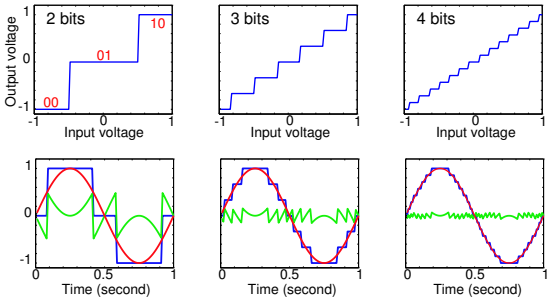
6.003: Signals and Systems

Quantization and Modulation

May 4, 2010

Last Time

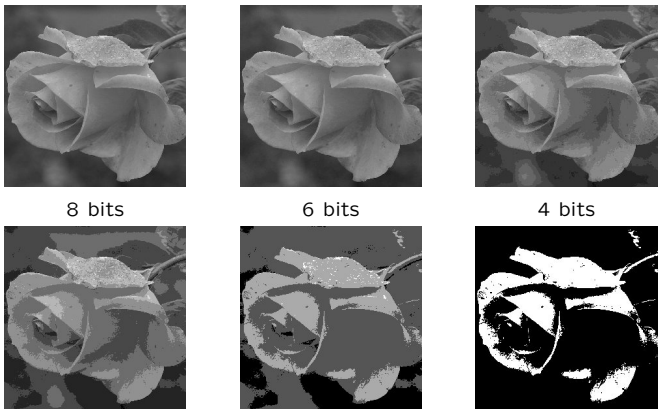
Quantization: discrete representation of amplitudes.



Digital representations of signals require discrete representations of both the independent variable (time, space, ...) and dependent variable (amplitude).

Quantizing Images

Quantizing creates distortion.



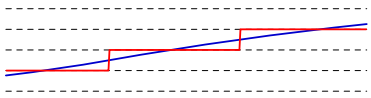
8 bits 6 bits 4 bits

3 bits 2 bits 1 bit

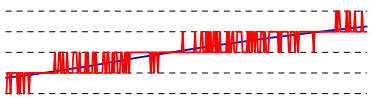
Quantization

Adding noise (dither) can reduce the perceptual effect of quantization.

Quantization: $y = Q(x)$

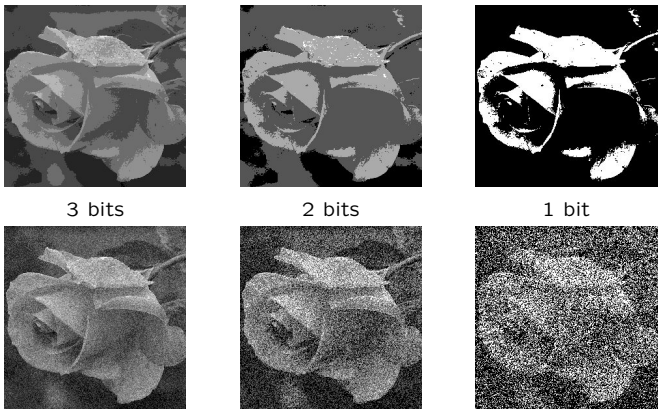


Quantization with dither: $y = Q(x + n)$



Quantizing Images

Adding noise (dither) reduces "banding" in images.



3 bits 2 bits 1 bit

3 bits dither 2 bits dither 1 bit dither

Quantizing Sound

Adding noise (dither) can reduces "distortions" in sound.

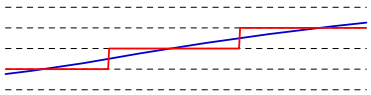
- 16 bits/sample
- 4 bits/sample
- 4 bits with dither/sample
- 3 bits/sample
- 3 bits with dither/sample
- 2 bit/sample
- 2 bit with dither/sample

J.S. Bach, Sonata No. 1 in G minor Mvmt. IV. Presto
Nathan Milstein, violin

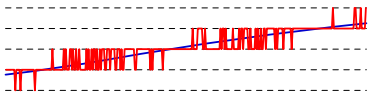
Robert's Technique

Robert's technique decorrelates quantization errors from the signal.

