6.003 (Spring 2010)

Quiz #2

April 7, 2010

Name:

Kerberos Username:

Please circle your section number:

Section	*Instructor*	*Time*
1	Peter Hagelstein	10 am
2	Peter Hagelstein	11 am
3	Rahul Sarpeshkar	1 pm
4	Rahul Sarpeshkar	2 pm

Grades will be determined by the correctness of your answers (explanations are not required).

Partial credit will be given for ANSWERS that demonstrate some but not all of the important conceptual issues.

You have **two hours.**

Please put your initials on all subsequent sheets.

Enter your answers in the boxes.

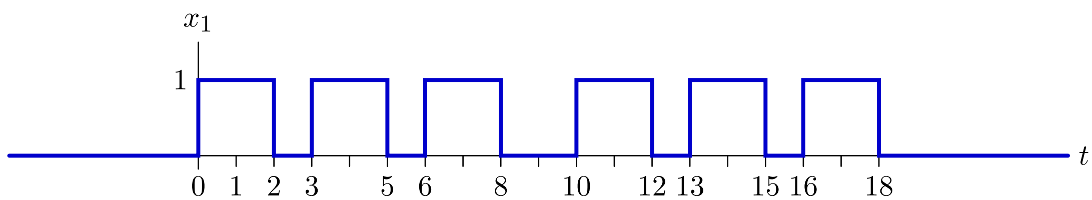
This quiz is closed book, but you may use two 8.5×11 sheets of paper (four sides total).

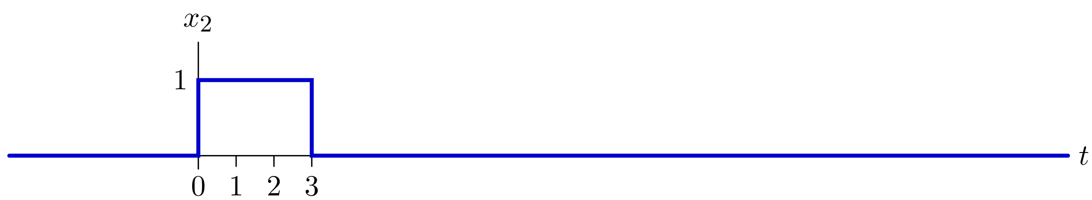
No calculators, computers, cell phones, music players, or other aids.

1	/20
2	/30
3	/20
4	/30
Total	/100

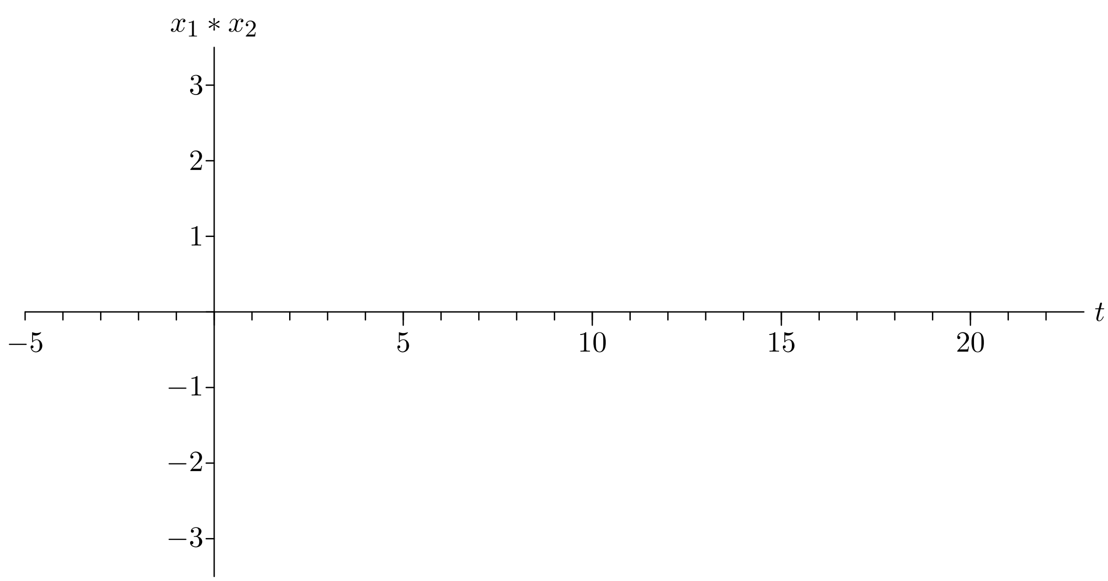
1. Convolution *[20 points]*

Signals $x_1(t)$ and $x_2(t)$ are shown in the plots below, and are zero outside the indicated intervals.





