6.003 Recitation, March 10, Sections 3 & 4.

1.

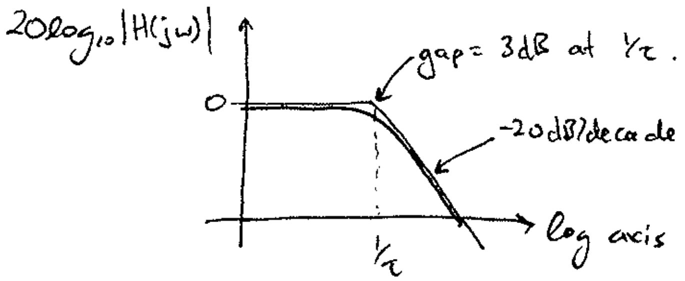
▷ Today:

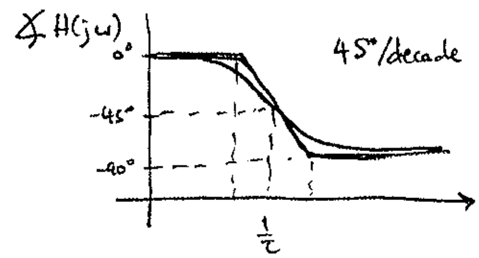
1. Bode plot of a single-pole first-order system
2. " " " " two-pole second-order system.
3. Bode plot example.

(1)

$$x(t) \rightarrow \boxed{h(t) = \frac{1}{\tau} e^{-t/\tau}} \rightarrow y(t)$$

$$H(j\omega) = \frac{1}{\tau j\omega + 1}, \quad |H(j\omega)| = \frac{1}{\sqrt{1+\omega^2\tau^2}}, \quad \measuredangle H(j\omega) = -\tan^{-1}(\omega\tau)$$





(2)

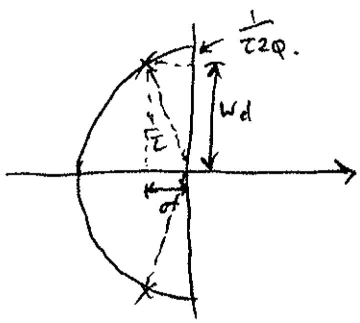
$$x(t) \rightarrow \boxed{h(t) = \frac{\omega_0^2}{\omega\tau} e^{-\sigma t} \sin(\omega_d t) u(t).} \rightarrow y(t)$$

$$X(s) \rightarrow \boxed{H(s) = \frac{1}{\tau^2 s^2 + \frac{\tau s}{Q} + 1}} \rightarrow Y(s).$$

$\omega_0 = 1/\tau.$

$\omega_d = \omega_0 \sqrt{1 - \frac{1}{4Q^2}}$

$\sigma = \frac{\omega_0}{2Q}$

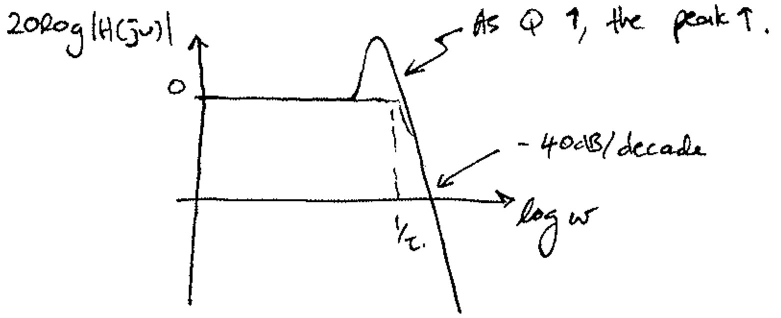


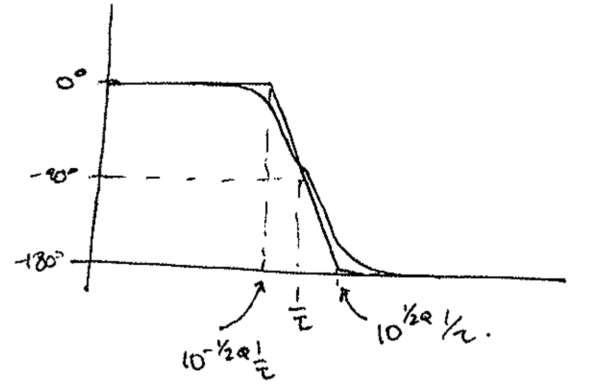
$$H(j\omega) = \frac{1}{1 - \omega^2\tau^2 + \frac{j\omega\tau}{Q}}$$

If $Q > 1/2$, the step response is underdamped (complex poles).

$$|H(j\omega)| = \frac{1}{\sqrt{(1-\omega^2\tau^2)^2 + \frac{\omega^2\tau^2}{Q^2}}}$$

$$\measuredangle H(j\omega) = -\tan^{-1}\left(\frac{\omega\tau/Q}{1-\omega^2\tau^2}\right)$$





$$\omega_{peak} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$$

$$H_{peak} = \frac{Q}{\sqrt{1 - 1/2Q^2}}$$

A peak only occurs if $Q > 1/\sqrt{2}$.

As $Q \to \infty$, $10^{-1/2Q}, 10^{1/2Q} \to 1$, and the phase transition becomes infinitely sharp.

(3) $$H(s) = \frac{10^6}{(s+1)(10^{-5}s+1)\left(10^{-14}s^2 + \frac{10^{-7}s}{100} + 1\right)}$$

$$20\log|H(j\omega)| = 120 - 20\log|j\omega+1| - 20\log|10^{-5}j\omega+1| - 20\log\left|10^{-14}j^2\omega^2 + \frac{10^{-7}j\omega}{100} + 1\right|$$

$$\angle H(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}(10^{5}\omega) - \tan^{-1}\left(\frac{10^{-7}\omega}{10^2} \Big/ (1-\omega^2 10^{-14})\right)$$