6.003: Signals and Systems

Signals and Systems

February 2, 2010

6.003: Signals and Systems

Today's handouts: Single package containing

- Slides for Lecture 1
- Subject Information & Calendar

Lecturer: Denny Freeman (**freeman@mit.edu**)

Instructors: Peter Hagelstein (**phagelstein@aol.com**)
Rahul Sarpeshkar (**rahuls@mit.edu**)

TAs: Sefa Demirtas (**sefa@mit.edu**)
Ulric Ferner (**uferner@mit.edu**)
Alison Laferriere (**alaferri@mit.edu**)

Website: **mit.edu/6.003**

Text: *Signals and Systems* – Oppenheim and Willsky

6.003: Homework

Doing the homework is essential for understanding the content.

- where subject matter is/isn't learned
- equivalent to "practice" in sports or music

Weekly Homework Assignments

- Conventional Homework Problems plus
- **Engineering Design Problems** (Python/Matlab)

Open Office Hours !

- Stata Basement (32-044)
- Mondays and Tuesdays, afternoons and early evenings

Collaboration Policy

- **Discussion** of concepts in homework is encouraged
- **Sharing** of homework or code is not permitted and will be reported to the COD

Firm Deadlines

- Homework must be submitted in recitation on due date
- Each student can submit one late homework assignment without penalty.
- Grades on other late assignments will be multiplied by 0.5 (unless excused by an Instructor, Dean, or Medical Official).

6.003 At-A-Glance

	Tuesday		Wednesday	Thursday	Friday
Feb 2	L1: Signals and Systems		R1: Continuous & Discrete Systems	L2: Discrete-Time Systems	R2: Difference Equations
Feb 9	L3: Feedback, Cycles, and Modes	HW1 due	R3: Feedback, Cycles, and Modes	L4: CT Operator Representations	R4: CT Systems
Feb 16	**Presidents Day: Monday Schedule**	HW2 due	R5: CT Operator Representations	L5: Second-Order Systems	R6: Second-Order Systems
Feb 23	L6: Laplace and Z Transforms	HW3 due	R7: Laplace and Z Transforms	L7: Transform Properties	R8: Transform Properties
Mar 2	L8: Convolution; Impulse Response	EX4	**Exam 1** no recitation	L9: Frequency Response	R9: Convolution and Freq. Resp.
Mar 9	L10: Bode Diagrams	HW5 due	R10: Bode Diagrams	L11: DT Feedback and Control	R11: Feedback and Control
Mar 16	L12: CT Feedback and Control	HW6 due	R12: CT Feedback and Control	L13: CT Feedback and Control	R13: CT Feedback and Control
Mar 23	**Spring Week**				
Mar 30	L14: CT Fourier Series	HW7	R14: CT Fourier Series	L15: CT Fourier Series	R15: CT Fourier Series
Apr 6	L16: CT Fourier Transform	EX8 due	**Exam 2** no recitation	L17: CT Fourier Transform	R16: CT Fourier Transform
Apr 13	L18: DT Fourier Transform	HW9 due	R17: DT Fourier Transform	L19: DT Fourier Transform	R18: DT Fourier Transform
Apr 20	**Patriots Day Vacation**	HW10	R19: Fourier Transforms	L20: Fourier Relations	R20: Fourier Relations
Apr 27	L21: Sampling	EX11 due	**Exam 3** no recitation	L22: Sampling	R21: Sampling
May 4	L23: Modulation	HW12 due	R22: Modulation	L24: Modulation	R23: Modulation
May 11	L25: Applications of 6.003	EX13	R24: Review	Breakfast with Staff	Study Period
May 18	**Final Examination Period**				

6.003: Signals and Systems

Weekly meetings with **class representatives**

- help staff understand student perspective
- learn about teaching

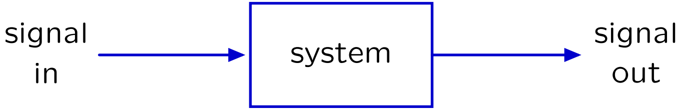
One representative from each section (4 total)

Tentatively meet on Thursday afternoon

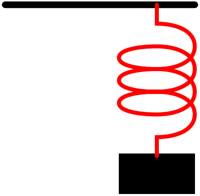
Interested? ... Send email to **freeman@mit.edu**

The Signals and Systems Abstraction

Describe a **system** (physical, mathematical, or computational) by the way it transforms an **input signal** into an **output signal**.



