Recitation # 2

6.003

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~~Rev~~
Today: ~~Linear constant co-efficient difference~~
Difference equations [Linear, constant co-efficients].
~~Block diagrams~~ - Delay operator.
~~Step Sample by sample solutions~~ Stability, convergence.
- Even & odd functions.

Difference equations:

$$\cancel{y[n] = \underbrace{\sum_{i=0}^{\infty} \beta_i x[n-i]}_{\text{memory}} + \overbrace{\sum_{j=0}^{\infty} \alpha_j y[n-j]}^{\text{feedback}}}$$ ~~- Linear, constant co-eff.~~

$$y[n] = \beta_0 x[n] + \underbrace{\sum_{i=1}^{\infty} \beta_i x[n-i]}_{\text{memory}} + \underbrace{\sum_{j=1}^{\infty} \alpha_j y[n-j]}_{\text{feedback}}.$$ - Linear, constant co-eff.

Using delay operators

~~$y[n] = \beta_0 x[n]$~~

$$Y = \beta_0 X + \sum_{i=1}^{\infty} \beta_i R^i X + \sum_{j=1}^{\infty} \alpha_j R^j Y.$$ $\rightarrow R$

R will be used a lot later in the course to introduce "fancy" techniques for analyzing systems.
For now, let us play with difference equations in their original form ~~to gain intuition~~ which is useful when R gets you stuck.

Stability & Convergence:

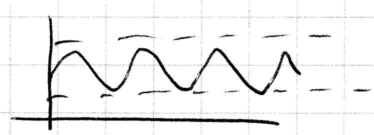
(2) Convergence: ~~if the response fluctuations in the response of a system~~
~~Suppose a system is given a bounded input~~

Suppose a system at rest is given a bounded input,
If the fluctuations in the ~~response~~ systems response go to zero,
then the output is said to converge.

(1) Stability: Suppose a system at rest is given any ~~a~~ bounded input.
If the output of the system is bounded, then the system is
said to be stable.

Stable $\overset{?}{\Rightarrow}$ Converge

Converge $\overset{?}{\Rightarrow}$ Stable ✓



Even & odd functions

A signal is even ~~odd~~ if $\forall n \quad x[-n] = x[n]$.

A signal is odd if $\forall n \quad x[-n] = -x[n]$.

Problems:

the size of the us ~~bad~~ financial bailout

① Suppose ~~a system~~ is governed by the following input-output relation:

$$y[n] = \beta x[n] + \alpha_1 y[n-1] + \alpha_2 y[n-2] + \cdots$$

to 1 crisis

The unit impulse response of the system is given by the following

(Days)	n	-1	0	1	2	...				
	$x[n]$	0	1	0	0	...				
(Billions \$)	$y[n]$	0	1	1	3	5	11	21	43	85 ...

~~(2a)~~ Find the constants $\beta, \alpha_1, \alpha_2, \ldots$

~~Sample~~

Stable? ~~Maybe not~~ Probably not.

Converging? Nope.

How many terms? ∞? No, but need feedback!

$y[0] = \beta x[0] \Rightarrow \beta = 1.$

$y[1] = x[1] + \alpha_1 y[0]$

$1 = 0 + \alpha_1 \cdot 1 \Rightarrow \alpha_1 = 1.$

$y[2] = \cancel{x[2]} + y[1] + \alpha_2 y[0].$

$3 = 1 + \alpha_2 1 \Rightarrow \alpha_2 = 2.$

Are we done? Yup.

② Consider the system $y[n] = \alpha x[n] + \beta x[n-1] - y[n-2].$

Let $x[n] = u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & o/w \end{cases}.$

Find $y[119]$.

System stable?

Does it converge? $\quad y[\infty] = \alpha + \beta - y[\infty]$.

$$y[\infty] = \frac{\alpha+\beta}{2}.$$

$$\frac{y[n+1]}{y[n]} \xrightarrow[n\to\infty]{?} 1$$

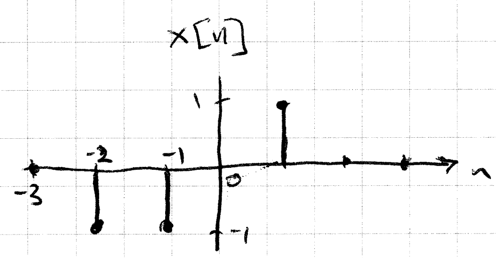
$y[0]$	α
$y[1]$	$\alpha+\beta$
$y[2]$	$\cancel{\alpha+\beta}$ α
$y[3]$	$\cancel{\alpha}$ $\cancel{+\beta}$ $\cancel{\alpha}$ 0
$y[4]$	$\cancel{\alpha+\beta-\beta-\alpha}$ α
$y[5]$	$\alpha+\beta$.

$T_0 = 4$

$$\left.\begin{aligned} y[4i] &= \alpha \\ y[4i+1] &= \alpha+\beta \\ y[4i+2] &= \alpha \\ y[4i+3] &= 0. \end{aligned}\right\}$$

$y[119] = y[29\times 4+3] = 0.$

(3)



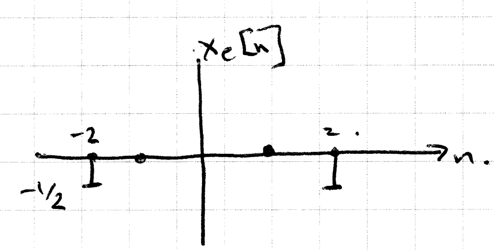
~~x~~ $x[n]$ can be written as the sum of an even + odd function.

$x[n] = x_e[n] + x_o[n]$. — (1)

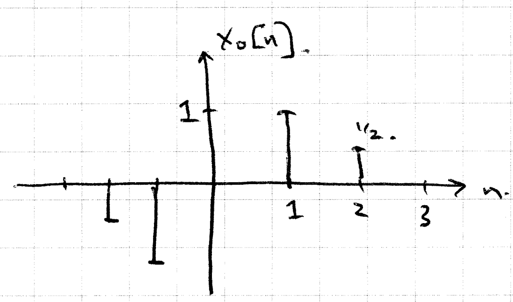
$x[-n] = x_e[-n] + x_o[-n]$

$= x_e[n] - x_o[n]$. — (2)

$\cancel{x}$ $x_e[n] = \dfrac{x[n] + x[-n]}{2}$ $\Rightarrow$



$x_o[n] = \dfrac{x[n] - x[-n]}{2}$. $\Rightarrow$



~~Prove $x_0[n]$ is unique.~~

Amendment to Q1.

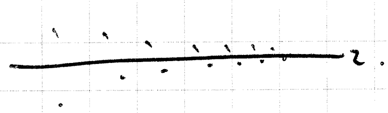
0
1 , 1, 3, 5 11 21 43 85 171 341 683 ⋯ 1365

The sequence grows approximately geometrically

$y[n] \approx ab^n$ for large n. Find a + b.

$$\frac{y[n]}{y[n-1]} \longrightarrow b.$$

$1/1 = 1$
$3/1 = 3$
$5/3$
$11/5 = 2.2$
$21/11 = \cancel{2}1.9$
$43/21 = 2.05$
$85/43 = 1.98$
$\vdots$
$\downarrow$
$2.$



$\Rightarrow b = 2.$

$a\,2^{11} \approx 1365.$

$a \approx \frac{1365}{2^{11}} = \frac{1365}{2048} = 2/3.$

$y[n] \approx 2/3\,(2)^n.$