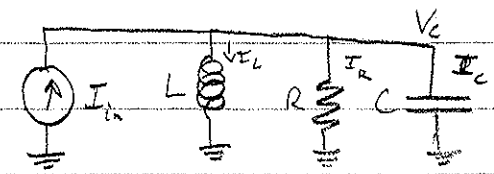
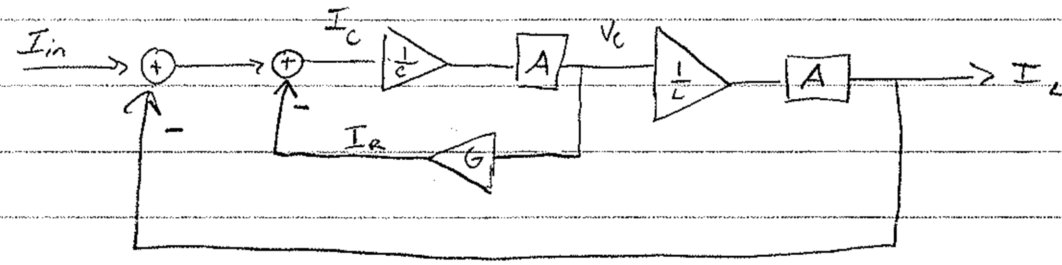
6.003 Recitation 6, Sections 3 & 4

1. Block Diagrams for Parallel LCR Circuits
2. The Impulse Response of Second-Order Systems
3. Examples of Physical second-Order Systems



$$I_{in} = I_L + I_R + I_C$$
$$I_L = \frac{1}{L}\int V_c\,dt$$
$$I_R = \frac{V_c}{R}$$
$$I_c = C\,\frac{dV_c}{dt}$$



$$\frac{I_L}{I_{in}} = \frac{\left(\frac{\frac{A}{C}}{1+\frac{A}{RC}}\right)\frac{A}{L}}{1+\left(\frac{\frac{A}{C}}{1+\frac{A}{RC}}\right)\frac{A}{L}}$$

$$\frac{I_R}{I_{IN}} = \frac{1}{A}\left(\frac{L}{R}\right)\left(\frac{I_L}{I_{in}}\right)$$

$$\frac{I_c}{I_{in}} = \left(\frac{L}{A}\right)\left(\frac{C}{A}\right)\left(\frac{I_L}{I_{in}}\right)$$

$$\frac{I_L}{I_{in}} = \frac{\frac{\frac{A^2}{LC}}{1+\frac{A}{RC}}}{1+\frac{\frac{A^2}{LC}}{1+\frac{A}{RC}}} = \frac{\frac{A^2}{LC}}{1+\frac{A}{RC}+\frac{A^2}{LC}}$$

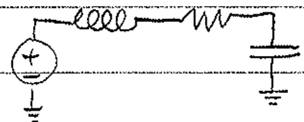
$$\omega_o^2 = \frac{1}{LC}$$

$$Q = \frac{R}{\sqrt{\frac{L}{C}}}$$

$$\boxed{\frac{I_L}{I_{in}} = \frac{\omega_o^2 A^2}{1+\frac{\omega_o A}{Q}+\omega_o^2 A^2}}$$

$$\frac{I_R}{I_{in}} = \frac{\frac{\omega_0 A}{Q}}{1+\frac{\omega_0 A}{Q}+\omega_0^2 A^2} \qquad \frac{I_C}{I_{in}} = \frac{1}{1+\frac{\omega_0 A}{Q}+\omega_0^2 A^2}$$

Series



$$\omega_0^2 = \frac{1}{LC}$$

$$Q = \frac{\sqrt{\frac{L}{C}}}{R}$$

$$\frac{V_c}{V_{in}} = \frac{\omega_0^2 A^2}{1+\frac{\omega_0 A}{Q}+\omega_0^2 A^2} \qquad \frac{V_R}{V_{in}} = \frac{\frac{\omega_0 A}{Q}}{1+\frac{\omega_0 A}{Q}+\omega_0^2 A^2} \qquad \frac{V_c}{V_{in}} = \frac{1}{1+\frac{\omega_0}{Q}A+\omega_0^2 A^2}$$