Example: Mass and Spring



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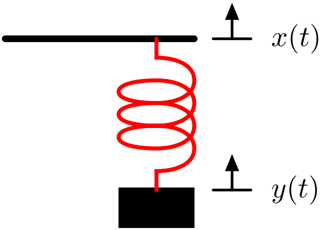

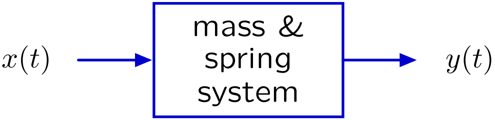

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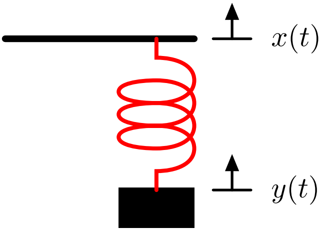
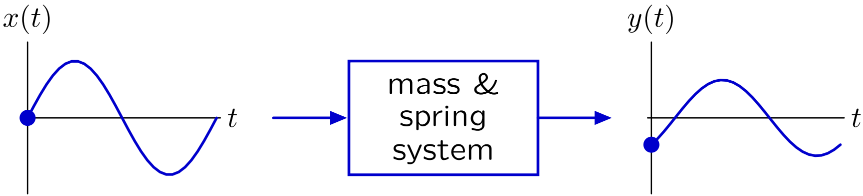
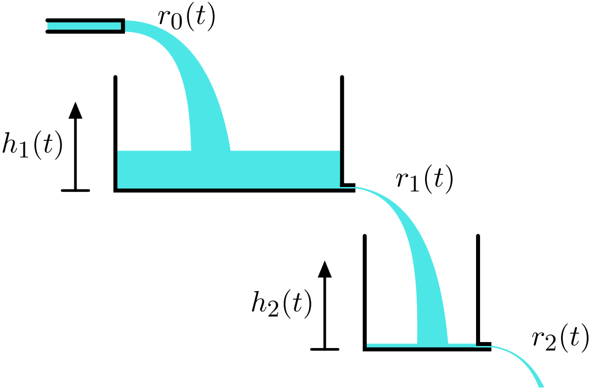
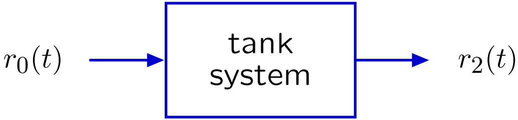




Example: Tanks





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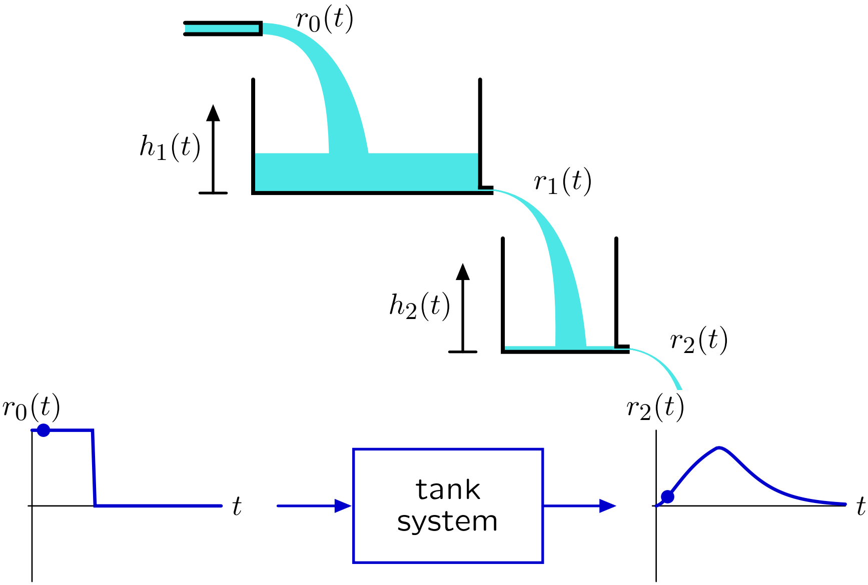
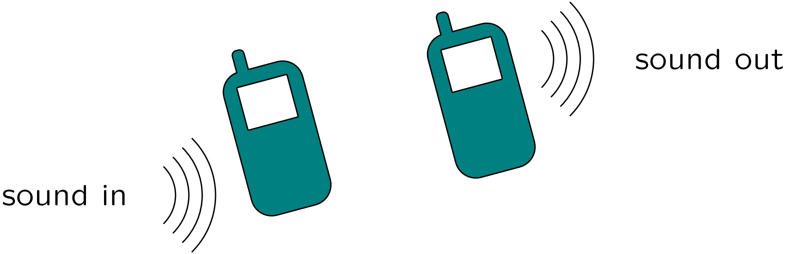
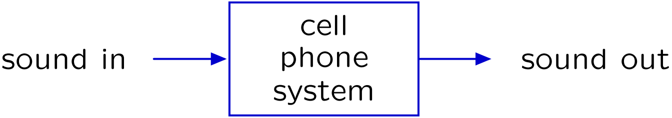


