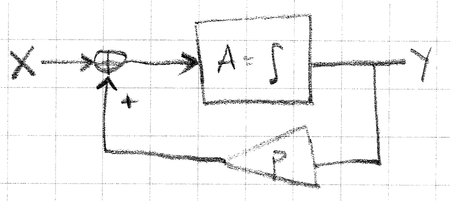
2/12/10 6.003 Recitation #3

TODAY:

1) Review of DT & CT canonical operator feedback topologies
2) CT operator analysis of a motor
3) DT simulation of a CT differential equation.

Part 1:

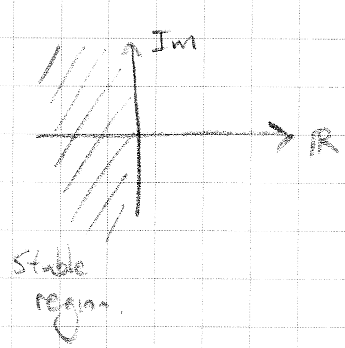
CT



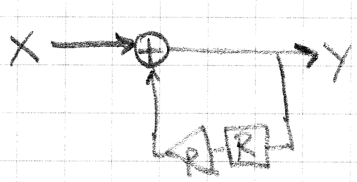
Black's formula.

$$\frac{Y}{X} = \frac{A}{1-pA} = \underbrace{\frac{\text{feedthrough transmission}}{1 - \text{loop transmission}}}$$

$$X = \delta(t) \Rightarrow Y = A(1 + pA + p^2A^2 + \cdots)\,\delta(t)$$
$$\Rightarrow y(t) = u(t)\,e^{pt}.$$

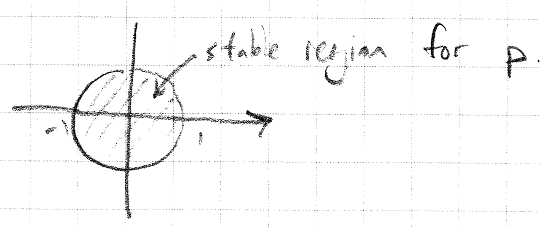


DT

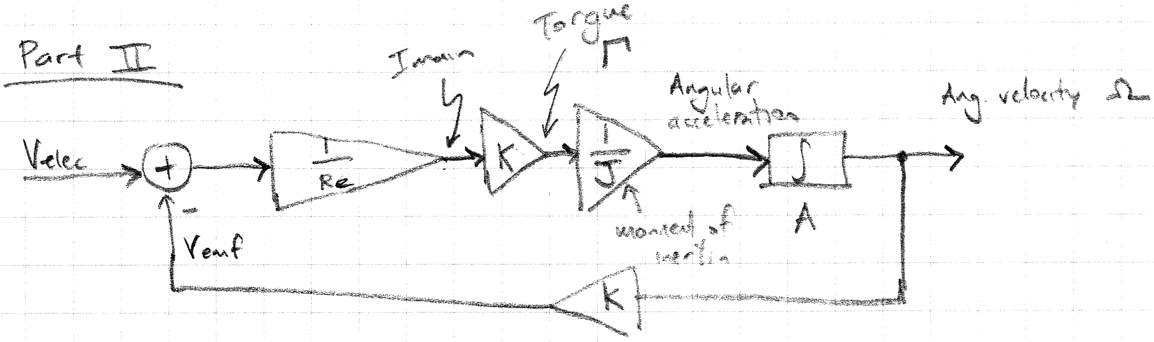


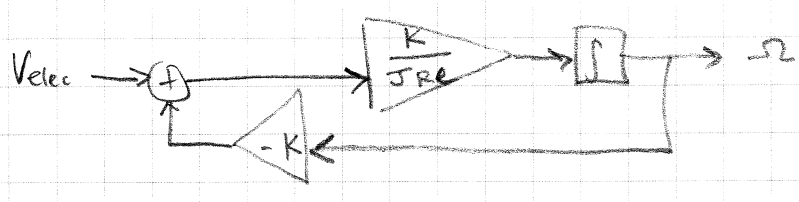
$$\frac{Y}{X} = \frac{1}{1-pR}$$

$$X = \delta[n] \Rightarrow Y = (1 + pR + p^2R^2 + \quad)\,\delta[n]$$
$$\Rightarrow y[n] = p^n u[n].$$



