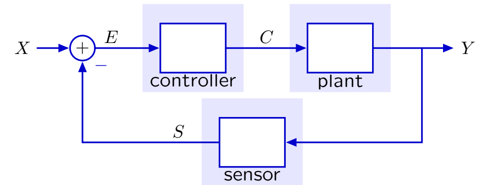
6.003: Signals and Systems

CT Feedback and Control

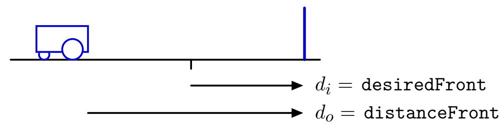
March 16, 2010

Feedback and Control

Feedback: simple, elegant, and robust framework for control.



Last time: robotic driving.



Feedback and Control

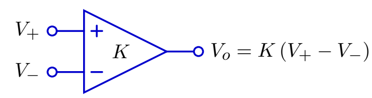
This week: using feedback to enhance performance.

Examples:

- increasing speed and bandwidth
- controlling position instead of speed
- reducing sensitivity to parameter variation
- reducing distortion
- stabilizing unstable systems
  - magnetic levitation
  - inverted pendulum

Op-amps

An "ideal" op-amp has many desireable characteristics.

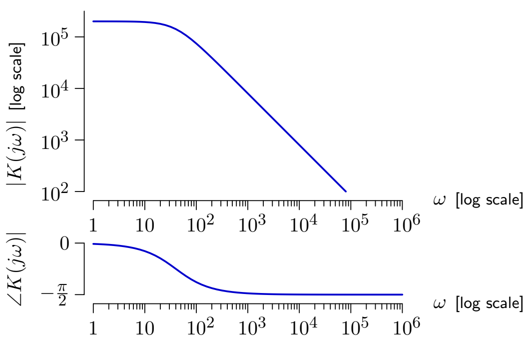


- high speed
- large bandwidth
- high input impedance
- low output impedance
- ...

It is difficult to build a circuit with all of these features.

Op-Amp

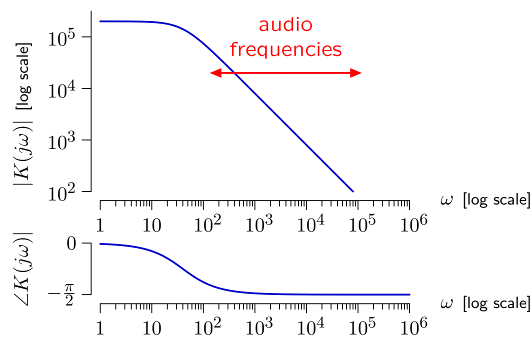
The gain of an op-amp depends on frequency.



Frequency dependence of LM741 op-amp.

Op-Amp

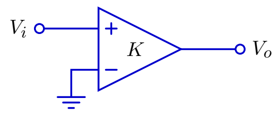
Low-gain at high frequencies limits applications.



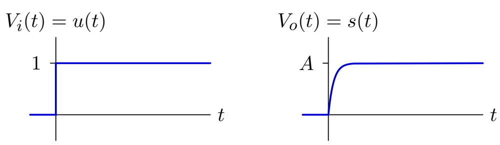
Unacceptable frequency response for an audio amplifier.

Op-Amp

An ideal op-amp has fast time response.

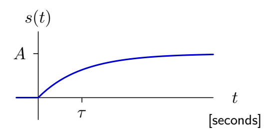


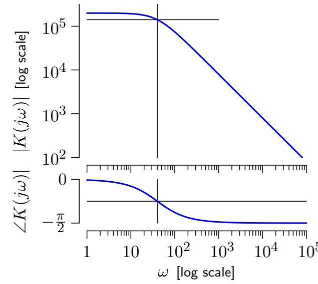
Step response:



Check Yourself

Determine τ for the unit-step response $s(t)$ of an LM741.