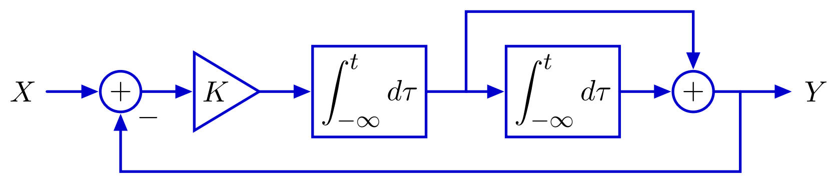
Plot the result of convolving $x_1(t)$ with $x_2(t)$.
Make sure that the important break-points are clear.



Worksheet (intentionally blank)

2. Impulse response *[30 points]*

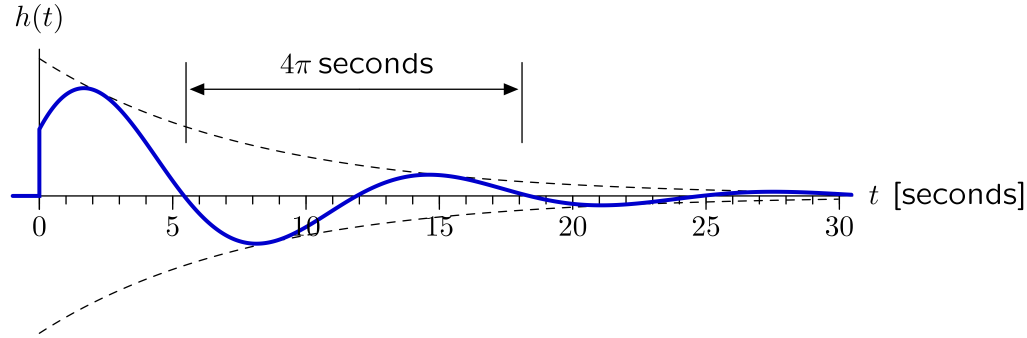
Consider the following control system where the gain K is a real-valued constant.



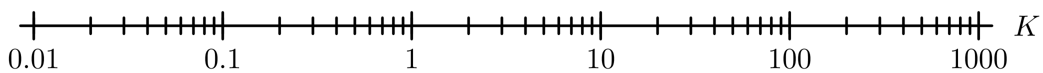
Part a. *[10 points]* Determine the system function $H(s) = \frac{Y(s)}{X(s)}$ as a function of K.

$H(s) =$

Part b. *[20 points]* The following plot shows the impulse response $h(t)$ of the closed-loop system for a particular value of K. [The dashed curves are exponential functions of time, shown for reference.]

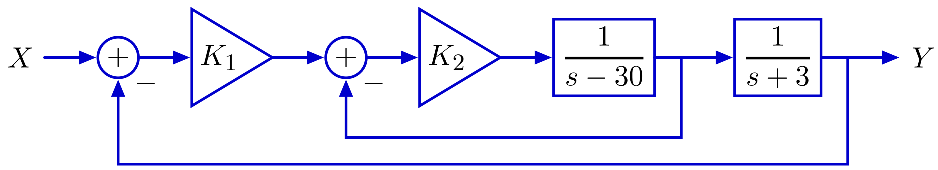


Determine K and indicate its value by placing an × on the following scale.

