6.003 Homework 8

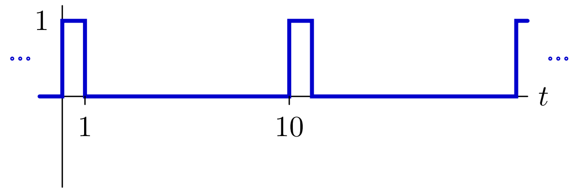
*Please do the following problems by **Wednesday, April 7, 2010**. You need not submit your answers: they will NOT be graded. Solutions will be posted.*

Problems

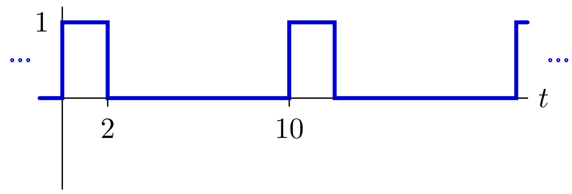
1. Fourier Series

Determine the Fourier series coefficients for each of the following periodic CT signals.

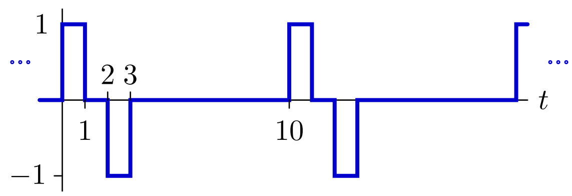
$x_1(t) = x_1(t+10)$



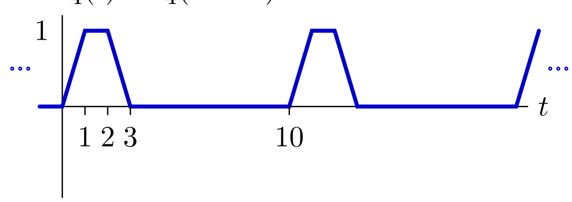
$x_2(t) = x_2(t+10)$



$x_3(t) = x_3(t+10)$



$x_4(t) = x_4(t+10)$



2. Inverse Fourier series

Determine the CT signals with the following Fourier series coefficients. Assume that the signals are periodic in $T = 4$.

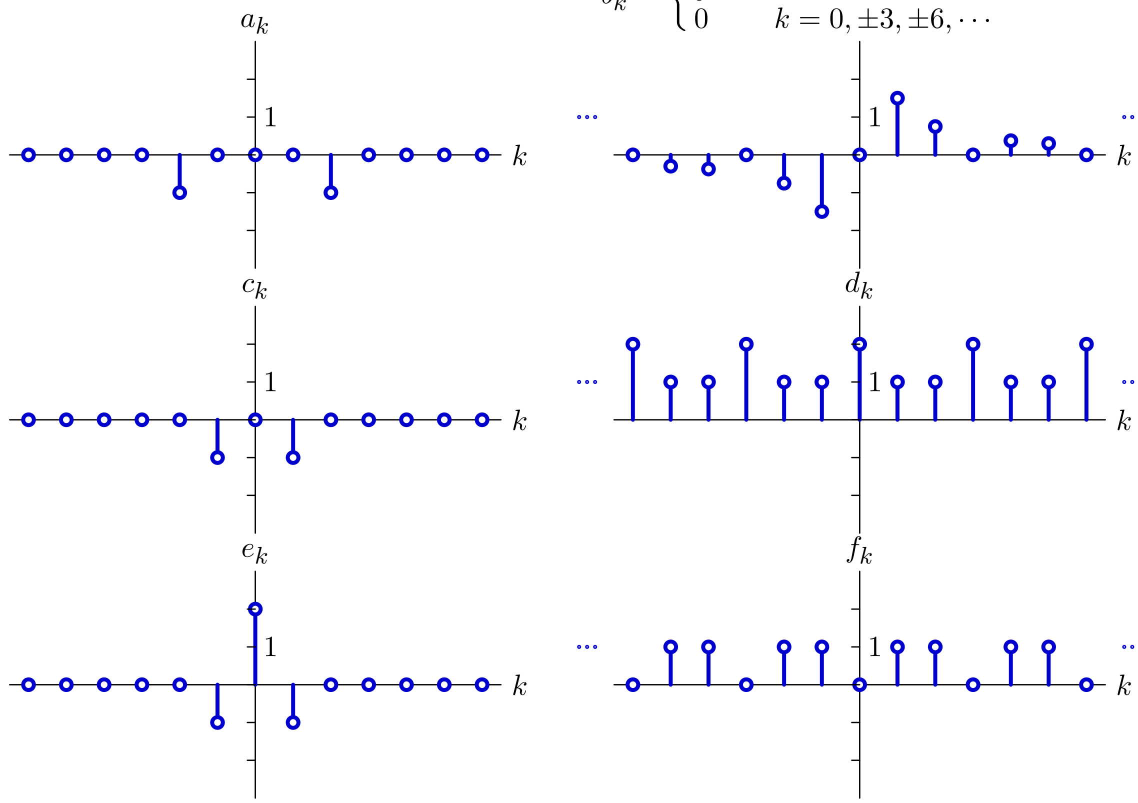
a. $a_k = \begin{cases} jk; & |k| < 3 \\ 0 & \text{otherwise} \end{cases}$

