6.003R

Feb 5, 2009

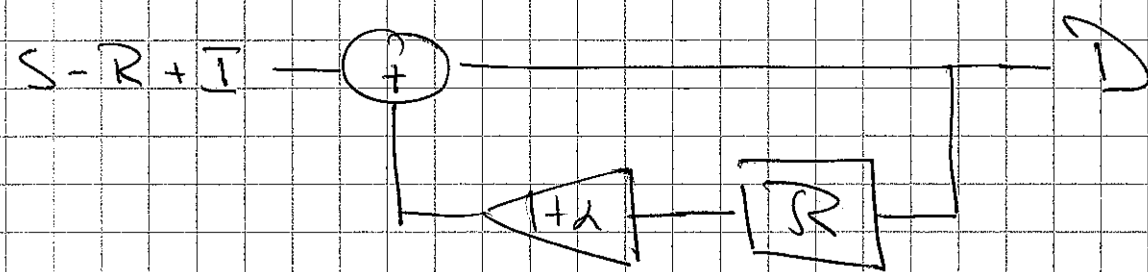
Conservation law ? for \$ — P. Hagelstein

Receipts : R

Spending : S

if R < S, then debt D

interest I



$$d[n] = (1+\alpha)\, d[n-1] + s[n] - r[n] + i[n]$$

	(Proj)			(Proj)	Wikipedia
n	$r[n]$	$s[n]$	$i[n]$	$d_{est}[n]$	$d_{est}[n]$
2007	2.568T	2.730T	0.433T	8.950T	8.950T
2008	2.521T	2.931T	0.454T	9.654T	9.985T
2009	2.699T	3.107T		10.413T	12.867T

$\alpha \sim 4.5\%$

Conclude numbers not consistent....

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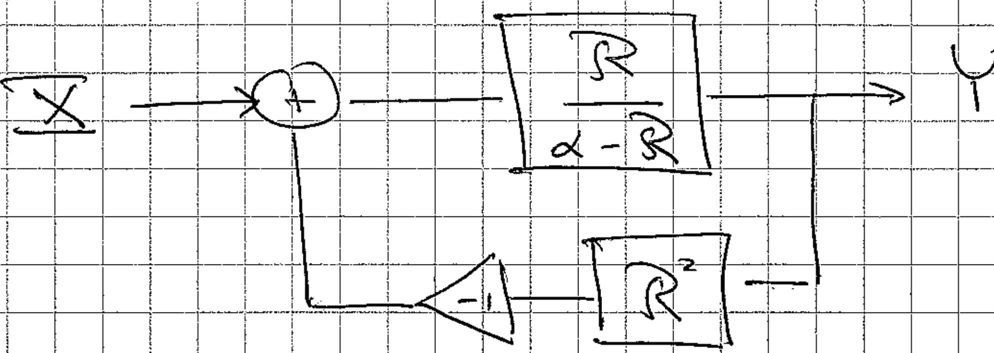
Example: Missing Parameter problem / like problem 5

Consider



Assume that system starts out at rest

Assume that $x[n] = \delta[n]$

Determine sequence of $y[n]$, or pick α so that $y[5] = 0$

How to solve?

To simplify life, Define

$$U = X \cancel{+} - R^2 Y$$

Then
$$Y = \frac{R}{\alpha - R} U$$

which is also $(\alpha - R) Y = R U$

Translate into equations for $x[n]$, $u[n]$, $y[n]$

$$u[n] = x[n] - y[n-2]$$
$$\alpha y[n] - y[n-1] = u[n-1]$$
$$\rightarrow \quad y[n] = \frac{y[n-1] + u[n-1]}{\alpha}$$

OK, make up table

n	$x[n]$	$u[n]$	$y[n]$
-2	0	0	0
-1	0	0	0
0	1	1	0
1	0	0	$\frac{1}{\alpha}$
2	0	0	$\frac{1}{\alpha^2}$
3	0	$-\frac{1}{\alpha}$	$\frac{1}{\alpha^3}$
4	0	$-\frac{1}{\alpha^2}$	$\frac{1}{\alpha^4} - \frac{1}{\alpha^2}$
5	0	$-\frac{1}{\alpha^3}$	$\frac{1}{\alpha^5} - \frac{2}{\alpha^3}$