Example: Cell Phone System





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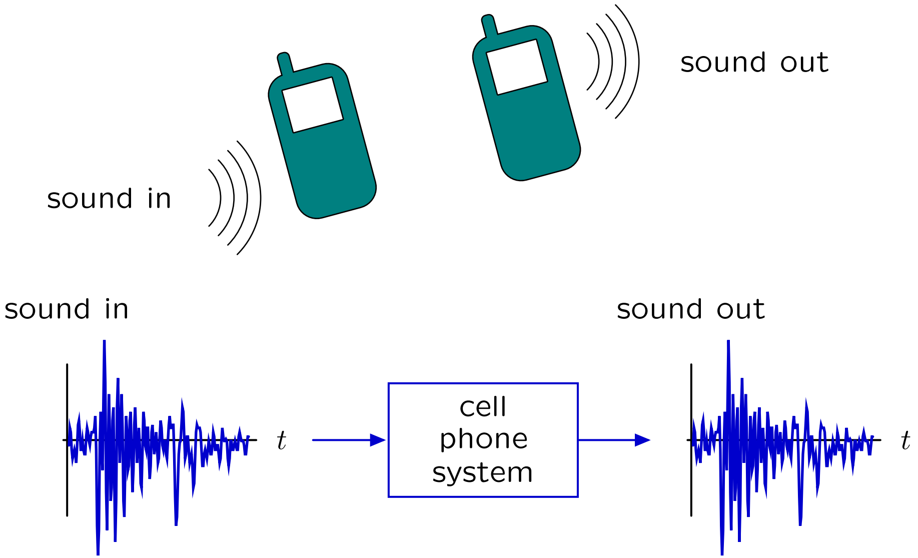
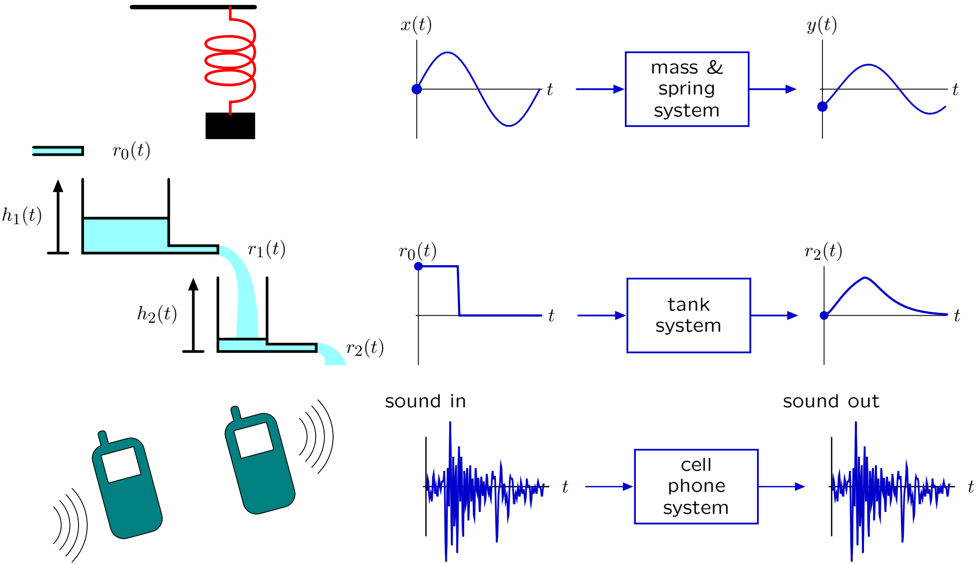