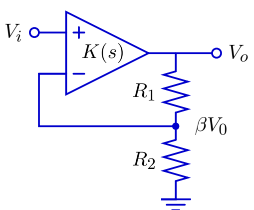
1. $40\,$s   2. $\frac{40}{2\pi}\,$s   3. $\frac{1}{40}\,$s   4. $\frac{2\pi}{40}\,$s   5. $\frac{1}{2\pi\times 40}\,$s

0. none of the above

Op-Amp

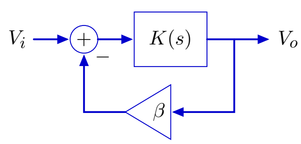
We can use feedback to improve performance of op-amps.

circuit



$$V_- = \beta V_o = \left(\frac{R_2}{R_1+R_2}\right) V_o$$

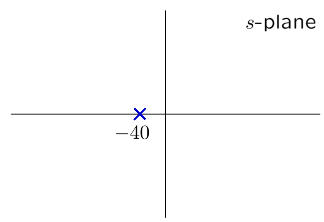
6.003 model



$$\frac{V_o}{V_i} = \frac{K(s)}{1+\beta K(s)}$$

Dominant Pole

Op-amps are designed to have a dominant pole at low frequencies:
→ simplifies the application of feedback.

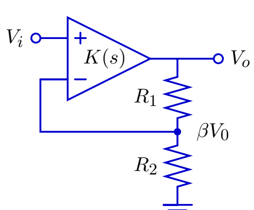


$$\alpha = 40\,\text{rad/s} = \frac{40\,\text{rad/s}}{2\pi\,\text{rad/cycle}} \approx 6.4\,\text{Hz}$$

Improving Performance

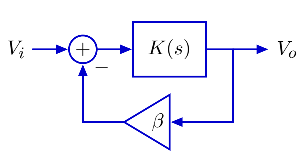
Using feedback to improve performance parameters.

circuit



$$V_- = \beta V_o = \left(\frac{R_2}{R_1+R_2}\right) V_o$$

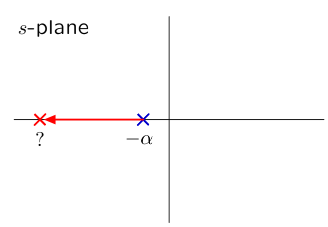
6.003 model



$$\frac{V_o}{V_i} = \frac{K(s)}{1+\beta K(s)} = \frac{\frac{\alpha K_0}{s+\alpha}}{1+\beta\frac{\alpha K_0}{s+\alpha}} = \frac{\alpha K_0}{s+\alpha+\alpha\beta K_0}$$

Check Yourself

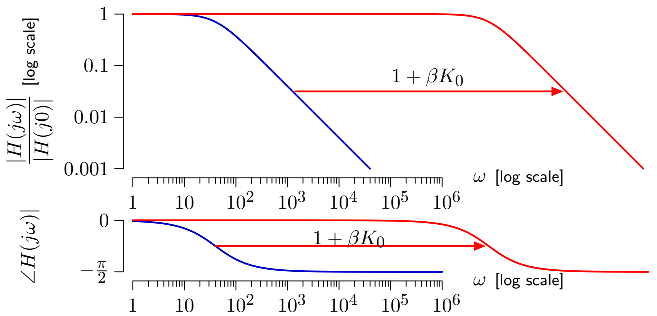
What is the most negative value of the closed-loop pole that can be achieved with feedback?



1. $-\alpha(1+\beta)$ 2. $-\alpha(1+\beta K_0)$
3. $-\alpha(1+K_0)$ 4. $-\infty$
5. none of the above

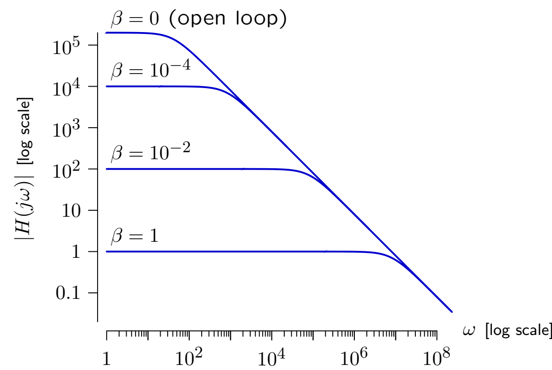
Improving Performance

Feedback extends frequency response by a factor of $1+\beta K_0$ ($K_0 = 2\times10^5$).