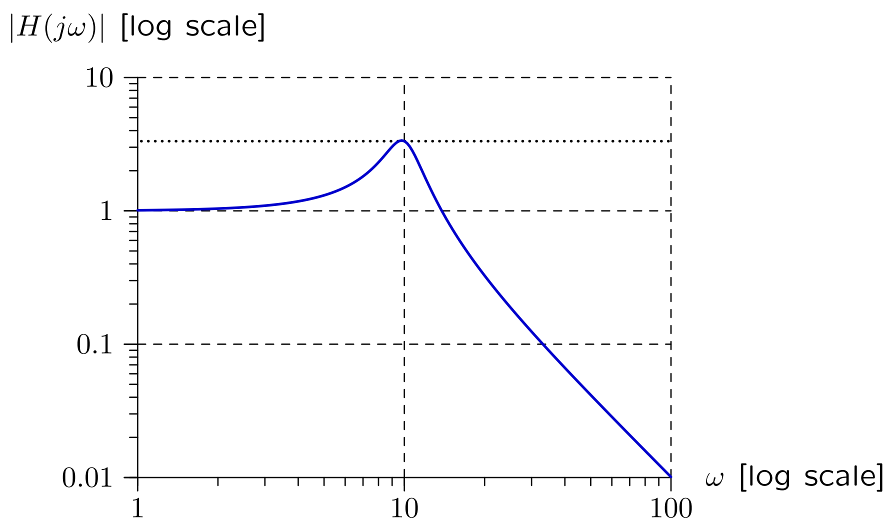


3. Frequency Response *[20 points]*

Let $H(s) = Y(s)/X(s)$ for the following system.



Find K_1 and K_2 so that $|H(j\omega)|$ matches the plot below.



Enter numbers (or numerical expressions) for K_1 and K_2 in the boxes.

$K_1 =$

$K_2 =$

Worksheet (intentionally blank)

4. Fourier series *[30 points]*

Let $x(t)$ represent a periodic signal (period $T = 8$ seconds) whose Fourier series coefficients are

$$a_k = \begin{cases} \frac{1}{j\pi k} & \text{for integers } k \neq 0 \\ 0 & k = 0. \end{cases}$$

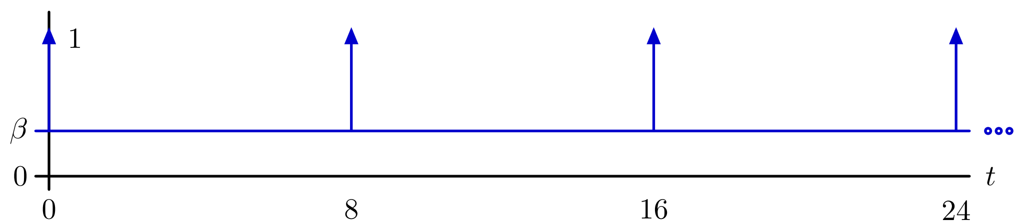
When $x(t)$ is the input to a linear, time-invariant system with system function $H(s)$

$$x(t) \longrightarrow \boxed{H(s)} \longrightarrow y(t)$$

the output $y(t)$ is the sum of a constant β plus a periodic train of impulses with area 1,

$$y(t) = \beta + \sum_{k=-\infty}^{\infty} \delta(t - 8k)$$

as shown below.



Part a. *[15 points]* Determine β.

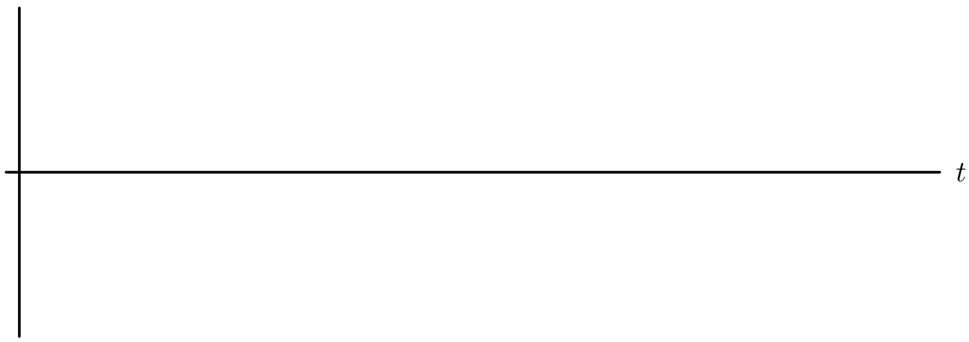
$\beta = \boxed{}$

Part b. *[15 points]* Consider the output of the same system if the period of the input signal is changed to $T = 4$ seconds while keeping the Fourier series coefficients a_k unchanged.

Is it possible to determine the new output signal from the information provided?

possible? (**Yes** or **No**)

If **Yes**, sketch and fully label the new output signal on the axes below.



If **No**, briefly explain why not.

Worksheet (intentionally blank)

Worksheet (intentionally blank)