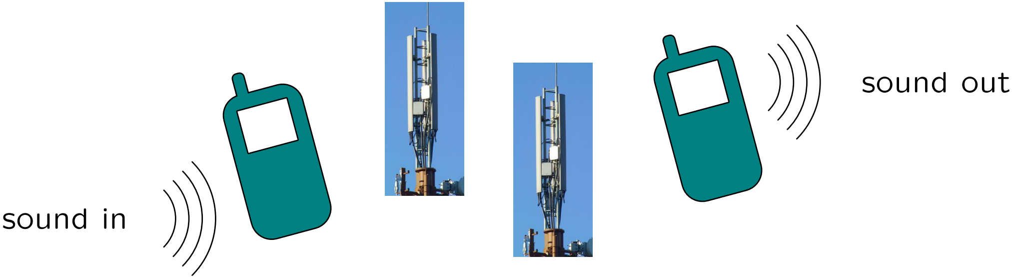
Signals and Systems: Widely Applicable

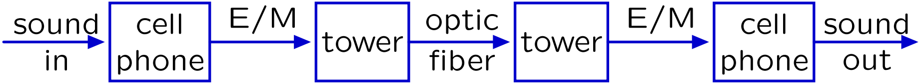
The Signals and Systems approach has broad application: electrical, mechanical, optical, acoustic, biological, financial, ...



Signals and Systems: Modular

The representation does not depend upon the physical substrate.



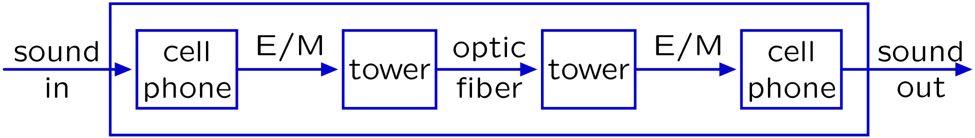


focuses on the flow of **information**, abstracts away everything else

Signals and Systems: Hierarchical

Representations of component systems are easily combined.

Example: cascade of component systems



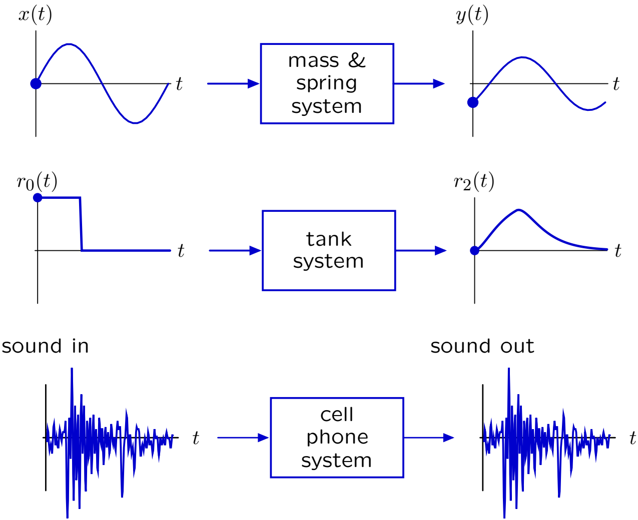
Composite system

Component and composite systems have the same form, and are analyzed with same methods.

Signals and Systems

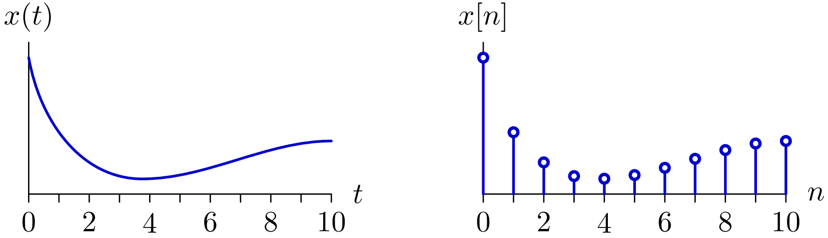
Signals are mathematical functions.

- independent variable = time
- dependent variable = voltage, flow rate, sound pressure



Signals and Systems

continuous "time" (CT) and discrete "time" (DT)



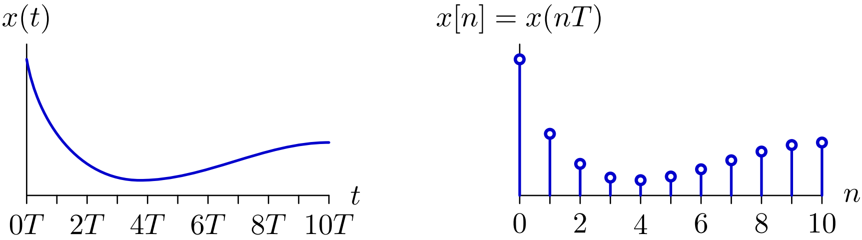
Many physical systems operate in continuous time.