b. $b_k = \begin{cases} 1; & k \text{ odd} \\ 0; & k \text{ even} \end{cases}$

3. Matching

Consider the following sets of Fourier series coefficients.

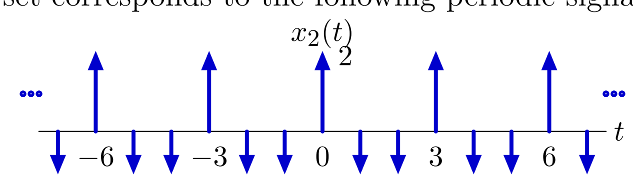
$$b_k = \begin{cases} \frac{3}{j2k} & k = \pm1, \pm2, \pm4, \pm5, \pm7, \ldots \\ 0 & k = 0, \pm3, \pm6, \cdots \end{cases}$$



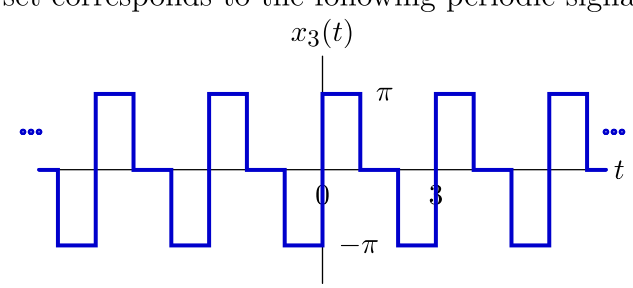
a. Which (if any) set corresponds to the following periodic signal?

$$x_1(t) = 2 - 2\cos\left(\frac{2\pi}{3}t\right)$$

b. Which (if any) set corresponds to the following periodic signal with period $T = 3$?



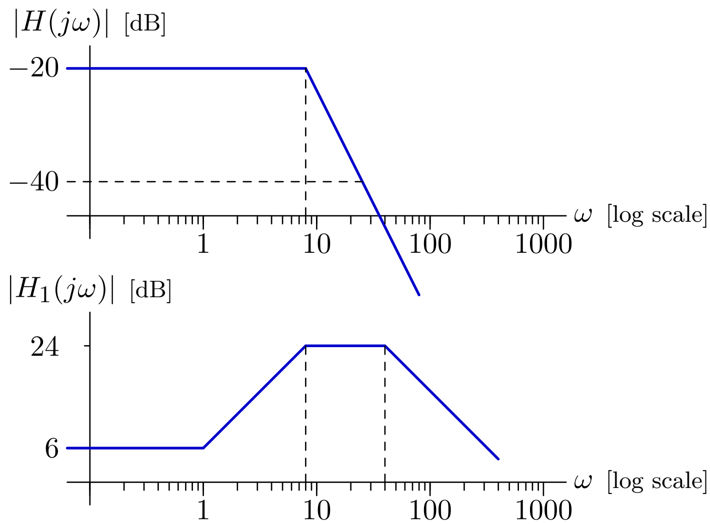
c. Which (if any) set corresponds to the following periodic signal with period $T = 3$?



