period for this K?

b. How do your results change if R is increased to $10\,\text{k}\Omega$?

Engineering Design Problem

6. Robotic steering

Design a steering controller for a car that is moving forward with constant velocity V.



You can control the steering-wheel angle $w(t)$, which causes the angle $\theta(t)$ of the car to change according to

$$\frac{d\theta(t)}{dt} = \frac{V}{d}w(t)$$

where d is a constant with dimensions of length. As the car moves, the transverse position $p(t)$ of the car changes according to

$$\frac{dp(t)}{dt} = V\sin\big(\theta(t)\big) \approx V\theta(t)\,.$$

Consider three control schemes:

a. $w(t) = Ke(t)$

b. $w(t) = K_v\dot{e}(t)$

c. $w(t) = Ke(t) + K_v\dot{e}(t)$

where $e(t)$ represents the difference between the desired transverse position $x(t) = 0$ and the current transverse position $p(t)$. Describe the behaviors that result for each control scheme when the car starts with a non-zero angle ($\theta(0) = \theta_0$ and $p(0) = 0$). Determine the most acceptable value(s) of K and/or K_v for each control scheme or explain why none are acceptable.