- mass and spring
- leaky tank

Digital computations are done in discrete time.

- state machines: given the current input and current state, what is the next output and next state.

Sampling: converting CT signals to DT



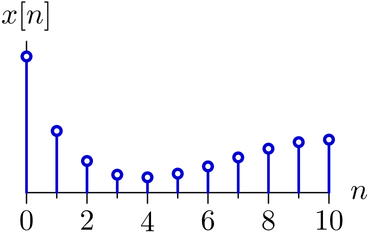
$T =$ sampling interval

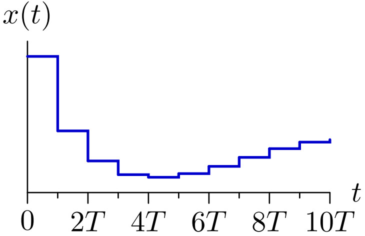
Important for computational manipulation of physical data.

- digital representations of audio signals (e.g., MP3)
- digital representations of pictures (e.g., JPEG)

Reconstruction: converting DT signals to CT

zero-order hold



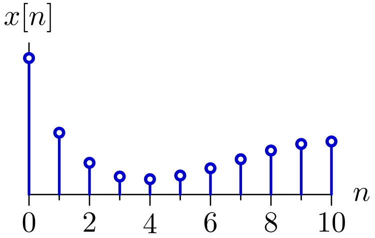


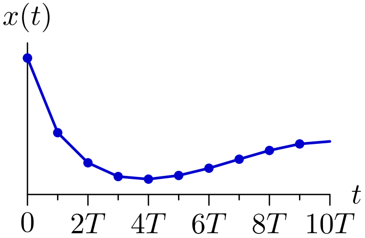
$T =$ sampling interval

commonly used in audio output devices such as CD players

Reconstruction: converting DT signals to CT

piecewise linear



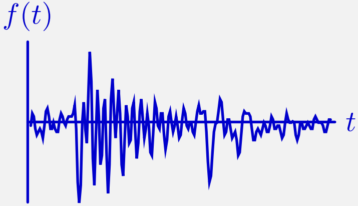


$T =$ sampling interval

commonly used in rendering images

Check Yourself

Computer generated speech (by Robert Donovan)



Listen to the following four manipulated signals:

$$f_1(t),\ f_2(t),\ f_3(t),\ f_4(t).$$

How many of the following relations are true?

- $f_1(t) = f(2t)$
- $f_2(t) = -f(t)$
- $f_3(t) = f(2t)$
- $f_4(t) = 2f(t)$

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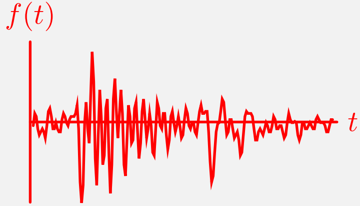

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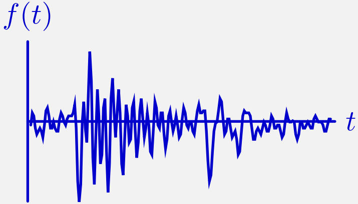

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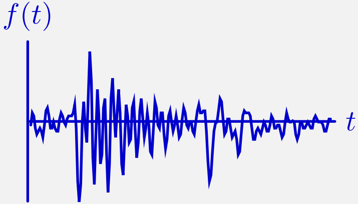

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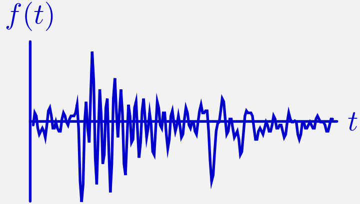

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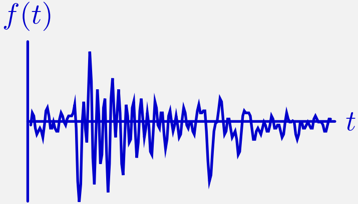

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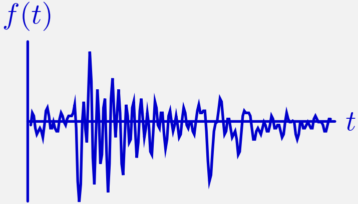

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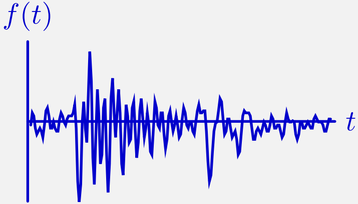