Impulse Response

$$\frac{I_L}{I_{in}} = \frac{\omega_0^2 A^2}{1+\frac{\omega_0 A}{Q}+\omega_0^2 A^2} = \frac{\omega_0^2}{p_0-p_1}\left[\frac{A}{1-p_0 A} - \frac{A}{1-p_1 A}\right]$$

roots p_0 & p_1 derived by solving for roots of quadratic

$$1+\frac{\omega_0 A}{Q}+\omega_0^2 A^2 = 0$$

ie. I can factorize $\left(1+\frac{\omega_0 A}{Q}+\omega_0^2 A^2\right) \Rightarrow (1-p_0 A)(1-$

Impulse Response: $\frac{\omega_0^2}{p_0-p_1}\left[e^{p_0 t} - e^{p_1 t}\right]u(t)$

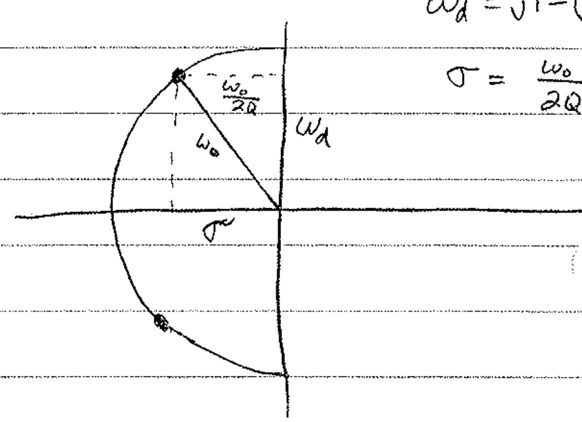
IF $Q > \frac{1}{2}$

$$p_0 = \frac{-\omega_0}{2Q} + j\underbrace{\left(\sqrt{1-\left(\tfrac{1}{2Q}\right)^2}\right)\omega_0}_{}^{\omega_d}$$

$$p_1 = \frac{-\omega_0}{2Q} - j\left(\sqrt{1-\left(\tfrac{1}{2Q}\right)^2}\right)\omega_0$$

$$\omega_d = \sqrt{1-\left(\tfrac{1}{2Q}\right)^2}\,\omega_0$$

$$\sigma = \frac{\omega_0}{2Q}$$



IF $Q < \frac{1}{2}$

$$p_0 = \frac{-\omega_0}{2Q} + \sqrt{1-\left(\tfrac{1}{2Q}\right)^2}\,\omega_0$$

$$p_1 = -\frac{\omega_0}{2Q} - \sqrt{1-\left(\tfrac{1}{2Q}\right)^2}\,\omega_0$$

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Impulse Response:

$$h(t) = \frac{\omega_0^2}{p_0 - p_1}\left[e^{p_0 t} - e^{p_1 t}\right]u(t)$$

IF $Q < \frac{1}{2}$: $\frac{\omega_0^2}{\omega_d}\left[e^{p_0 t} - e^{p_1 t}\right]u(t)$

IF $Q > \frac{1}{2}$: $\frac{\omega_0}{\sqrt{1-\left(\frac{1}{2Q}\right)^2}}\, e^{-\frac{\omega_0}{2Q}t} \sin\left(\omega_0\sqrt{1-\left(\frac{1}{2Q}\right)^2} \cdot t\right)$

Derivation of Impulse Response for $Q > \frac{1}{2}$

$$\frac{\omega_0^2}{p_0 - p_1}\left[e^{p_0 t} - e^{p_1 t}\right]$$

$$\frac{\omega_0^2}{2j\omega_d}\left[e^{-\sigma t}\right]\left[e^{j\omega_d t} - e^{-j\omega_d t}\right]$$

$$\frac{\omega_0^2}{\omega_d} e^{-\sigma t}\left[\frac{e^{j\omega_d t} - e^{-j\omega_d t}}{2j}\right]$$

$$\frac{\omega_0^2}{\omega_d} e^{-\sigma t} \cdot \sin(\omega_d t)$$