Part II



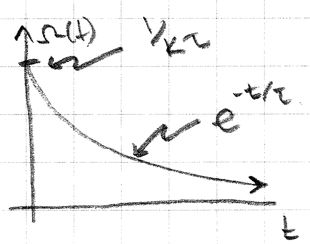


$$\frac{\Omega}{V_{elec}} = \frac{\frac{KA}{JRe}}{1 + \frac{K^2 A}{JRe}} \quad . \quad \text{Let } \frac{K^2}{JRe} \triangleq \frac{1}{\tau}.$$

$$\Rightarrow \frac{\Omega}{V_{elec}} = \frac{1}{K}\left(\frac{A/\tau}{1 + A/\tau}\right)$$

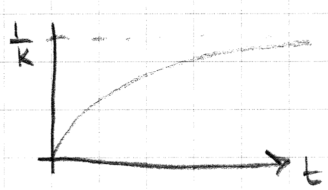
If $V_{elec} = \delta(t)$, $\Rightarrow \Omega(t) = \frac{1}{K\tau} e^{-t/\tau} u(t)$

Impulse response



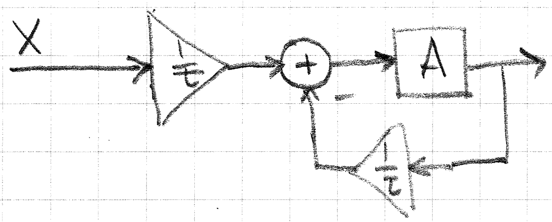
We can obtain the step response from the impulse response by integrating, since $u(t) = \int_0^t \delta(\tau)\, d\tau$

Step response:

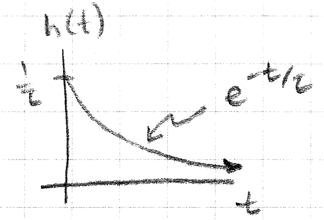


Part III Note that $t = nT$.

$$\frac{dy}{dt} + \frac{y}{\tau} = \frac{x}{\tau}.$$



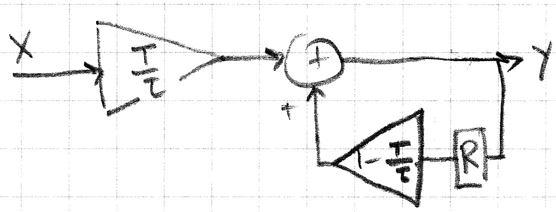
$\Rightarrow$



In DT, $\dfrac{y[n+1]-y[n]}{T} + \dfrac{y[n]}{\tau} = \dfrac{x[n+1]}{\tau}$ (Note, $T \ll \tau$)

$$\Rightarrow y[n] = y[n-1]\left(1-\frac{T}{\tau}\right) + \frac{T}{\tau}x[n]$$

$\Rightarrow$



$$\Rightarrow h[n] = \frac{T}{\tau}\left(1-\frac{T}{\tau}\right)^n u[n].$$

Taking limits to compare $h[n]$, & $h(t)$:

$$\lim_{T\to 0} \frac{T}{\tau}\left(1-\frac{t}{\tau}\right)^n = \frac{T}{\tau}\left(1-\frac{T}{\tau}\right)^{t/T} = \frac{T}{\tau}e^{-t/\tau} = T\,h(t)$$

$$\int_0^\infty h(t)\,dt = 1 \Rightarrow \int_0^\infty \frac{1}{\tau}e^{-t/\tau}\,dt = 1.$$

Additionally, $\sum_n h[n] = 1 \Rightarrow \sum_{n=0}^{\infty} \frac{T}{\tau}(1-T/\tau)^n = 1$