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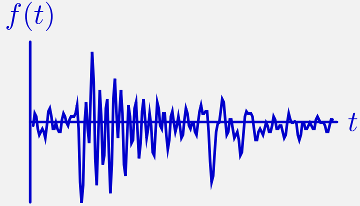


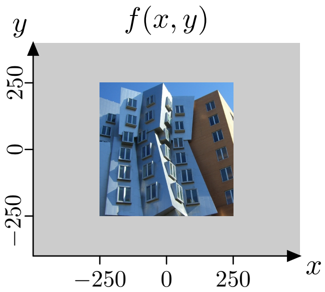
Listen to the following four manipulated signals:

$$f_1(t),\ f_2(t),\ f_3(t),\ f_4(t).$$

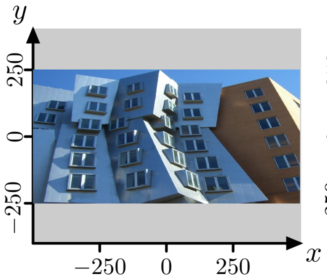
How many of the following relations are true? 2

- $f_1(t) = f(2t)$ ✓
- $f_2(t) = -f(t)$ ✗
- $f_3(t) = f(2t)$ ✗
- $f_4(t) = 2f(t)$ ✓

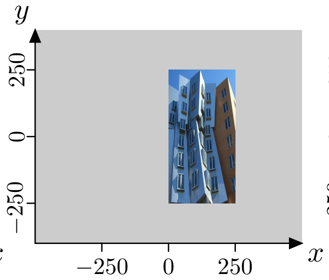
Check Yourself



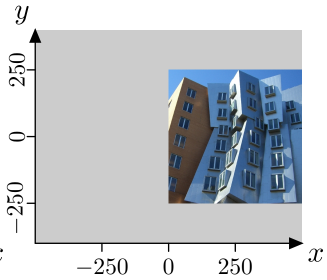
How many images match the expressions beneath them?



$f_1(x,y)=f(2x,y)$?

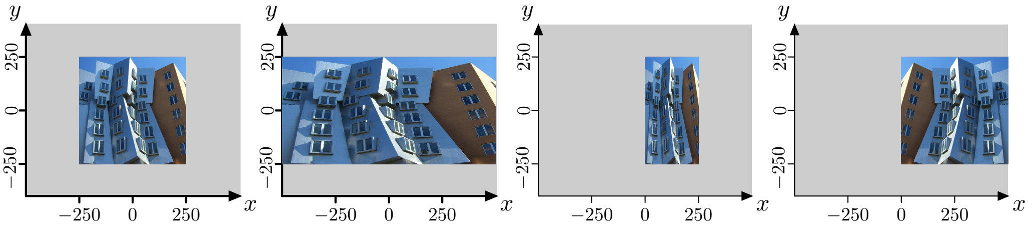


$f_2(x,y)=f(2x-250,y)$?



$f_3(x,y)=f(-x-250,y)$?

Check Yourself



$f(x,y)$ — $f_1(x,y) = f(2x,y)$? — $f_2(x,y) = f(2x-250,y)$? — $f_3(x,y) = f(-x-250,y)$?

$x = 0 \quad \rightarrow f_1(0,y) = f(0,y)$ ✓

$x = 250 \quad \rightarrow f_1(250,y) = f(500,y)$ ✗

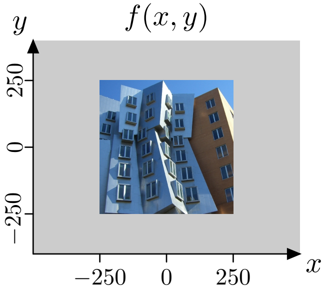
$x = 0 \quad \rightarrow f_2(0,y) = f(-250,y)$ ✓

$x = 250 \quad \rightarrow f_2(250,y) = f(250,y)$ ✓

$x = 0 \quad \rightarrow f_3(0,y) = f(-250,y)$ ✗

$x = 250 \quad \rightarrow f_3(250,y) = f(-500,y)$ ✗

Check Yourself



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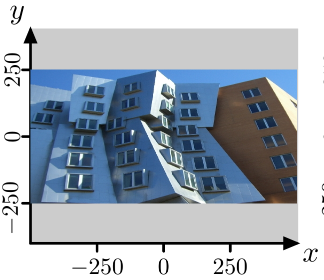
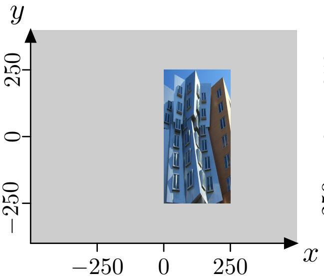
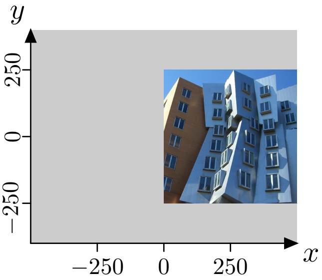