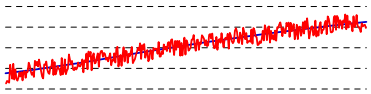
Quantization: $y = Q(x)$



Quantization with dither: $y = Q(x+n)$



Quantization with Robert's technique: $y = Q(x+n) - n$



Quantizing Images

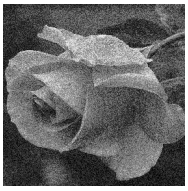
Robert's technique decorrelates quantization errors from the signal.

3 bits dither

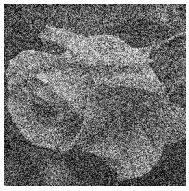
2 bits dither

1 bit dither

3 bits Robert's



2 bits Robert's



1 bit Roberts

Quantizing Sound

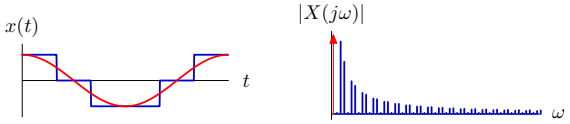
Robert's technique decorrelates quantization errors from the signal.

- 16 bits/sample
- 2 bits/sample
- 2 bits with dither/sample
- 2 bits with Robert's technique/sample

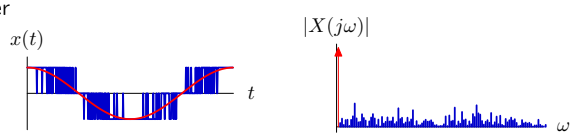
J.S. Bach, Sonata No. 1 in G minor Mvmt. IV. Presto
Nathan Milstein, violin

Fourier Analysis

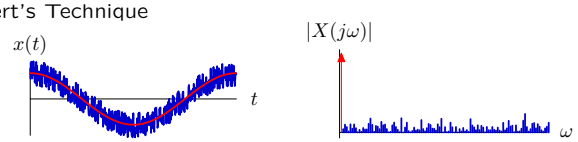
Quantization



Dither



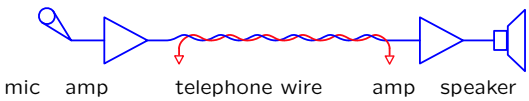
Robert's Technique



Modulation

Applications of signals and systems in communication systems.

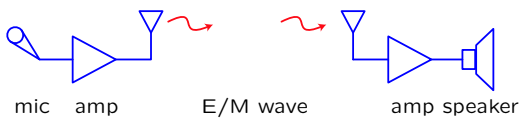
Example: Transmit voice via telephone wires (copper)



Works well: basis of local land-based telephones.

Wireless Communication

In cellular communication systems, signals are transmitted via electromagnetic (E/M) waves.



For efficient transmission and reception, antenna length should be on the order of the wavelength.

Telephone-quality speech contains frequencies from 200 to 3000 Hz.

How long should the antenna be?

Check Yourself

For efficient transmission and reception, the antenna length should be on the order of the wavelength.

Telephone-quality speech contains frequencies between 200 Hz and 3000 Hz.

How long should the antenna be?

1. < 1 mm
2. $\sim$ cm
3. $\sim$ m
4. $\sim$ km
5. > 100 km

Check Yourself

What frequency E/M wave is well matched to an antenna with a length of 10 cm (about 4 inches)?

1. < 100 kHz
2. 1 MHz
3. 10 MHz
4. 100 MHz
5. > 1 GHz

Wireless Communication

Speech is not well matched to the wireless medium.

Many applications require the use of signals that are not well matched to the required media.

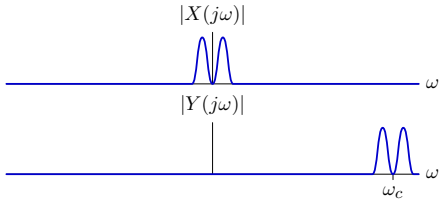
signal	applications
audio	telephone, radio, phonograph, CD, cell phone, MP3
video	television, cinema, HDTV, DVD
internet	coax, twisted pair, cable TV, DSL, optical fiber, E/M

We can often modify the signals to obtain a better match.