$y[5] = 0$ if $\alpha^2 = \frac{1}{2}$, $\alpha = \pm\frac{1}{\sqrt{2}}$

Taylor series expansion (Thinking about problem 3)

$$f(x_0+\Delta x) = f(x_0) + \Delta x \left(\frac{df}{dx}\right)_{x_0} + \frac{1}{2}\Delta x^2 \left(\frac{d^2f}{dx^2}\right)_{x_0}$$

$$+ \frac{1}{3!}\Delta x^3 \left(\frac{d^3f}{dx^3}\right)_{x_0} + \frac{1}{4!}\Delta x^4 \left(\frac{d^4f}{dx^4}\right)_{x_0} + \cdots$$

Example $\quad f(x) = e^x$

$$\frac{df}{dx} = e^x$$

$$\vdots$$

§ $e^{\Delta x} = 1 + \Delta x + \frac{\Delta x^2}{2} + \frac{\Delta x^3}{3!} + \frac{\Delta x^4}{4!} + \cdots$

Example $\quad f(x) = \cos x$

$$\frac{df}{dx} = -\sin x$$

$$\frac{d^2f}{dx^2} = -\cos x$$

$$\vdots$$

$$\cos(\Delta x) = 1 - \frac{1}{2}\Delta x^2 + \frac{1}{4!}\Delta x^4 - \frac{1}{6!}\Delta x^6 + \cdots$$

Try $f(x) = \dfrac{1}{1+x}$

$$\frac{df}{dx} = -\frac{1}{(1+x)^2}$$

$$\frac{d^2f}{dx^2} = \frac{2}{(1+x)^3}$$

$$\frac{d^3f}{dx^3} = -\frac{3!}{(1+x)^4}$$

$$\frac{1}{1+\Delta x} = 1 - \Delta x + \Delta x^2 - \Delta x^3 + \Delta x^4 \cdots$$

Convergence issues

let $\Delta x = 0.1$

$$\frac{1}{1.1} = 1 - 0.1 + 0.01 - 0.001 + 0.0001 + \cdots$$

looks like it converges

let $\Delta x = 2$

$$\frac{1}{3} \stackrel{?}{=} 1 - 2 + 4 - 8 + 16 \cdots$$

Hmmm

$\Delta x = -2$

$$-1 \stackrel{?}{=} 1 + 2 + 4 + 8 + 16 + \cdots$$

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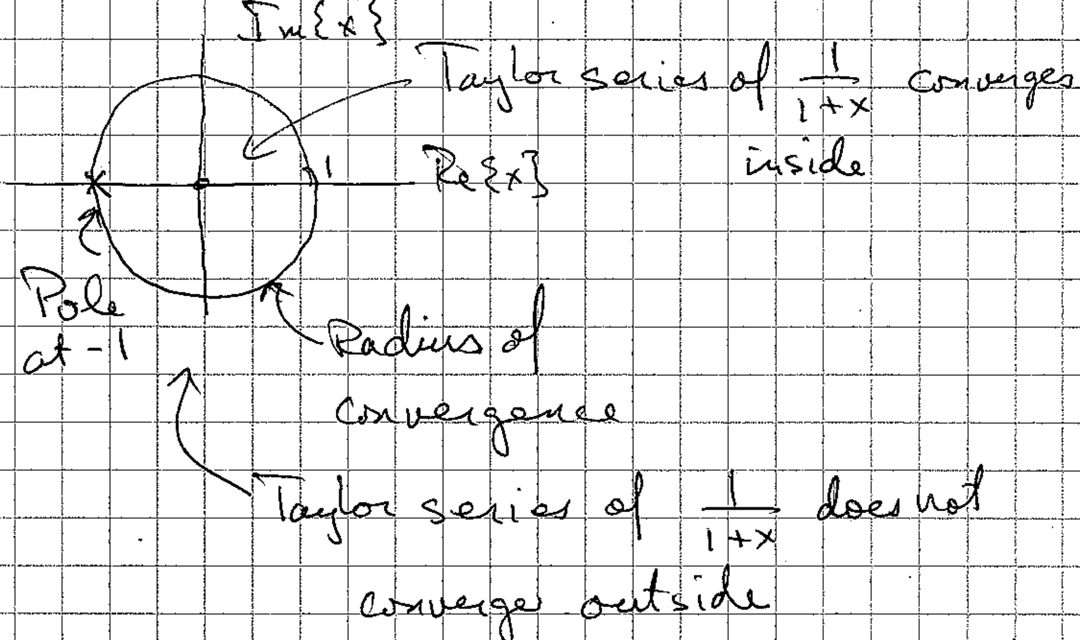
Does not converge

(Advanced, not 6.003 at least yet)

Convergence depends on distance to nearest pole in complex plane

$\frac{1}{1+x}$



$f(x) = e^x$ does not have poles near $x=0$, so it's Taylor series converges for finite x,