6.003 Recitation, Sections 3 & 4, March 5, 2010.

TODAY:

(1) 2 physical interpretations of convolution

(2) Example of the 'flip, slide, and overlap' method.

(3) Frequency response & its relationship to convolution.

(1) CONVOLUTION

DT

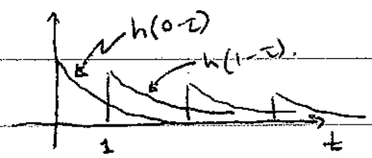
$$(x*h)[n] = \sum_k x[k]\, h[n-k]$$

CT

$$(x*h)(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau.$$

(a)

An interpretation:



← Each impulse response is weighted by $x(t)$.

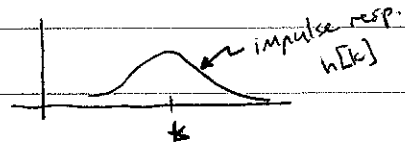
⇒ Convolution: "input-weighted 'sum' of impulses"

(b)

Another interpretation:

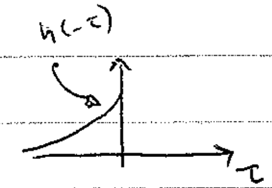
$$(h*x)[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$$(h*x)(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau.$$

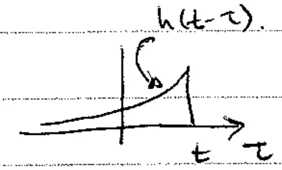


⇒ Convolution: "impulse-response weighted 'sum' of inputs" ⇒ "sliding window"

(2) $h(t-\tau) = h(-(\tau - t)) \Rightarrow$

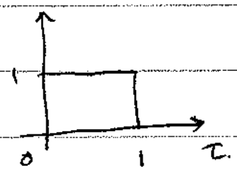


$\Rightarrow$

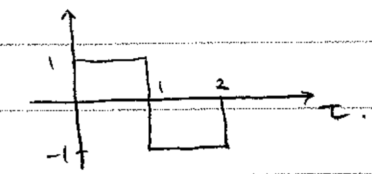


<u>Example:</u>

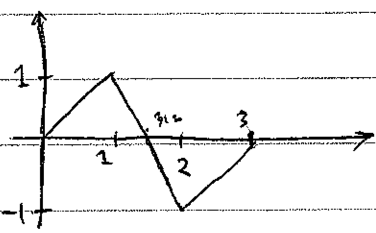
$h(t) =$



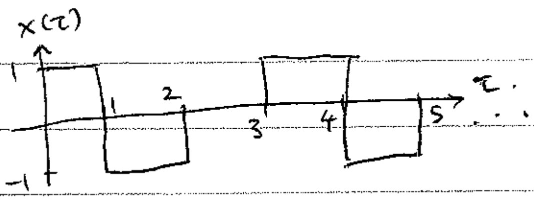
$x(\tau) =$



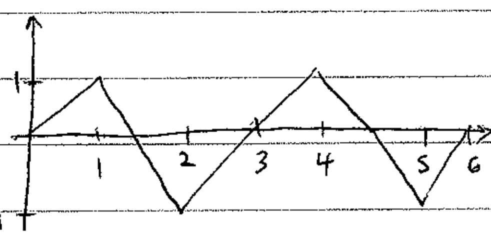
$(x * h)(t) = ?$



Now suppose $x(\tau)$ is periodic,



then $(x * h)(\tau) =$



(8) $e^{st} \longrightarrow \boxed{h(t)} \longrightarrow ?$

$$\int h(\tau) e^{s(t-\tau)} d\tau = \left(\int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau \right) e^{st} = H(s) e^{st}$$

$\Rightarrow$ The Laplace transform of the impulse response is the 'eigenvalue' of e^{st}.

$\Rightarrow$ $e^{j\omega t} \longrightarrow \boxed{h(t)} \longrightarrow H(j\omega) e^{j\omega t}$

$e^{-j\omega t} \longrightarrow \boxed{h(t)} \longrightarrow H(-j\omega) e^{-j\omega t} = H^*(j\omega) e^{-j\omega t}.$

$$\frac{e^{j\omega t} + e^{-j\omega t}}{2} \longrightarrow \boxed{h(t)} \longrightarrow \tfrac{1}{2}H(j\omega)e^{j\omega t} + \tfrac{1}{2}H^*(j\omega)e^{-j\omega t}.$$

Now let $H(j\omega) = |H(j\omega)|\,e^{\angle H(j\omega)}$

$$\cos\omega t \longrightarrow \boxed{h(t)} \; |H(j\omega)|\cos($$

Suppose

$$\sin\omega t \longrightarrow \boxed{h(t) = \tfrac{1}{\tau}e^{-t/\tau}} \longrightarrow ?$$

$$\mathcal{L}\{\tfrac{1}{\tau}e^{-t/\tau}\} = \frac{1}{\tau s+1} = H(s), \quad H(j\omega) = \frac{1}{j\omega\tau+1} = \frac{1}{\sqrt{1+\omega^2\tau^2}}\,e^{-j\tan^{-1}(\omega\tau)}$$

then $$y(t) = \frac{\sin(\omega t - \tan^{-1}(\omega\tau))}{\sqrt{1+\omega^2\tau^2}}$$