Today we will introduce simple matching strategies based on **modulation**.

Check Yourself

Construct a signal Y that codes the audio frequency information in X using frequency components near 2 GHz.

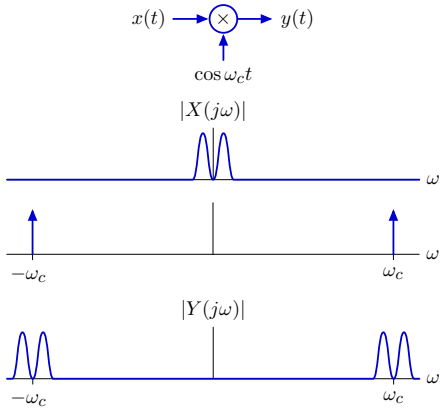


Determine an expression for Y in terms of X.

1. $y(t) = x(t)\, e^{j\omega_c t}$
2. $y(t) = x(t) * e^{j\omega_c t}$
3. $y(t) = x(t)\cos(\omega_c t)$
4. $y(t) = x(t) * \cos(\omega_c t)$
5. none of the above

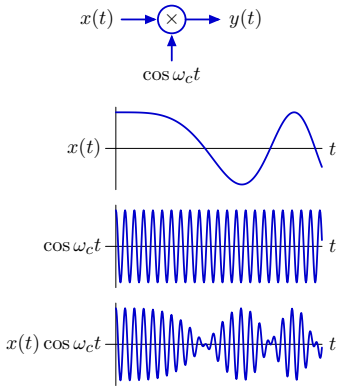
Amplitude Modulation

Multiplying a signal by a sinusoidal **carrier** signal is called amplitude modulation (AM). AM shifts the frequency components of X by $\pm\omega_c$.



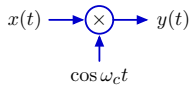
Amplitude Modulation

Multiplying a signal by a sinusoidal **carrier** signal is called amplitude modulation. The signal "modulates" the amplitude of the carrier.



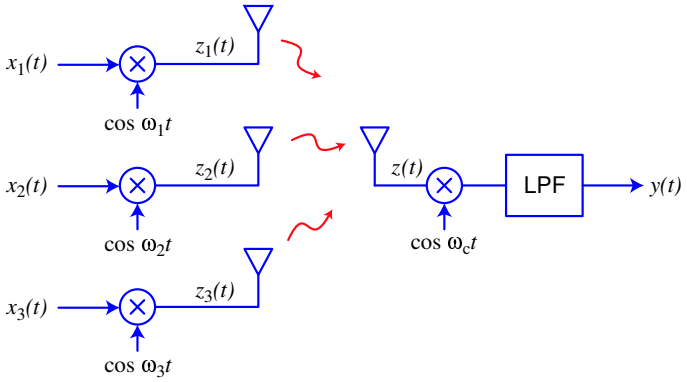
Amplitude Modulation

How could you recover $x(t)$ from $y(t)$?



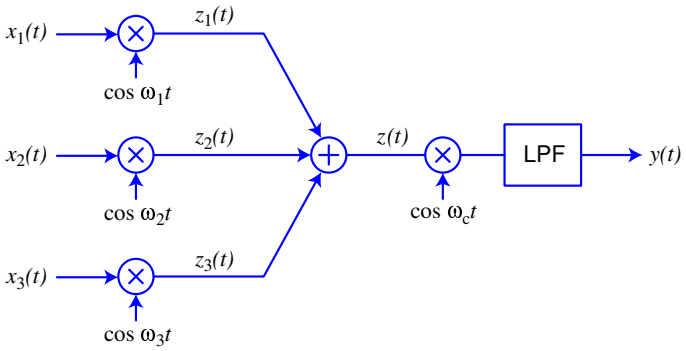
Frequency-Division Multiplexing

Multiple transmitters can co-exist, as long as the frequencies that they transmit do not overlap.