~~$f_1(x, y) = f(2x, y)$?~~



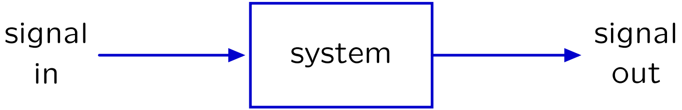
$f_2(x, y) = f(2x - 250, y)$?



~~$f_3(x, y) = f(-x - 250, y)$?~~

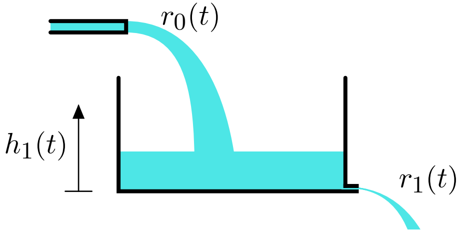
The Signals and Systems Abstraction

Describe a **system** (physical, mathematical, or computational) by the way it transforms an **input signal** into an **output signal**.



Example System: Leaky Tank

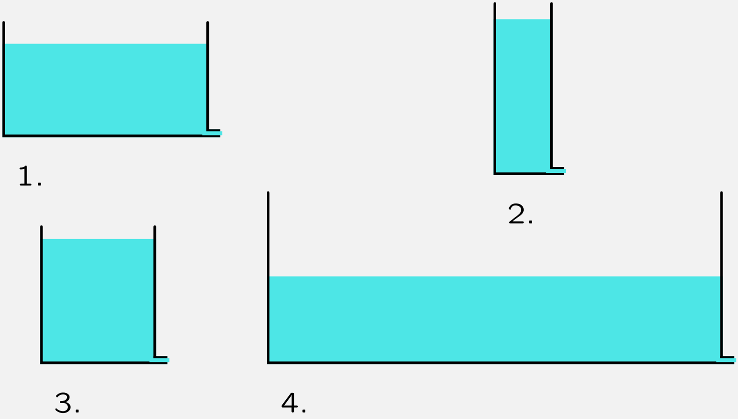
Formulate a mathematical description of this system.



What determines the leak rate?

Check Yourself

The holes in each of the following tanks have equal size. Which tank has the largest leak rate $r_1(t)$?



Check Yourself

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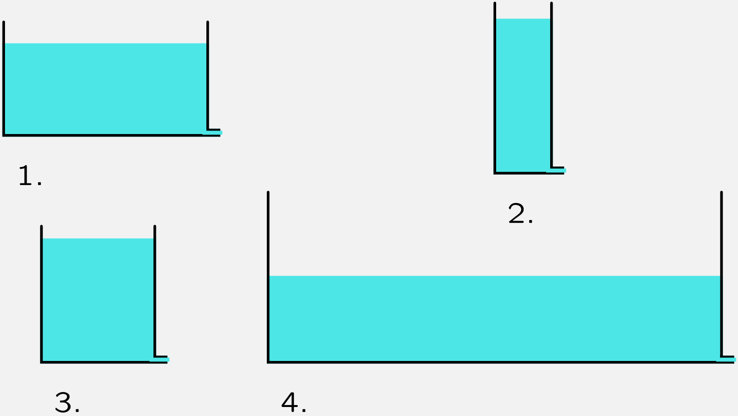

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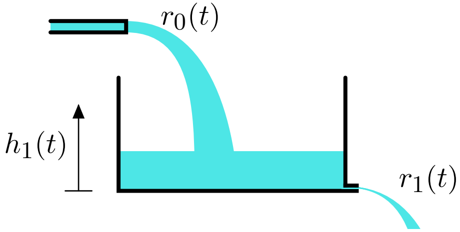
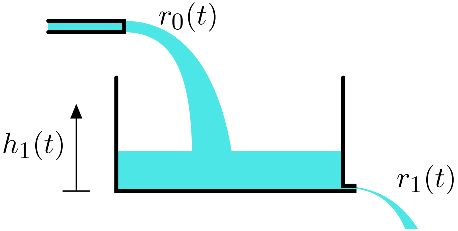


Assume linear leaking: $r_1(t) \propto h_1(t)$

What determines the height $h_1(t)$?