Review Problems

4. Bode Plots

Our goal is to design a stable CT LTI system H by cascading two causal CT LTI systems: H_1 and H_2. The magnitudes of $H(j\omega)$ and $H_1(j\omega)$ are specified by the following straight-line approximations. We are free to choose other aspects of the systems.

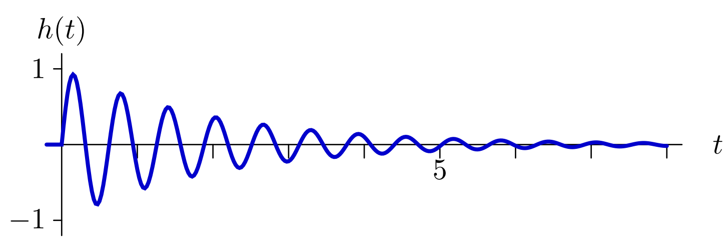


a. Determine all system functions $H_1(s)$ that are consistent with these design specifications, and plot the straight-line approximation to the phase angle of each (as a function of ω).

b. Determine all system functions $H_2(s)$ that are consistent with these design specifications, and plot the straight-line approximation to the phase angle of each (as a function of ω).

5. Relation between time and frequency responses

The impulse response of an LTI system is shown below.



If the input to the system is an eternal cosine, i.e., $x(t) = \cos(\omega t)$, then the output will have the form

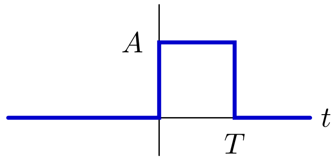
$$y(t) = C\cos(\omega t + \phi)$$

a. Determine ω_m, the frequency ω for which the constant C is greatest. What is the value of C when $\omega = \omega_m$?

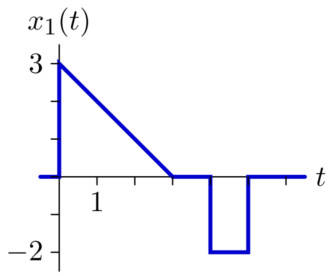
b. Determine ω_p, the frequency ω for which the phase angle ϕ is $-\frac{\pi}{4}$. What is the value of C when $\omega = \omega_p$?

6. CT responses

We are given that the impulse response of a CT LTI system is of the form



where A and T are unknown. When the system is subjected to the input



the output $y_1(t)$ is zero at $t = 5$. When the input is

$$x_2(t) = \sin\left(\frac{\pi t}{3}\right) u(t),$$

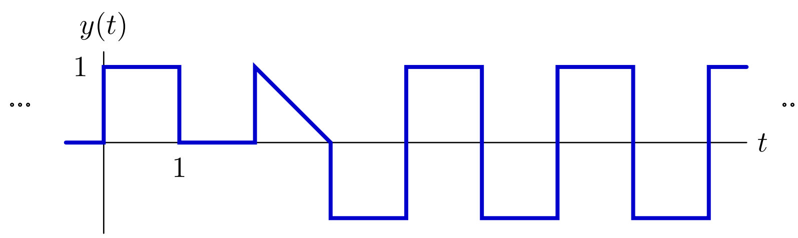
the output $y_2(t)$ is equal to 9 at $t = 9$. Determine A and T. Also determine $y_2(t)$ for all t.

7. Impulse response

The response of a causal LTI system to the input $x(t)$ which is given by

$$x(t) = \sum_{k=0}^{\infty} \delta(t-k)$$

is $y(t)$ which is given by



where $y(t)$ is 0 for $t < 0$ and $y(t) = y(t-2)$ for $t > 6$. Sketch the impulse response $h(t)$ of the system.