Improving Performance

Feedback produces higher bandwidths by **reducing** the gain at low frequencies. It trades gain for bandwidth.

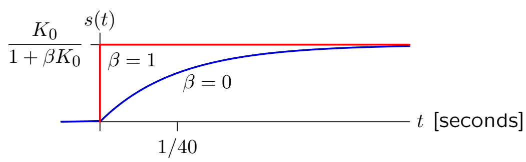


Improving Performance

Feedback makes the time response faster by a factor of $1+\beta K_0$ ($K_0 = 2\times10^5$).

Step response

$$s(t) = \frac{K_0}{1+\beta K_0}\left(1-e^{-\alpha(1+\beta K_0)t}\right)u(t)$$

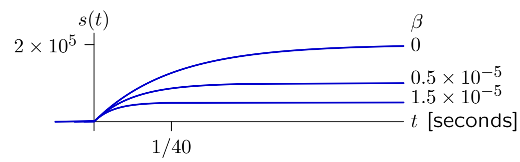


Improving Performance

Feedback produces faster responses by **reducing** the final value of the step response. It trades gain for speed.

Step response

$$s(t) = \frac{K_0}{1+\beta K_0}\left(1-e^{-\alpha(1+\beta K_0)t}\right)u(t)$$



The maximum rate of voltage change $\left.\frac{ds(t)}{dt}\right|_{t=0+}$ is not increased.

Improving Performance

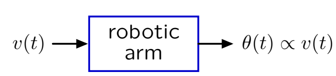
Feedback improves performance parameters of op-amp circuits.

- can extend frequency response
- can increase speed

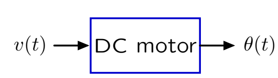
Performance enhancements are achieved through a reduction of gain.

Motor Controller

We wish to build a robot arm (actually its elbow). The input should be voltage $v(t)$, and the output should be the elbow angle $\theta(t)$.



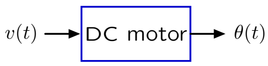
We wish to build the robot arm with a DC motor.



This problem is similar to the head-turning servo in 6.01 !

Check Yourself

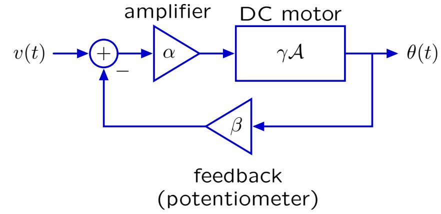
What is the relation between $v(t)$ and $\theta(t)$ for a DC motor?



1. $\theta(t) \propto v(t)$
2. $\cos\theta(t) \propto v(t)$
3. $\theta(t) \propto \dot{v}(t)$
4. $\cos\theta(t) \propto \dot{v}(t)$
5. none of the above

Motor Controller

Use proportional feedback to control the angle of the motor's shaft.

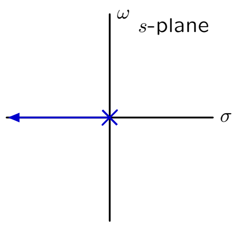


$$\frac{\Theta}{V} = \frac{\alpha\gamma\mathcal{A}}{1+\alpha\beta\gamma\mathcal{A}} = \frac{\alpha\gamma\frac{1}{s}}{1+\alpha\beta\gamma\frac{1}{s}} = \frac{\alpha\gamma}{s+\alpha\beta\gamma}$$

Motor Controller

The closed loop system has a single pole at $s=-\alpha\beta\gamma$.

$$\frac{\Theta}{V} = \frac{\alpha\gamma}{s+\alpha\beta\gamma}$$



As α increases, the closed-loop pole becomes increasingly negative.