Example System: Leaky Tank

Formulate a mathematical description of this system.



Assume linear leaking: $r_1(t) \propto h_1(t)$

Assume water is conserved: $\dfrac{dh_1(t)}{dt} \propto r_0(t) - r_1(t)$

Solve: $\dfrac{dr_1(t)}{dt} \propto r_0(t) - r_1(t)$

Check Yourself

What are the dimensions of constant of proportionality C?

$$\frac{dr_1(t)}{dt} = C\Big(r_0(t) - r_1(t)\Big)$$

Check Yourself

What are the dimensions of constant of proportionality C?
inverse time (to match dimensions of dt)

$$\frac{dr_1(t)}{dt} = C\Big(r_0(t) - r_1(t)\Big)$$

Analysis of the Leaky Tank

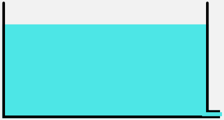
Call the constant of proportionality $1/\tau$.

Then τ is called the **time constant** of the system.

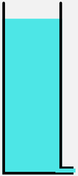
$$\frac{dr_1(t)}{dt} = \frac{r_0(t)}{\tau} - \frac{r_1(t)}{\tau}$$

Check Yourself

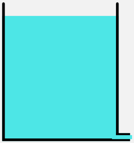
Which of the following tanks has the largest time constant τ?



1.



2.

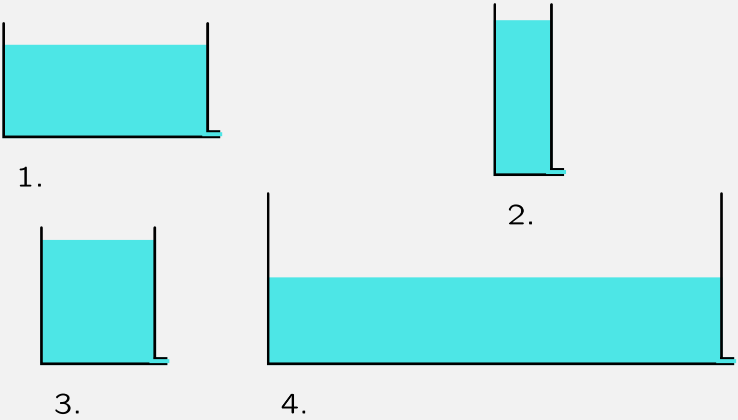


3.

4.

Check Yourself

Which of the following tanks has the largest time constant τ? 4



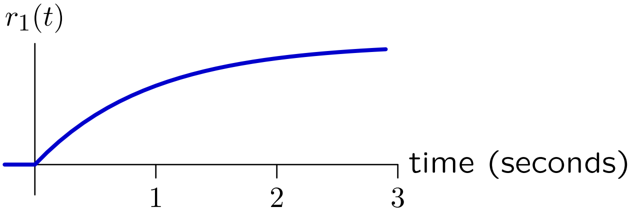
Analysis of the Leaky Tank

Call the constant of proportionality $1/\tau$.

Then τ is called the **time constant** of the system.

$$\frac{dr_1(t)}{dt} = \frac{r_0(t)}{\tau} - \frac{r_1(t)}{\tau}$$

Assume that the tank is initially empty, and then water enters at a constant rate $r_0(t) = 1$. Determine the output rate $r_1(t)$.



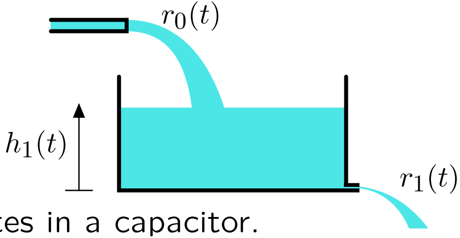
Explain the shape of this curve mathematically.

Explain the shape of this curve physically.

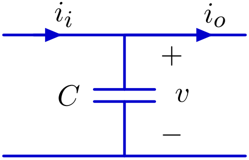
Leaky Tanks and Capacitors

Although derived for a leaky tank, this sort of model can be used to represent a variety of physical systems.

Water accumulates in a leaky tank.



Charge accumulates in a capacitor.



$$\frac{dv}{dt} = \frac{i_i - i_o}{C} \propto i_i - i_o \qquad \text{analogous to} \qquad \frac{dh}{dt} \propto r_0 - r_1$$