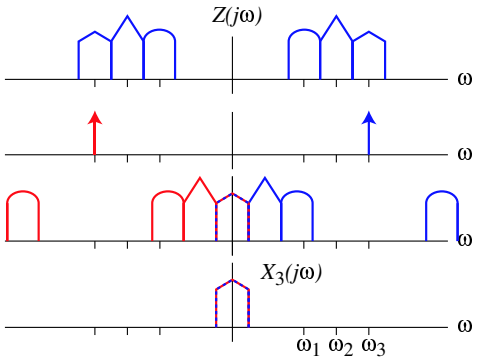
Frequency-Division Multiplexing

Multiple transmitters simply sum (to first order).



Frequency-Division Multiplexing

The receiver can select the transmitter of interest by choosing the corresponding demodulation frequency.



Broadcast Radio

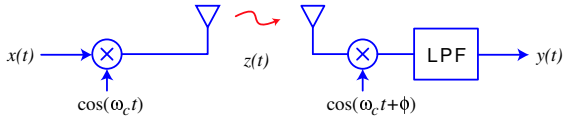
"Broadcast" radio was championed by David Sarnoff, who previously worked at Marconi Wireless Telegraphy Company (point-to-point).

- envisioned "radio music boxes"
- analogous to newspaper, but at speed of light
- receiver must be cheap (as with newsprint)
- transmitter can be expensive (as with printing press)

Sarnoff (left) and Marconi (right)

Check Yourself

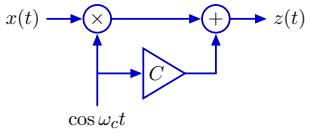
The problem with making an inexpensive radio receiver is that you must know the carrier signal exactly!



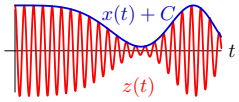
What happens if there is a phase shift ϕ between the signal used to modulate and that used to demodulate?

AM with Carrier

One way to synchronize the sender and receiver is to send the carrier along with the message.



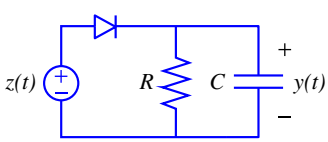
$$z(t) = x(t)\cos\omega_c t + C\cos\omega_c t = (x(t) + C)\cos\omega_c t$$

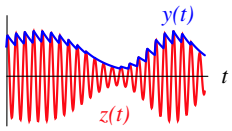


Adding carrier is equivalent to shifting the DC value of $x(t)$.
If we shift the DC value sufficiently, the message is easy to decode: it is just the envelope (minus the DC shift).

Inexpensive Radio Receiver

If the carrier frequency is much greater than the highest frequency in the message, AM with carrier can be demodulated with a peak detector.





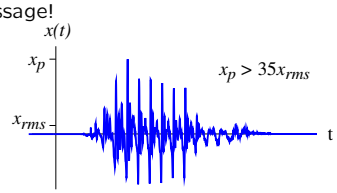
In AM radio, the highest frequency in the message is 5 kHz and the carrier frequency is between 500 kHz and 1500 kHz.

This circuit is simple and inexpensive.

But there is a problem.

Inexpensive Radio Receiver

AM with carrier requires more power to transmit the carrier than to transmit the message!



Speech sounds have high crest factors (peak value divided by rms value). The DC offset C must be larger than x_p for simple envelope detection to work.

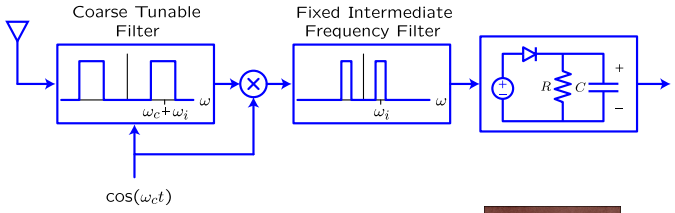
The power needed to transmit the carrier can be $35^2 \approx 1000\times$ that needed to transmit the message.

Okay for broadcast radio (WBZ: 50 kwatts).

Not for point-to-point (cell phone batteries wouldn't last long!).

Superheterodyne Receiver

Edwin Howard Armstrong invented the superheterodyne receiver, which made broadcast AM practical.



Edwin Howard Armstrong also invented and patented the "regenerative" (positive feedback) circuit for amplifying radio signals (while he was a junior at Columbia University). He also invented wide-band FM.