Motor Controller

Find the impulse and step response.

The system function is

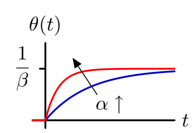
$$\frac{\Theta}{V} = \frac{\alpha\gamma}{s+\alpha\beta\gamma}\,.$$

The impulse response is

$$h(t) = \alpha\gamma e^{-\alpha\beta\gamma t}u(t)$$

and the step response is therefore

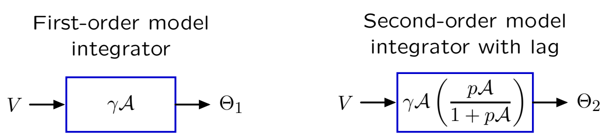
$$s(t) = \frac{1}{\beta}\left(1-e^{-\alpha\beta\gamma t}\right)u(t)\,.$$



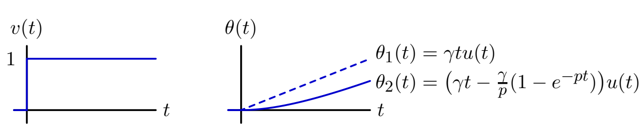
The response is faster for larger values of α.
Try it: Demo.

Motor Controller

The speed of a DC motor does not change instantly if the voltage is stepped. There is lag due to rotational inertia.

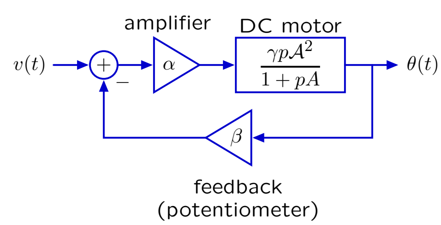


Step response of the models:



Motor Controller

Analyze second-order model.

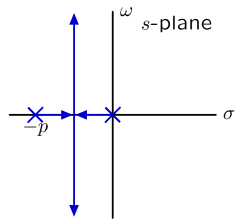


$$\frac{\Theta}{V} = \frac{\frac{\alpha\gamma p\mathcal{A}^2}{1+p\mathcal{A}}}{1+\frac{\alpha\beta\gamma p\mathcal{A}^2}{1+p\mathcal{A}}} = \frac{\alpha\gamma p\mathcal{A}^2}{1+p\mathcal{A}+\alpha\beta\gamma p\mathcal{A}^2} = \frac{\alpha\gamma p}{s^2+ps+\alpha\beta\gamma p}$$

$$s = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - \alpha\beta\gamma p}$$

Motor Controller

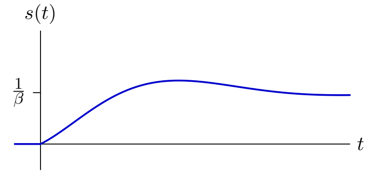
For second-order model, increasing α causes the poles at 0 and $-p$ to approach each other, collide at $s = -p/2$, then split into two poles with imaginary parts.



Increasing the gain α does not increase speed of convergence.

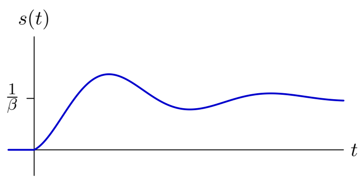
Motor Controller

Step response.



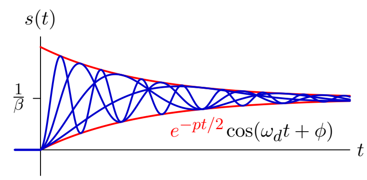
Motor Controller

Step response.



Motor Controller

Step response.



Feedback and Control: Summary

CT feedback is useful for many reasons. Today we saw two:

- increasing speed and bandwidth
- controlling position instead of speed

Next time we will look at several others:

- reduce sensitivity to parameter variation
- reduce distortion
- stabilize unstable systems
  - magnetic levitation
  - inverted pendulum