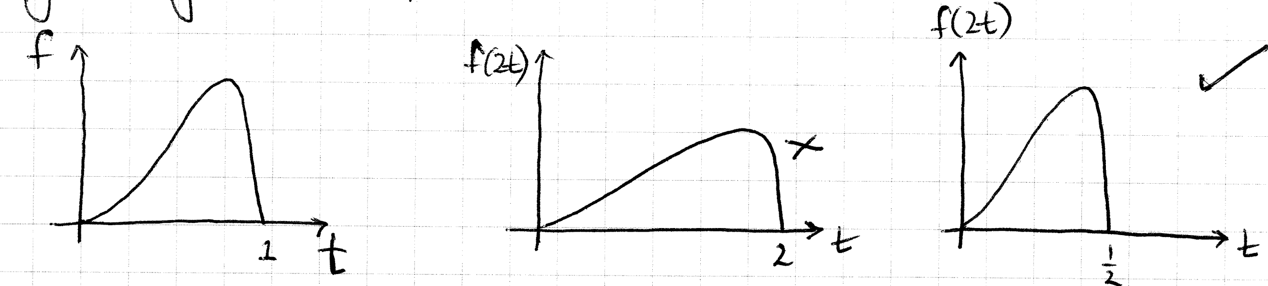
Recitation #1, 6.003, Spring 2010.

2/2/2010.

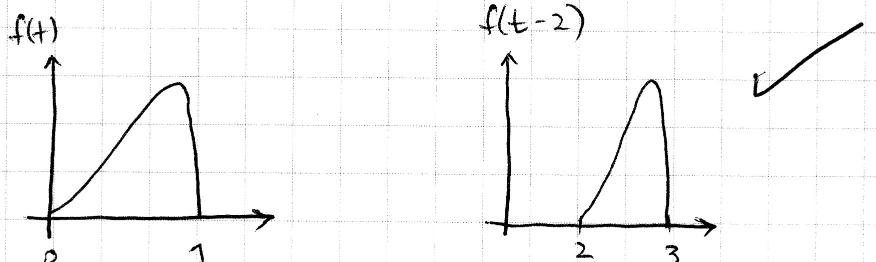
TODAY

1. Review function transformations (translation & scaling)
2. Leaky tanks and RC circuits (unifying mathematics)
3. Drug dosing, discrete systems, and RC circuits.

Part 1:

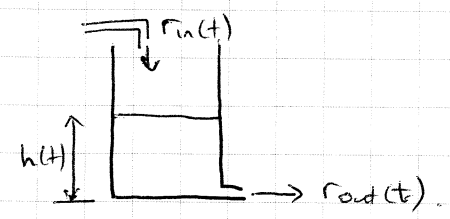


⇒ A ~~scaling~~ doubling transformation on the argument of a function compresses that function by a factor of ½.



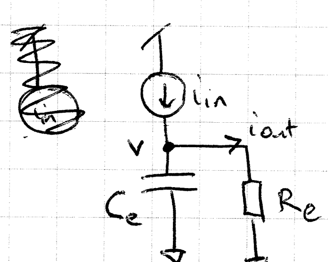
⇒ A ~~translational~~ subtractive transformation of the argument of a function translates it to the right.

Part 2:



r_{in} : volume/sec of water in.
r_{out} : volume/sec of water out.

Equivalently,



i_{in} : Charge/sec of electrons in.

i_{out} : Charge/sec of electrons out.

rle AT MIT
www.rle.mit.edu

area of tank.

Water volume = $A \times h$.

$\left\{ \text{Capacitor charge} = C_e \times V \right\}$ capacitance

$$\frac{d(Ah)}{dt} = r_{in} - r_{out}. \qquad \frac{d(C_e V)}{dt} = i_{in} - i_{out}.$$

constant related to viscosity of the fluid

$$A\frac{dh}{dt} = r_{in} - \gamma h \qquad C_e \frac{dV}{dt} = i_{in} - V/R_e.$$

$$= r_{in} - h/R_F.$$

Now let us denote

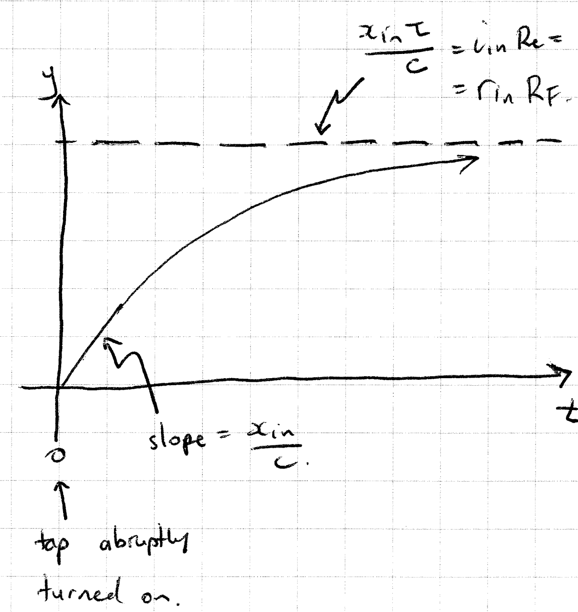
$$V, h \longleftrightarrow y.$$
$$C_e, A \longleftrightarrow C$$
$$R_F, R_e \longleftrightarrow R.$$
$$r_{in}, i_{in} \longleftrightarrow x_{in}.$$

$$\Rightarrow \quad C\frac{dy}{dt} = x_{in} - \frac{y}{R}.$$

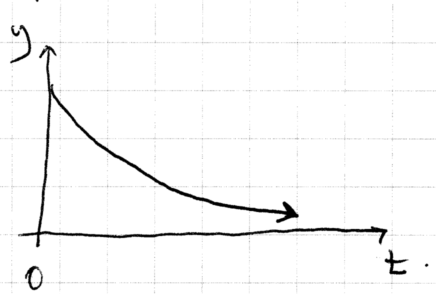
$$\frac{dy}{dt} = \frac{x_{in}}{C} - \frac{y}{RC}$$

$$= \frac{x_{in}}{C} - \frac{y}{\tau}, \quad \text{where } \tau = RC.$$

Note, i_{in} and r_{in} are constant.



Now assume the system is in steady state and the tap is abruptly turned off.



$$\frac{dy}{dt} = \frac{-y}{\tau}. \qquad \frac{dy}{y} = \frac{-dt}{\tau}.$$

$$\ln \frac{y(t)}{y(0)} = -\frac{t}{\tau}.$$

$$\Rightarrow y(t) = y(0)\, e^{-t/\tau}.$$

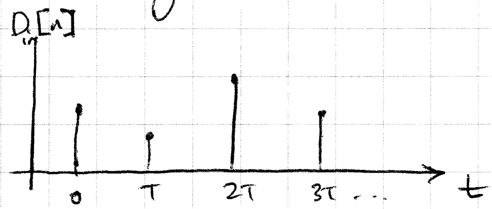
"The natural response"

Part 3 Drug dosing

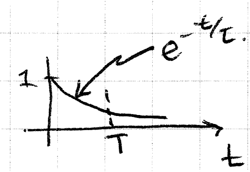
[An example of a DT system and how it relates to a CT system].

Let $D_{in}[n]$ be the amount of drug (the dose) taken on the nth day. Assume:

1) The drug is instantly absorbed into the blood.
2) One dose of drug per day at a pre-defined time.
3) The drug is broken down every day with some time constant τ.



The drugs in the body are broken down as:



Let $d[n]$ be the amount of drug in the body in the nth day.

$$\Rightarrow d[n] = D_{in}[n] + d[n-1]\, e^{-T/\tau}.$$

As a difference equation with constant co-efficients:

$$d[n] = \alpha\, d[n-1] + D_{in}[n].$$

If we assume a constant dosage each day, so $D_{in}[n] = D_{in}$, and that $d[0] = 0$.

Asymptotically, $d[\infty] = \alpha\, d[\infty] + D_{in}$

$$\Rightarrow d[\infty] = \frac{D_{in}}{1-\alpha}.$$